MARCELLIN COLLEGE RANDWICK - YR12 - ASS. 1 - 2004

CALCULUS ONLY

Question 1

a. Differentiate the following:

i) $f(x) = 3x^5 - \frac{1}{2}x^2$ **1**

ii) $y = 6x\sqrt{x}$ **2**

iii) $\frac{5x^6 + 4x^5}{3x^3}$ **2**

b. Find the derivative of $y = 10x^2\sqrt{2x-1}$. **3**

c. i) Find the equation of the tangent to $y = \sqrt{x}$ at $(1,1)$. **2**

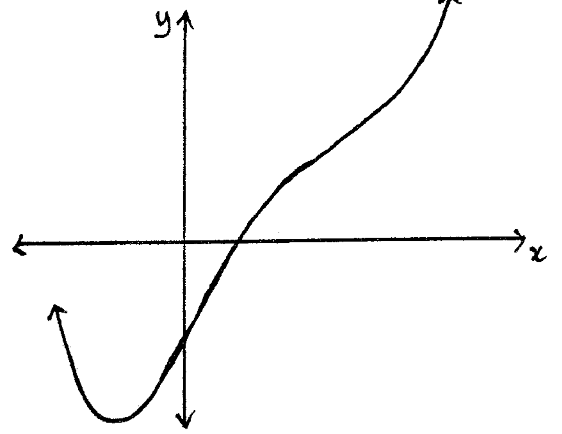
ii) Sketch the function $y = \sqrt{x}$ also showing the equation of the tangent at $(1,1)$ on your diagram. **2**

d. A curve has equation $y = 2x^3 + ax^2 + bx$. At the point P(1,5) on the curve, there is a point of inflexion. **2**

Find the values of a and b.

Question 2

a. Sketch the derivative of the following function. 2



b. Consider the curve $y = 4x^3 + 6x^2$.

i) Find the coordinates of any turning points and determine their nature. 2

ii) Find the x coordinate of any points of inflexion. 1

iii) Sketch the curve for the domain $-2 \le x \le 1$. 2

iv) What is the maximum value of $4x^3 + 6x^2$ in the domain $-2 \le x \le 1$? 1

c. A closed water tank in the shape of a right cylinder is to be constructed with a surface area of $54\pi\ cm^2$. The height of the cylinder is $h\ cm$ and the base radius is $r\ cm$.

i. Show that the height of the water tank in terms of r is given by $h = \dfrac{27}{r} - r$. 2

ii. Show that the volume V that can be contained in the tank is given by $V = 27\pi r - \pi r^3$. 1

iii. Find the radius $r\ cm$ which will give the cylinder its greatest possible volume. Justify your answer. 3

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Qu. 1

[a] (i) $f(x) = 3x^5 - \frac{1}{2}x^2$

$f'(x) = 15x^4 - 1x$

(ii) $y = 6x\sqrt{x}$

$dy/dx = 6x \times x^{1/2}$
$= 6x^{1\frac{1}{2}}$
$= 6x^{3/2}$ $\Rightarrow = 9x^{1/2}$
$= 9\sqrt{x}$

(iii) $\frac{5x^6 + 4x^5}{3x^3}$

$d/dx = \frac{vu' - uv'}{v^2}$

$= \frac{(3x^3)(30x^5+20x^4)-(5x^6+4x^5)(9x^2)}{(3x^3)^2}$

$= \frac{90x^8 + 60x^7 - 45x^8 - 36x^7}{(3x^3)^2}$

$= \frac{45x^8 + 24x^7}{9x^6}$

$= \frac{15x^8 + 8x^7}{3x^6}$

$= \frac{15x^2 + 8x}{3}$

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[b] $y = 10x^2\sqrt{2x-1}$

$dy/dx = 10x^2 \times (2x-1)^{1/2}$
$= vu' + uv'$
$= (2x-1)^{1/2}(20x) + (10x^2)(2x-1)^{-1/2}$

