J.M.J.

MARCELLIN COLLEGE RANDWICK

YEAR 11

MATHEMATICS

HSC PRELIMINARY EXAMINATION

ASSESSMENT TASK 3

2009

Weighting: 40% of assessment Mark.

STUDENT NAME: ______________________ **MARK:** /84

TEACHER: ______________________

Time Allowed: 2 Hours

Directions:

- Show all necessary working.
- Underline all answers.
- Marks may be deducted for untidy or poorly presented work.
- Papers will be collected in two parts.
- Write your name and your teacher's name on the top right of each page.
- **Attach the appropriate cover sheet to the front of your answers to each Part.**

Outcomes Examined:

- P1 Demonstrates confidence in using Mathematics to obtain realistic solutions to problems.
- P2 Provides reasoning to support conclusions which are appropriate to the context.
- P3 Performs routine arithmetic and algebraic manipulations involving surds and simple rational expressions.
- P4 Chooses and applies appropriate arithmetic and algebraic techniques.
- P5 Understands the concept of a function and the relationship between a function and its graph.

Total Marks – 84
Attempt Questions 1 to 7
ALL questions are of equal value
Start each question on a SEPARATE page.

PART A

QUESTION 1 (12 marks) — Marks

(a) Simplify:

(i) $\sqrt{27}$ — 1

(ii) $3x(2-x)-(4x+2)$ — 1

(b) Evaluate: $\sqrt{\frac{4.8+2.5}{6.7-1.9}}$ correct to 2 decimal places — 2

(c) Expand and simplify: $(a-3)(a^2+3a+9)$ — 2

(d) Expand and simplify $(1-\sqrt{5})(1+\sqrt{5})$ — 2

(e) Simplify $\frac{7x-21}{x^2-9}\times\frac{x^2-9x}{x^2-7x-18}$ — 2

(f) Express $\frac{4\sqrt{3}+3}{3\sqrt{3}-1}$ with a rational denominator. Simplify your answer — 2

QUESTION 2 (12 marks) Start a new page.

(a) Factorise fully the following expressions

(i) $-x+3x^2+x^3$ — 1

(ii) $9x^2+15x+4$ — 2

(iii) $ax+ba+by+xy$ — 2

(b) Find the exact value of $\cos(-150°)$ — 2

(c) Simplify $\frac{3}{y+2}-\frac{2}{y+3}$ — 3

(d) Find the domain and range of $y=\frac{1}{(x-2)^2}$ — 2

QUESTION 3 (12 marks) Start a new page. Marks

(a) Find values for a, b and c if $x^2 - x \equiv a(x+2)^2 + bx + c - 2$ 3

(b) Find the Discriminant and determine the nature of the roots for
$4x^2 - 3x + 2 = 0$ 2

(c) Solve: $4^{4-x} = 8^{2x}$ 2

(d) Find all values of k for which $2x^2 + x + k + 2 = 0$ has real roots. 2

(e) Evaluate: 3

$$\lim_{x \to 2}\left(\frac{x^2 + x - 6}{x^2 - 4}\right)$$

QUESTION 4 (12 marks) Start a new page

(a) Find the equation of the line which passes through the point (15,2) and which is parallel to the line $7x + y + 8 = 0$ 2

(b) Sketch and shade the region given by $x^2 + y^2 \le 9$ 2

(c) Sketch the function $y = \frac{1}{x+2}$ 1

(d) Two points, A and B, on the number plane have coordinates: $A(3,4)$ and $B(11,8)$.

(i) Find the coordinates of the midpoint of AB. 1

(ii) Show that the equation of the perpendicular bisector of AB is given by: $2x + y - 20 = 0$. 2

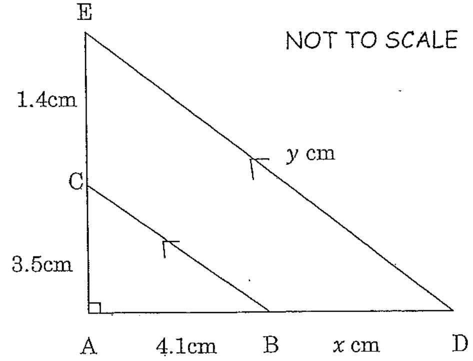
iii A third point, G, is on the line $x = 3$ and is $\sqrt{20}$ units from a fourth point, $H(5,7)$. Find the coordinates of the point G given that the y- coordinate of G is greater than the y-coordinate of A. 2

(iv) Find the shortest distance of G from $2x + y - 20 = 0$, correct to 2 decimal places. 2.

