J.M.J.

MARCELLIN COLLEGE RANDWICK

YEAR 11 HSC

ASSESSMENT TASK # 1

EXTENSION TWO

MATHEMATICS

2007

Weighting: 15% of HSC Assessment Mark.

STUDENT NAME: _____ **MARK:** / 26

PERCENTAGE: %

RANK ON THIS TASK: / 11

Time Allowed: 50 minutes

Directions:
* Answer all questions on separate lined paper.
* Begin each question on a new page.
* Show all necessary working.
* Marks may not be awarded for careless or badly arranged work.

Question One (Total - 18 marks) Begin your answers on a new page.

(a) If $z = 3+2i$, plot on the same Argand diagram:

(i) z and $\bar{z}$ **1**

(ii) iz **1**

(iii) $z(1+i)$ **1**

(b) (i) Find all the pairs of integers a and b such that $(a+ib)^2 = 8+6i$ **1**

(ii) Hence solve: $z^2 + 2z(1+2i) - (11+2i) = 0$ **2**

(c) (i) If $z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$, find the value of z^6 **2**

(ii) Plot on an Argand diagram all complex numbers that are the solutions of $z^6 = 1$ **2**

(d) Sketch the locus of the each of the following on separate diagrams:

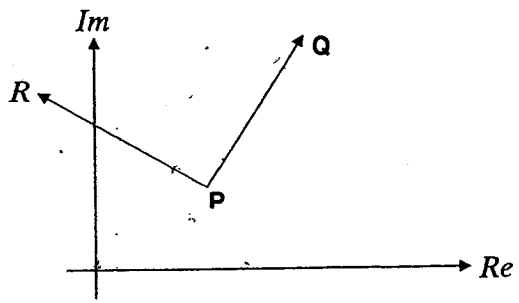
(i) $\arg(z-1-2i) = \frac{\pi}{4}$ **1**

(ii) $z\bar{z} - 3(z+\bar{z}) \le 0$ **2**

(iii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ **2**

Question One comtinued:

(e) In the diagram below, P represents the complex number $3+2i$ and Q represents $7+8i$.

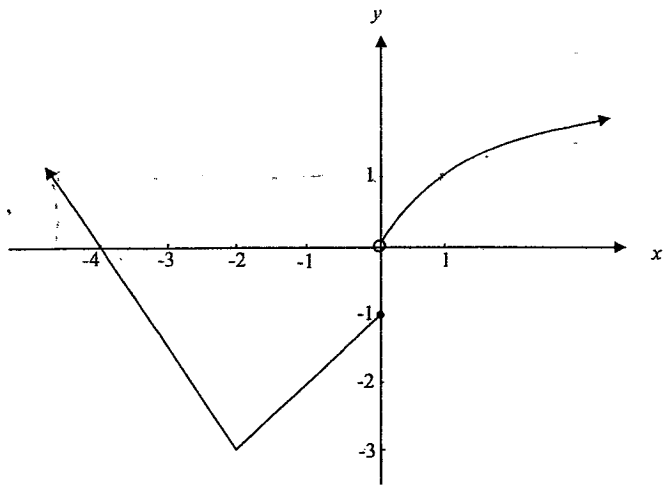


(i) What complex number is represented by the vector PQ? **1**

(ii) Suppose that R is the image of Q under an anticlockwise rotation of $\frac{\pi}{2}$ about P. Write the complex number represented by the point R. **2**

Question Two (Total - 8 marks) Begin your answers on a new page.

The diagram below shows the discontinuous function $y=f(x)$.