$= (2x-1)^{-1/2}(10x)\left[(2x-1)(2) + (2x)(1)\right]$

$= 10x(2x-1)^{-1/2}(4x-2+2x)$

$= 10x(2x-1)^{-1/2}(6x-2)$

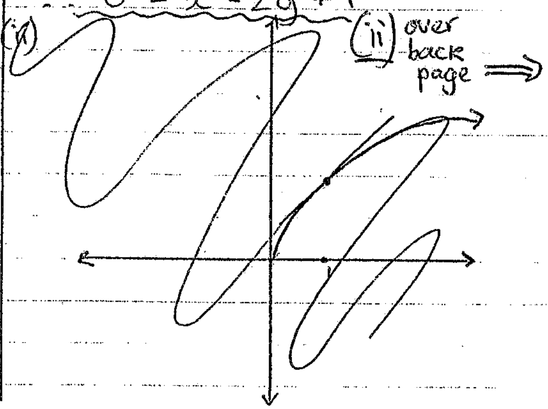
$= \frac{10x(6x-2)}{\sqrt{(2x-1)}} = \frac{60x^2-20x}{\sqrt{(2x-1)}}$

[c] (i) $y = \sqrt{x}$ pt. $(1,1)$
$y = x^{1/2}$
$y' = \frac{1}{2}x^{-1/2}$
$= \frac{1}{2\sqrt{x}}$

$\therefore m = \frac{1}{2}$.

$y - y_1 = m(x - x_1)$
$y - 1 = \frac{1}{2}(x - 1)$
$2y - 2 = x - 1$
$\therefore 0 = x - 2y + 1$

(ii)



(ii) over back page $\Longrightarrow$

Qu.1. cont...

[d] $y = 2x^3 + ax^2 + bx$

$P(1,5)$

$y' = 6x^2 + 2ax + b$
$y'' = 12x + 2a$

$\therefore 0 = 12x + 2a$
x must equal 1
$\therefore 0 = 12(1) + 2a$
$-12 = 2a$
$a = -6$.

$\therefore$ when $x = 1$; $y = 5$.
$y = 2x^3 + ax^2 + bx$
$5 = 2(1)^3 - 6(1)^2 + b(1)$
$5 = 2 - 6 + b$
$5 = -4 + b$
$9 = b$.

$\therefore$ $a = -6$ and $b = 9$.

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(b) $y = 10x^2\sqrt{2x-1}$

Let $u = 10x^2$; $v = (2x-1)^{1/2}$
$u' = 20x$; $v' = \frac{1}{2}(2x-1)^{-1/2}\cdot 2$
$= \frac{1}{\sqrt{2x-1}}$

$\therefore \frac{dy}{dx} = vu' + uv'$

$= (2x-1)^{1/2}(20x) + 10x^2 \cdot \frac{1}{\sqrt{2x-1}}$

$= \frac{(2x-1)(20x) + 10x^2}{\sqrt{2x-1}}$

$= \frac{40x^2 - 20x + 10x^2}{\sqrt{2x-1}}$

$= \frac{50x^2 - 20x}{\sqrt{2x-1}}$

$= \frac{10x(5x-2)}{\sqrt{2x-1}}$

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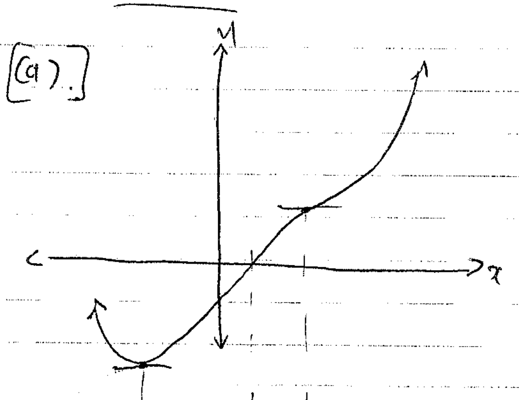
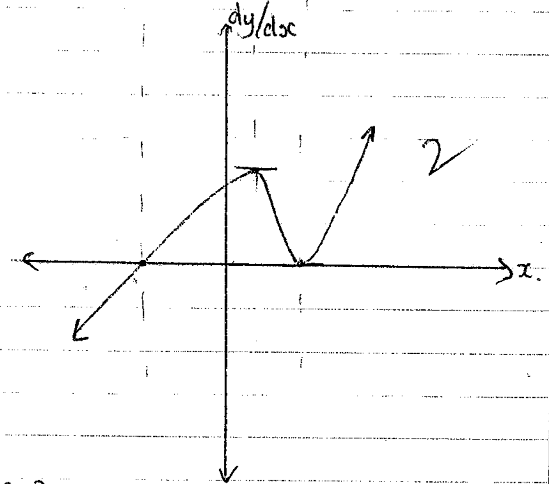
Qu. 2.