PART B

QUESTION 5 (12 marks) Start a new page Marks

(a)



Copy the diagram above onto your answer page.

$AB = 4.1cm$; $BD = x\ cm$; $AC = 3.5cm$; $EC = 1.4cm$; $ED = y\ cm$

(i) Prove ΔABC is similar to ΔADE 2

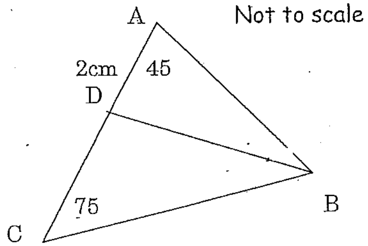
(ii) Find the values of x and y to one decimal place. 2

(b) Solve $\sin\theta = \frac{-1}{\sqrt{2}}$ for $0° \le \theta \le 360°$ 2

(c) (i) Find the equation of the parabola with Vertex (2,5) and Focus (2,9) 2

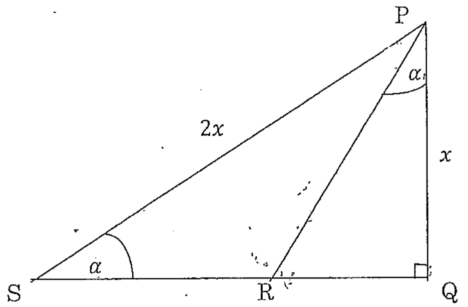
(ii) Sketch the above parabola. 1

(d) In ΔABC, $\angle A = 45°$, and $\angle C = 75°$
D is a point on AC such that $AD = 2cm$ and DB bisects $\angle ABC$.
Find the *exact* length of DB. 3



QUESTION 6 (12 marks) Start a new page

(a) In the diagram, $PS = 2 \times PQ$ and $\angle PSR = \angle RPQ$



(i) By writing an expression for $\sin\alpha$, show that $\alpha = 30°$ 1

(ii) Hence find the size of $\angle SPR$ 2

(iii) If $SR = 3cm$, find the length of PQ in exact form. 2

(b) If α and β are the roots of the quadratic equation: $x^2 - 3x - 1 = 0$ find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

(c) A parabola has equation $(x-1)^2 = 8y$ find the:

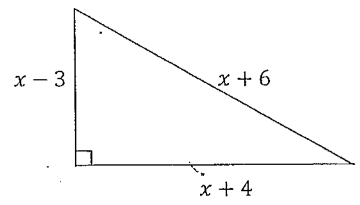
(i) coordinates of its vertex and focus. 2

(ii) equation of its directrix 1

(iii) Sketch the curve, showing the coordinates of the Vertex, Focus and the Directrix. 2

QUESTION 7 (12marks) Start a new page

(a) Find all possible values of x: 3



(b) Given $f(x) = \begin{cases} ax + b & for\ x < 2 \\ bx - a & for\ 2 \le x \le 4 \\ x^2 - 4 & for\ x > 4 \end{cases}$

Find a and b if $f(-3) = f(2) = f(5)$ 3

(c) Show that $cosec\,\theta \tan\theta = \sec\theta$ 2

(d) (i) Find the coordinates of the Vertex, Focus and the equation of the directrix for the parabola: $x^2 - 2x - 8y + 33 = 0$ 3

(ii) Sketch this parabola. 1

END OF EXAM

HSC Prelim Exam Solutions

Question 1

(a) (i) $\sqrt{27} = 3\sqrt{3}$

(ii) $3x(2-x) - 4(x+2) = -3x^2 + 2x - 2$

(b) Ans $= 1.23$ to 2 dec pl.