Draw separate sketches of each of the following:

(a) $y=|f(x-1)|$ **2**

(b) $y=\frac{1}{f(x)}$ **2**

(c) $\sqrt{-f(x)}$ **2**

(d) $y=\ln(f(x))$ **2**

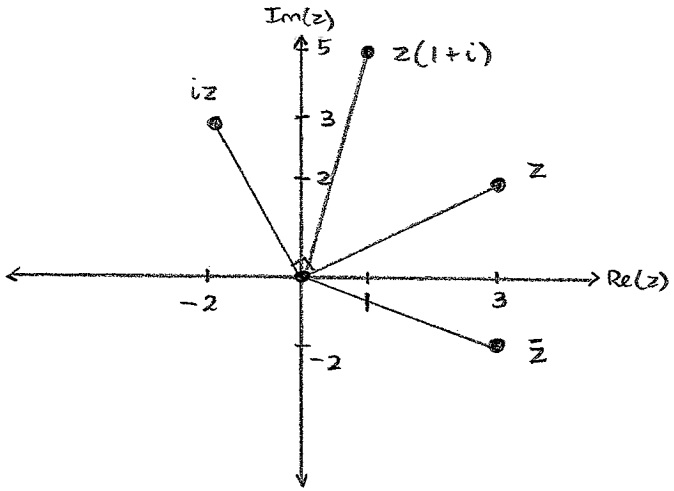
END OF ASSESSMENT TASK

ASSESSMENT TASK NO. 1

SOLUTIONS

Question One

(a) $z = 3 + 2i$



1 mark for each correctly plotted point.

(iii) $z(1+i) = (3+2i)(1+i)$

$= 3 + 3i + 2i - 2$

$= 1 + 5i$

(b)(i) $(a+ib)^2 = 8 + 6i$

$\therefore a^2 - b^2 + 2abi = 8 + 6i$

Equating real and imag. parts:

$a^2 - b^2 = 8$ and $ab = 3$

$\therefore a = 3, b = 1$

and $a = -3, b = -1$) 1 mark

Q1.(b)(ii) $z^2 + 2z(1+2i) - (11+2i) = 0$

Using QF: $z = \dfrac{-2(1+2i) \pm \sqrt{4(1+2i)^2 + 4(11+2i)}}{2}$

$\therefore z = \dfrac{-2(1+2i) \pm \sqrt{32 + 24i}}{2}$

$\therefore z = \dfrac{-2(1+2i) \pm 2\sqrt{8+6i}}{2}$ ← 1 mark

$\therefore z = -(1+2i) \pm (3+i)$

$\therefore z = -1 - 2i \pm (3+i)$

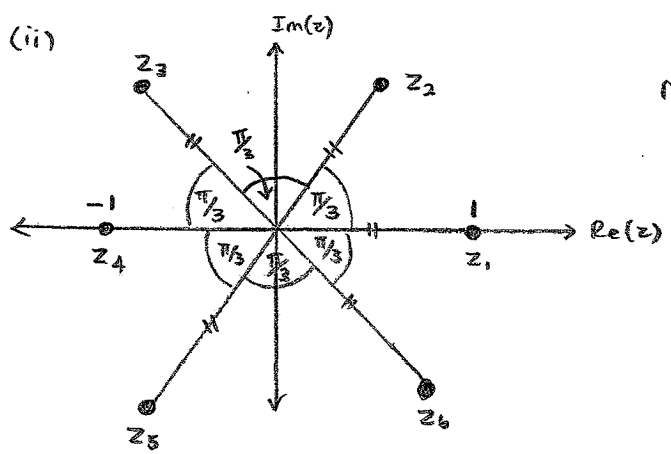
$\therefore z = 2 - i, -4 - 3i$ ← 1 mark

(c)(i) $z = \text{cis}\,\frac{\pi}{3}$ ← 1 mark

$\therefore z^6 = \text{cis}\,2\pi$ (Using de Moivre's Theorem)

$\therefore z^6 = 1$ ← 1 mark

(ii)



Marked same as Q1(d)

$z_1 = 1$

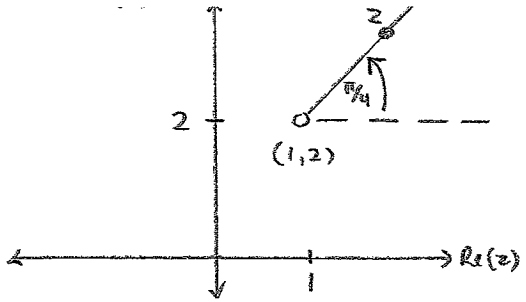
$z_2 = \text{cis}\,\pi/3$

$z_3 = \text{cis}\,2\pi/3$

$z_4 = -1$

$z_5 = \text{cis}(-2\pi/3)$

$z_6 = \text{cis}(-\pi/3)$



For Q1 (d) (i) – (iii), 1 mark deducted for each incorrectly drawn or omitted part of correct graph