(a).





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(b). $y = 4x^3 + 6x^2$

(i). $y' = 12x^2 + 12x$

$0 = 12x^2 + 12x$

$= 12x(x+1)$

$\therefore x = 0$ & -1

at $x = 0$; $y = 0$ $(0,0)$

at $x = -1$; $y = 2$. $(-1,2)$.

$y'' = 24x + 12$

at $x = 0$; $y'' = 24(0) + 12$

$= 12$

$\therefore > 0$

$\therefore$ minimum tn. pt.

at $x = -1$; $y'' = 24(-1) + 12$

$= -12$

$\therefore < 0$

$\therefore$ maximum tn. pt.

2

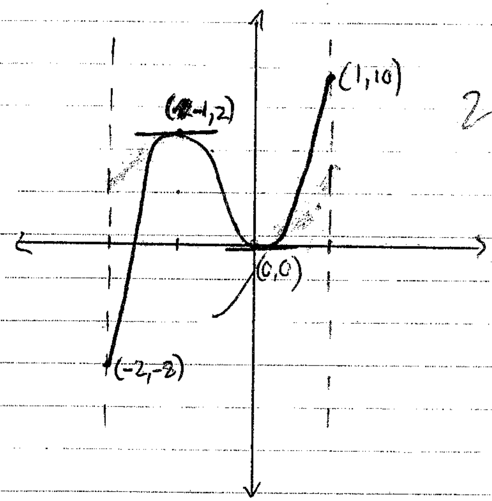
(ii) $0 = 24x + 12$

$0 = 12(2x + 1)$

$\therefore x = -\frac{1}{2}$

1

(iii) $-2 \leq x \leq 1$



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(iv) maximum value.

$(1, 10)$.

when $x = 1$; maximum value is 10 (for $-2 \leq x \leq 1$.

1

Qu. 2 cont...

(c)

(i) $h = \frac{27}{r} - r$

$54 = 2\pi r^2 + 2\pi rh$

$54\pi = 2\pi r^2 + 2\pi rh$

$27\pi = \pi r^2 + \pi rh$

$27 = r^2 + rh$

$\frac{27}{r} = r + h$.

$\frac{27}{r} - r = h$

$\therefore h = \frac{27}{r} - r$

2

(ii) $V = 27\pi r - \pi r^3$

$V = \pi r^2 h$

$= \pi r^2 \left[\frac{27}{r} - r\right]$

1

$\therefore V = 27\pi r - \pi r^3$

(iii) $V = 27\pi r - \pi r^3$

$\frac{dV}{dr} = 27\pi - 3\pi r^2$

$0 = 27\pi - 3\pi r^2$

$= 3\pi(9 - r^2)$

$\therefore r = \pm 9$

$\therefore r = 9$ (must be positive value).

$\boxed{r = 3}$

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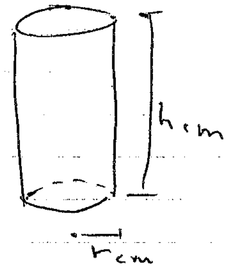

$SA = 54\pi\,cm^2$

$\frac{d^2V}{dr^2} = -6\pi r$

$= -6\pi(9)$

$= -169.6460033$

$\therefore < 0$

$\therefore$ maximum value.

$\therefore$ when radius = 9 cm; cylinder has greatest (max.) possible volume.