(c) $(a-3)(a^2+3a+9) = a^3 - 27$

(d) $(1-\sqrt{5})(1+\sqrt{5}) = 1^2 - (\sqrt{5})^2$

$= -4$

(e) $E = \dfrac{7(x-3)}{(x-3)(x+3)} \times \dfrac{x(x-9)}{(x-9)(x+2)}$

$= \dfrac{7x}{(x+3)(x+2)}$

(f) $\dfrac{4\sqrt{3}+3}{3\sqrt{3}-1} \times \dfrac{3\sqrt{3}+1}{3\sqrt{3}+1} = \dfrac{39+13\sqrt{3}}{27-1}$

$= \dfrac{39+13\sqrt{3}}{26}$ $(\div 13)$

$= \dfrac{3+\sqrt{3}}{2}$

Question 2

(a) (i) $-x+3x^2+x^3 = x(x^2+3x-1)$

(ii) $9x^2+15x+4 = (3x+4)(3x+1)$

(iii) $ax + ba + by + xy = a(x+b) + y(x+b)$

$= (a+y)(x+b)$

(b) $\cos(-150^\circ) = -\cos(-180^\circ+30^\circ)$

$= -\cos 30^\circ$

$= \dfrac{-\sqrt{3}}{2}$

(c) $E = \dfrac{3(y+3) - 2(y+2)}{(y+2)(y+3)}$

$= \dfrac{y+5}{(y+2)(y+3)}$

(d) Domain: All real values of x, except $x = 2$.

Range: $y > 0$

Question 3

(a) RHS $= a(x^2+4x+4) + bx + c - 2$

$= ax^2 + 4ax + 4a + bx + c - 2$

$= ax^2 + (4a+b)x + (4a+c-2)$

LHS $= x^2 - x$ (ie: $x^2 - x + 0$)

$\therefore a = 1$ ---(1)

$4a + b = -1$ --(2) $\Rightarrow b = -5$

$4a + c - 2 = 0$ ...(3) $\Rightarrow c = -2$

$\therefore a = 1,\ b = -5,\ c = -2$

(b) $\Delta = 9 - 4(4)(2) = -23$.

Roots are unreal

(c) $4^{4-x} = 8^{2x}$

$(2^2)^{4-x} = (2^3)^{2x}$

$2^{8-2x} = 2^{6x}$

$\therefore 8 - 2x = 6x \Rightarrow 8 = 8x \Rightarrow x = 1$

(d) $\Delta = 1 - 4(2)(k+2)$

$= 1 - 8(k+2) = -8k - 15$

Real roots $\Rightarrow \Delta \geq 0$

$\therefore -8k - 15 \geq 0 \Rightarrow -8k \geq 15$

$\therefore k \leq -1\frac{7}{8}$

(e) $\lim\limits_{x\to 2} \dfrac{(x+3)(x-2)}{(x-2)(x+2)} = \lim\limits_{x\to 2} \dfrac{x+3}{x+2}$

$= \dfrac{5}{4} = 1\frac{1}{4}$

Question 4

(a) In line $7x + y + 8 = 0$, $m = -7$.

$\therefore$ Eqn of line, grad -7 through $(15, 2)$:

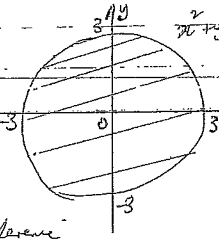
$y - 2 = -7(x - 15) \Rightarrow y - 2 = -7x + 105$

$\therefore 7x + y - 107 = 0$

-2-

Question 4.

(b)



Shaded area is where $x^2 + y^2 \leq 9$

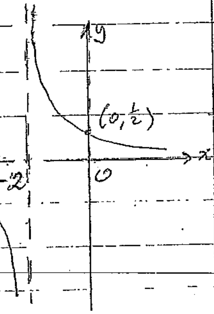
(NB: Circumference is included)

$(0,0)$ satisfied $x^2 + y^2 \leq 9$.

(c) $y = \dfrac{1}{x+2}$

Hyperbola undefined when $x = -2$,

Asymptotes: $x = -2,\ y = 0$.



(d) (i) MP $= \left(\dfrac{3+11}{2}, \dfrac{4+8}{2}\right) = (7, 6)$

(ii) $A(3,4)$ $B(11,8)$

$m_{AB} = \dfrac{8-4}{11-3} = \dfrac{4}{8} = \dfrac{1}{2}$.

