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MARCELLIN COLLEGE RANDWICK

POLY$^{\underline{LS}}$ 2 + PARAMETRICS

YEAR 12 PRELIMINARY

ASSESSMENT TASK # 2

EXTENSION I MATHEMATICS

2006

Weighting: 40% of Preliminary Assessment Mark.

STUDENT NAME: ____________________ MARK: / 27

PERCENTAGE: %

RANK ON THIS TASK: / 12

Time Allowed: 50 minutes

Directions:

- Answer all questions on separate answer paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Outcomes examined:

- **PE3** – Solves problems involving polynomials and parametric representations.

- **PE4** – Uses the parametric representation together with differentiation to identify geometric properties of parabolas.

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QUESTION ONE (2 MARKS)

Marks

$P(x) = x^3 - 13x - 12$

(a) Show that $x - 4$ is a root of $P(x)$ 1

(b) Hence factorize $P(x)$ in terms of its linear factors 1

QUESTION TWO (6 MARKS)

Marks

Consider the equation $x^3 - 7x + 4 = 0$

(a) Show a root exists between $x = 0.6$ and $x = 0.7$ 2

(b) Using the method of "halving the interval", determine whether $x = 0.6$ or $x = 0.7$ is the best approximation to the root correct to 1 decimal place 1

(c) Use Newton's Method once with an initial approximation of $x = 0.5$ to determine a better approximation for the root (correct to 1 decimal place) 3

QUESTION THREE (11 MARKS)

Marks

$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are variable points on the parabola $x^2 = 4ay$

(a) Show the equation of PQ is given by: 2

$$y = \frac{(p+q)x}{2} - apq$$

(b) If PQ is a focal chord, find the value of pq 1

(c) Show the equation of the normal to $x^2 = 4ay$ at P is given by: 3

$$py - ap^3 = 2ap - x$$

(d) State the equation of the normal at Q 1

(e) Find the locus of the point of intersection of the normals at P and Q 4

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QUESTION FOUR (2 MARKS)

Marks

Solve the equation $2x^3 + 7x^2 + 4x - 4 = 0$ given it has a root of multiplicity 2 (ie a double root) — 2

QUESTION FIVE (6 MARKS)

Marks

$P(6p, 3p^2)$ and $Q(6q, 3q^2)$ are variable points on the parabola $x^2 = 12y$.

The chord PQ when produced passes through the point (4, -3)

(a) Prove that $3pq - 2(p + q) - 3 = 0$ — 2

(b) The tangents at P and Q intersect at T. Show T has coordinates $[3(p + q), 3pq]$ — 2

(c) Hence find the equation of the locus of T — 2

SOLUTIONS/MARKING SCHEME – YR 12

EXTENSION I PRELIMINARY

ASSESSMENT TASK 2

Question One

(a) $P(4) = 0$ $\therefore$ by the Factor Theorem, $x - 4$ is a factor of $P(x)$ (1)

(b) $P(x) = (x-4)(x+3)(x+1)$ (1)

Question Two

let $P(x) = x^3 - 7x + 4$

(a) $P(0\cdot6) > 0$ and $P(0\cdot7) < 0$ (1)

Since $P(x)$ is continuous, $P(0\cdot6) > 0$ and $P(0\cdot7) < 0$, a root exists between $x = 0\cdot6$ and $0\cdot7$ (1)

(b) $P(0\cdot65) < 0$

$\therefore x = 0\cdot6$ is a better approximation to the root (1)

(c) $P(0\cdot5) = 0\cdot625$

$P'(x) = 3x^2 - 7$ (1)

$\therefore P'(0\cdot5) = -6\cdot25$

Now $a_1 = a_0 - \dfrac{P(a_0)}{P'(a_0)}$ (1)

$= 0\cdot5 + \dfrac{0\cdot625}{6\cdot25}$

$= 0\cdot6$ (1)

Question Three

(a) m of $PQ = \dfrac{p+q}{2}$ ①

Eqn of PQ: $y - ap^2 = \dfrac{p+q}{2}(x - 2ap)$

$\therefore 2y - 2ap^2 = px + qx - 2ap^2 - 2apq$

$\therefore 2y = (p+q)x - 2apq$ ①

$\therefore y = \dfrac{(p+q)x}{2} - apq$

(b) If PQ is a focal chord it passes thru $(0, a)$

$\therefore a = -apq$

$\therefore pq = -1$ ①

(c) $x^2 = 4ay$

$\therefore y = x^2/4a$

$\therefore y' = 2x/4a$

$\therefore y'(2ap) = p$ ①

$\therefore m$ of $T = p$

$\therefore m$ of $N = -1/p$ ①

Eqn of Normal at P:

$y - ap^2 = -\dfrac{1}{p}(x - 2ap)$ ①

$\therefore py - ap^3 = -x + 2ap$

ie. $py - ap^3 = 2ap - x$ as req'd

(d) $qy - aq^3 = 2aq - x$ ①

Question Three continued...