(ii) let $z = x + yi$

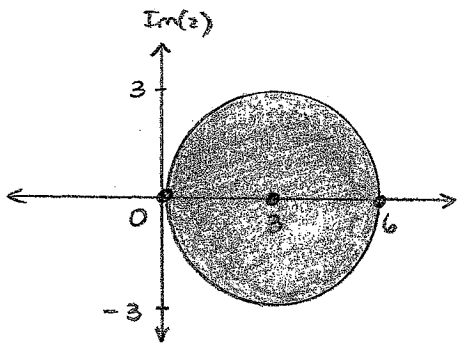
New $z\bar{z} = (x+yi)(x+yi)$

$= x^2 + y^2$

and $z + \bar{z} = 2x$

$\therefore z\bar{z} - 3(z+\bar{z}) = x^2 - 6x + y^2$

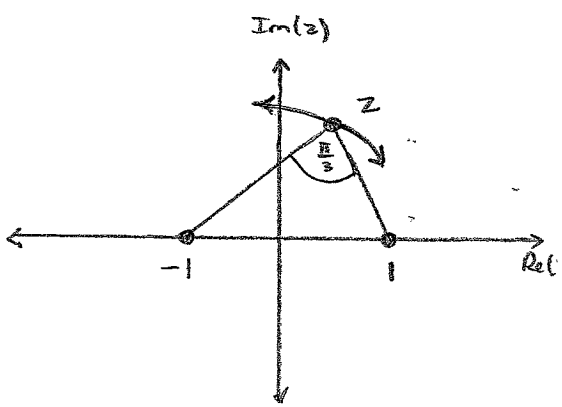
$= (x-3)^2 - 9 + y^2$



$(x-3)^2 + y^2 \leq 9$

(iii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

$\therefore \arg(z-1) - \arg(z+1) = \pi/3$



Q1(e)i) $\vec{PQ} = \vec{PO} + \vec{OQ}$

$= -3 - 2i + 7 + 8i$

$= 4 + 6i$ ← 1 mark

ii) $\vec{PR} = i\vec{PQ}$

$= i(4 + 6i)$

$= -6 + 4i$ ← 1 mark

$\vec{OR} = \vec{OP} + \vec{PR}$

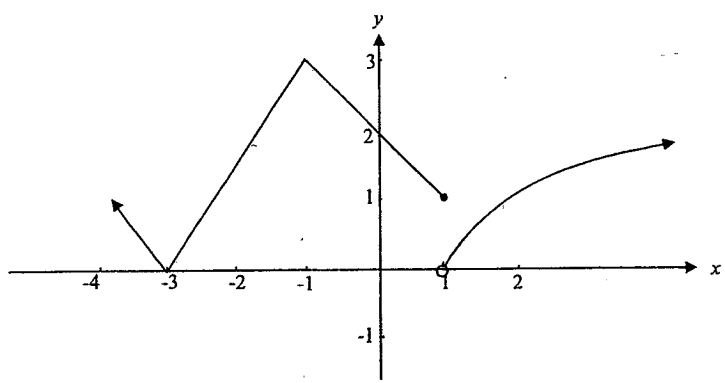