So Perp. Bisector has grad. -2, and passes through $(7,6)$

Equation is: $y - 6 = -2(x-7)$

$\Rightarrow y - 6 = -2x + 14 \Rightarrow 2x + y - 20 = 0$

(iii) G lies on $x = 3$, so $G = (3, y)$

Dist: $G(3,y)$ $H(5,7)$ is:

$d = \sqrt{(5-3)^2 + (7-y)^2}$

$= \sqrt{4 + (7-y)^2}$

But $d = \sqrt{20}$ from question

so $\sqrt{4 + (7-y)^2} = \sqrt{20}$

i.e. $4 + (7-y)^2 = 20$.

$\Rightarrow 4 + 49 - 14y + y^2 = 20$

$\Rightarrow y^2 - 14y + 33 = 0$

4(d)(iii) Solving for y: $(y-11)(y-3) = 0$

$\therefore y = 3$, or 11.

If y coord must be greater than ie:

so $y = 11$. $\therefore G = (3, 11)$

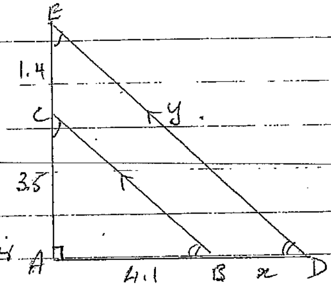
(iv) $d = \dfrac{|2x + y - 20|}{\sqrt{4+1}}$ $\Big|_{x=3,\ y=11}$

$= \dfrac{|6 + 11 - 20|}{\sqrt{5}}$

$= \dfrac{3}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5} = 1.34$ (To 2 dp)

Question 5

(a)



(i) $\hat{A}$ is common

$A\hat{C}B = A\hat{E}D$, $A\hat{B}C = A\hat{D}E$ }

Both corresponding angles.

so $\triangle ABC \parallel \triangle ADE$ (AAA)

(ii) By ratio of intercepts:

$\dfrac{x}{4.1} = \dfrac{1.4}{3.5} \Rightarrow x = \dfrac{4.1 \times 1.4}{3.5}$

$\therefore x = 1.6$ (To one d.p)

In $\triangle ADE$: $DE^2 = AE^2 + AD^2$ (Pythagoras)

$\therefore (4.9)^2 + (5.7)^2 = DE^2$

$y = DE = 7.5$ (7.6 also accepted)

(b) $\sin\theta = -\frac{1}{\sqrt{2}}$, θ lies in 3rd + 4th Quad.

$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore \theta = 180^\circ + 45^\circ,\ 360^\circ - 45^\circ$

$= 225^\circ,\ 315^\circ$

Solution

Question 5

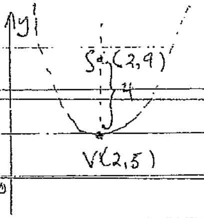
(c)(i) Vertex + focus lie on $x = 2$ (axis)

$a = 9 - 5 = 4$

Parabola is upright

Thus equation is: $(x-2)^2 = 16(y-5)$

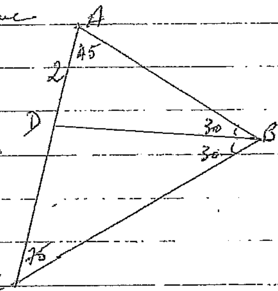
(ii) Sketch above



(d) $\hat{ABC} = 60°$

$(180° - (45 + 75))$.

By info given $\hat{ABD} = \hat{CBD} = 30°$



Now using sine rule in $\triangle ABD$:

$$\frac{2}{\sin 30} = \frac{DB}{\sin 45°}$$

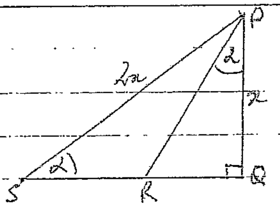
$$\Rightarrow DB = \frac{2 \sin 45°}{\sin 30°} \quad \because \sin 45° = \frac{1}{\sqrt{2}}, \ \sin 30° = \tfrac{1}{2}.$$

$$\therefore DB = \frac{2 \times \frac{1}{\sqrt{2}}}{\frac{1}{2}} \Rightarrow DB = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2}.$$

$$\therefore DB = 2\sqrt{2} \text{ cm}$$

Question 6

(a)(i) In $\triangle SPQ$:



$\sin \alpha = \frac{x}{2x}$

$= \frac{1}{2}$

$\therefore \alpha = 30°$.