(e) $py - ap^3 = 2ap - x \quad (1)$

$qy - aq^3 = 2aq - x \quad (2)$

$(1) - (2): \quad py - qy - ap^3 + aq^3 = 2ap - 2aq$

$\therefore py - qy = ap^3 - aq^3 + 2ap - 2aq$

$\therefore (p-q)y = a(p-q)(p^2 + pq + q^2) + 2a(p-q)$

$\therefore y = a(p^2 + pq + q^2) + 2a$

subst $y \rightarrow (1)$

$ap(p^2 + pq + q^2) + 2ap - ap^3 = 2ap - x$

$\therefore ap^3 + ap^2q + apq^2 + 2ap - ap^3 = 2ap - x$

$\therefore \quad x = -ap^2q - apq^2$

$\therefore x = -apq(p+q)$

$\therefore$ Point of Intersection of Normals has coords:

$\left[-apq(p+q), \; a(p^2 + pq + q^2 + 2)\right]$ (1) (1)

Now $x = -apq(p+q)$ and $y = a(p^2 + pq + q^2 + 2)$

$\therefore p + q = \dfrac{x}{-apq}$

$\therefore \dfrac{y}{a} = p^2 + q^2 + pq + 2$

But $pq = -1$

$\therefore \dfrac{y}{a} = (p+q)^2 - 2pq + pq + 2$ (1)

$\therefore p + q = x/a \longrightarrow \therefore \dfrac{y}{a} = \left(\dfrac{x}{a}\right)^2 + 3$

$\therefore \dfrac{y}{a} = \dfrac{x^2}{a^2} + 3$

$\therefore ay = x^2 + 3a^2$

$\therefore x^2 = ay - 3a^2$ (1)

Question Four

let $P(x) = 2x^3 + 7x^2 + 4x - 4$

$$P'(x) = 6x^2 + 14x + 4$$
$$= 2(3x^2 + 7x + 2)$$
$$= 2(3x+1)(x+2) \quad 1$$

Since $p(x)$ has a double root, it must be $x = -2$

$\therefore P(x) = (x+2)^2(2x-1)$

$\therefore$ Solns to $2x^3 + 7x^2 + 4x - 4 = 0$ are $x = -2, \frac{1}{2}$ 1

Question Five

(a) m of $PQ = \dfrac{3p^2 - 3q^2}{6p - 6q}$

$$= \frac{p+q}{2}$$

Eqn of PQ:

$$y = \frac{(p+q)x}{2} - 3pq \quad 1$$

Now if PQ passes thru $(4, -3)$, $x = 4$ and $y = -3$ satisfy the eqn of PQ

ie. $-3 = 2(p+q) - 3pq$

$\therefore 3pq - 2(p+q) - 3 = 0$ 1

b) $y = \dfrac{x^2}{12}$

$\therefore y' = \dfrac{x}{6}$

$\therefore y'(6p) = p$

$\therefore$ Eqn of T at P:

$y - 3p^2 = p(x - 6p)$

$\therefore y = px - 3p^2$

1

$y = px - 3p^2 \quad (1)$

$y = qx - 3q^2 \quad (2)$

$\underline{(1)-(2)}$: $px - qx = 3p^2 - 3q^2$

$\therefore x = 3(p+q)$

$\therefore y = 3p^2 + 3pq - 3p^2$

$= 3pq$

$\therefore$ Coords of T are:

$[3(p+q), 3pq]$ 1

Question Five continued ...

(c) $x = 3(p+q)$ and $y = 3pq$

$\therefore p+q = x/3$ $\quad pq = y/3$

Using eqn of PQ:

$3(y/3) - 2(x/3) - 3 = 0$ ①

$\therefore y - \frac{2x}{3} - 3 = 0$

$\therefore 3y - 2x - 9 = 0$

$\therefore 2x - 3y + 9 = 0$ ①