$= 3 + 2i - 6 + 4i$

$= -3 + 6i$

$\therefore$ the complex no. represented by R is $-3 + 6i$ ← 1 mark

Question Two

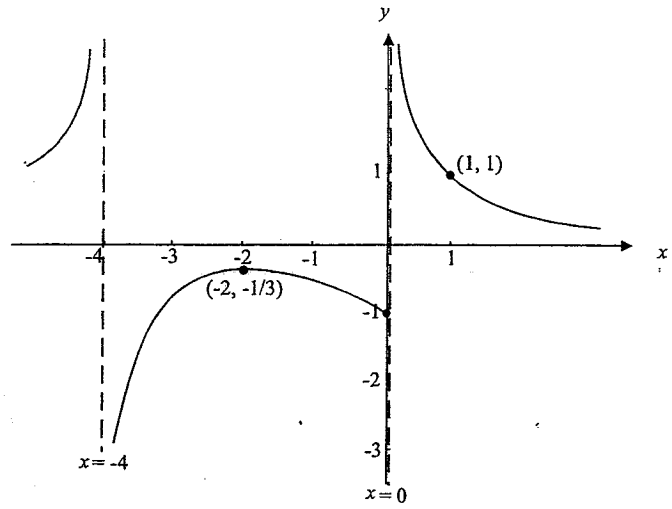
(a)



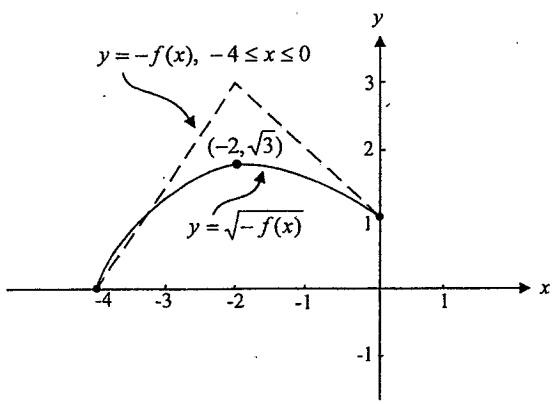
For all parts (a)–(d) of Q2, one mark deducted for each incorrectly drawn or omitted part of correct graph

Q2 continued

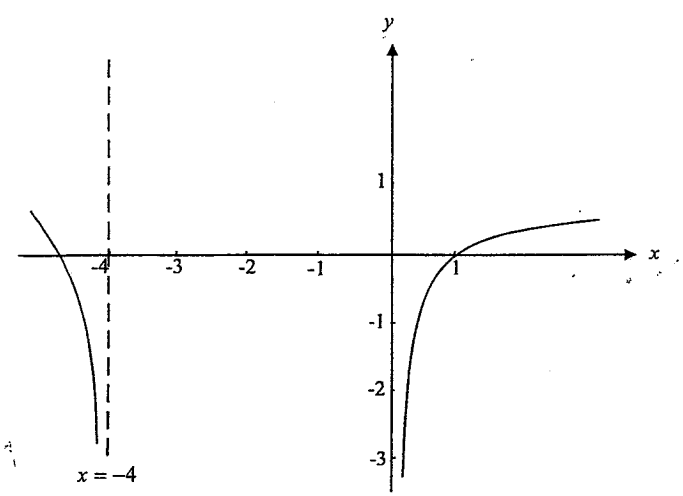
(b)



(c)



(d)



c) $z = \text{cis}\,\frac{\pi}{3}$

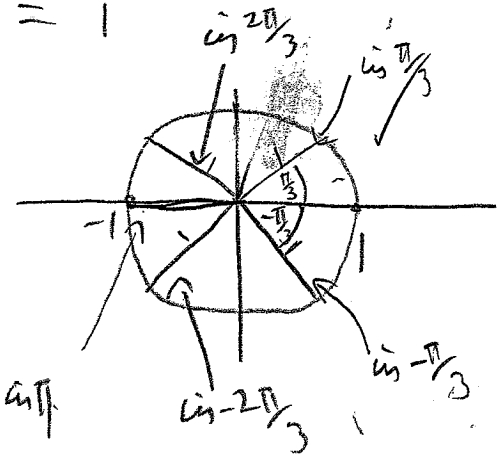
$z^6 = \left(\text{cis}\,\frac{\pi}{3}\right)^6$ (de M th)

$= \text{cis}\,2\pi$

$= \text{cis}\,0$

$= 1$ ✓

ii)



(1)