(ii) In $\triangle SPQ$: if $\alpha = 30°$, $\hat{SPQ} = 60°$.

$\therefore \hat{SPQ} = \hat{SPR} + \hat{RPQ}$

$60° = \hat{SPR} + 30°$

$\therefore \hat{SPR} = 30°$

Question 6

(iii) From previous work: $\hat{RSQ} = \hat{SPR} = 30°$

So $\triangle SPR$ is isosceles.

$SR = 3 \Rightarrow PR = 3$.

Now in $\triangle PRQ$:

$\cos 30 = \frac{x}{3} \Rightarrow x = 3\cos 30°$.

$= \frac{3 \times \sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2}$ cm (EXACT Form)

(b) For $x^2 - 3x - 1 = 0$,

$\alpha + \beta = -\left(\frac{-3}{1}\right) = 3$

$\alpha\beta = \frac{-1}{1} = -1$

Now: $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{3}{-1} = -3$

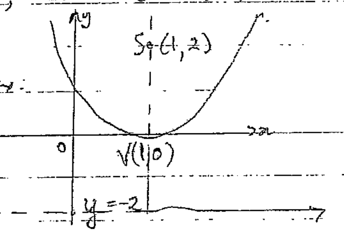
(c)(i) $(x-1)^2 = 8y$ (ie: $(x-1)^2 = 8(y-0)$)

$V(1,0)$, $a = 2$ ($4 \times 2 = 8$!)

$S(1,2)$

(ii) Directrix:

$y = -2$



Question 7

(a) By Pythagoras:

$(x-3)^2 + (x+4)^2 = (x+6)^2$

$x^2 - 6x + 9 + x^2 + 8x + 16 = x^2 + 12x + 36$.

$2x^2 + 2x + 25 = x^2 + 12x + 36$

$\therefore x^2 - 10x - 11 = 0$.

Solve for x:

$(x-11)(x+1) = 0 \Rightarrow x = 11, -1$.

When $x = -1$, sides become: $-4, 3, 5$.

Thus only possible solution is $x = 11$

Solution

Question 7

(b)
$$f(x) = \begin{cases} ax + b & x < 2 \\ bx - a & -2 \le x \le 4 \\ x^2 - 4 & x > 4 \end{cases}$$

$f(-3) = a(-3) + b = -3a + b$

$f(2) = b(2) - a = 2b - a$

$f(5) = 5^2 - 4 = 21$.

We can thus write:

$-3a + b = 21$ --- (1)

$2b - a = 21$ --- (2)

Solve simultaneously (use elimination)

$-3a + b = 21$ --- (1)

$-a + 2b = 21$ --- (2)

(1)×2 ⇒ $-6a + 2b = 42$ --- (1a)

$-a + 2b = 21$ --- (2)

(1a)−(2) ⇒ $-5a = 21$

$a = -\frac{21}{5} = -4\frac{1}{5}$.

Subs for a into (2):

$2b = 21 + (-4\frac{1}{5}) = \frac{84}{5}$

$2b = \frac{84}{5} \Rightarrow b = \frac{42}{5} \quad \therefore b = 8\frac{2}{5}$.

$\therefore a = -4\frac{1}{5}, \ b = 8\frac{2}{5}$

(c) LHS = $\operatorname{cosec}\theta \tan\theta$

$= \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

$=$ RHS

(d) $x^2 - 2x = 8y - 33$

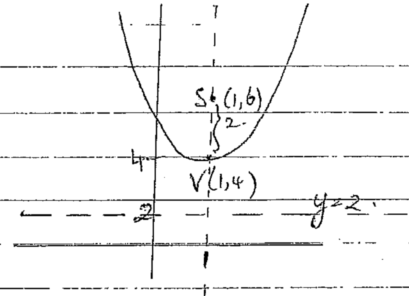
$\therefore x^2 - 2x + 1 = 8y - 33 + 1 = 8y - 32$

$(x-1)^2 = 8(y-4)$

Vertex is $(1,4)$

$a = 2$

Parabola is upright ($a > 0$).



Focus is $(1,6)$ $(6 = 4+2)$

Directrix: $y = 2$. $(2 = 4-2)$.

Sketch is above