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MARCELLIN COLLEGE RANDWICK

YEAR 11

ACCELERATED MATHEMATICS

Assessment Task #2

2012

Weighting: 20 % towards HSC mark

STUDENT NAME: ____________________ MARK: / 30

Time Allowed: 50 minutes.

Directions:

- Answer all questions.
- Begin each question on a new page.
- Show working where necessary.
- Marks may not be awarded for answers only.

Question 1 – 14 marks

a. The third term of an arithmetic series is 32 and the 6^{th} term is 17.

  i. Find the common difference and the first term. **2**

  ii. Find the sum of the first 20 terms. **1**

b. Consider the series $\frac{4}{\sqrt{3}+1}+\frac{2}{\sqrt{3}+1}+\frac{1}{\sqrt{3}+1}+\ldots$.

  i. Show that a limiting sum exists. **2**

  ii. Show that $S_\infty = 4(\sqrt{3}-1)$. **2**

c. Evaluate $\sum_{4}^{10} 5\times 2^{n-1}$. **2**

d. Mrs Smith has just retired on her $52nd$ birthday. She has saved $\$90000$ and plans to invest it the day after her birthday in an account at an interest rate of 6% pa compounding annually.
At the end of each year and after interest has been calculated, she plans to withdraw $\$6000$ until the account is empty.

Let A_n be the balance owing after the n^{th} repayment.

  i. Show that there $\$88764$ remains in the account at the end of the 2^{nd} year. **1**

  ii. Show that $A_n = 100000 - 10000(1\cdot 06)^n$. **2**

  iii. How old will Mrs Smith be when there is no money left in the account? **2**

Question 2 – 16 marks

Start a new page

a. Solve for x: $\log_3(x+7)-\log_3(x-1)=2$. **2**

b. Consider the curve $f(x)=-xe^{2x}$.

i. Find the coordinates of any stationary points and determine their nature. **3**

ii. Find the coordinates of any points of inflexion. **2**

iii. Sketch the curve showing all relevant points. **2**

c. Consider the function $y=e^{x}+1$.

i. Sketch the function and shade the region bounded by the curve, the x-axis and the lines $x=0$ and $x=\log_e 3$. **1**

ii. The region in part (i) is rotated around the x-axis. Find the volume of the resulting solid of revolution leaving your answer in simplest exact form. **3**

d. Find the derivative of $y=x\ln x$, and hence find $\int_1^2 \ln x\ dx$. **3** part ① Q1 14

1a i) $T_3 = a + (3-1)d = 32$
$= a + 2d = 32$
$T_6 = a + 5d = 17$
$3d = -15$
$d = -5$ ✓
Sub $d = -5$ into
$T_6 = a + 25 = 17$ ✓
$a = 42$
Ans $d = -5$
$a = 42$ ✓

ii) $S_{20} = 10\,(84 + (19 \times -5))$ 1
$= -110$ ✓

b i) $r = \frac{1}{2}$ ✓ 1
∴ since $|r| < 1$
and $-1 < r < 1$ ✓ 1
∴ a limiting sum exists.

ii) $S_\infty = \frac{a}{1-r} = \frac{4}{\sqrt{3}+1} \div \left(1 - \frac{1}{2}\right)$ ✓ $= \frac{4}{\sqrt{3}+1} \times 2 = \frac{8}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$
$= \frac{8(\sqrt{3}-1)}{3-1} = \frac{8(\sqrt{3}-1)}{2} = 4(\sqrt{3}-1)$ ✓ 2
$\therefore S_\infty = 4(\sqrt{3}-1)$

c) 40, 80, 160 ✓
$r = 2 \quad a = 40$
$S_7 = 40\,(2^7 - 1) = 5080$ ✓ 2

Q1 Part ②

1 d i) Year 1 $A_1 = 90000(1.06) - 6000$
2nd year → $A_2 = 90000(1.06)^2 - 6000(1.06) - 6000 = \$88764$ ✓ 1

ii) $A_3 = 90000(1.06)^3 - 6000(1.06)^2 - 6000(1.06) - 6000$
$= 90000(1.06)^3 - 6000(1 + 1.06 + 1.06^2 \ldots)$
$\therefore A_n = 90000(1.06)^n - 6000(1 + 1.06 + 1.06^2 \ldots + 1.06^{n-1})$
$= 90000(1.06)^n - 6000\left(\frac{1.06^n - 1}{0.06}\right)$ ✓
$= 90000(1.06)^n - 100{,}000(1.06^n - 1)$ 2
$= 90000(1.06)^n - 100000(1.06)^n + 100000$
$= 100{,}000 - 10000(1.06)^n$ ✓

iii) let $A_n = 0$
$100{,}000 - 10000(1.06)^n = 0$
$100{,}000 = 10{,}000(1.06)^n$
$10 = 1.06^n$
$\log_{1.06} 10 = n$ ✓
$\frac{\ln 10}{\ln 1.06} = n$ 2
$n = 39.5$
$52 + 39.5 = 91.5$ ✓
= 92 years old, there will be 0 dollars left.

Question 2 part 1 12

2a) $\log_3 \frac{x+7}{x-1} = 2$

$3^2 = \frac{x+7}{x-1}$

$9 = \frac{x+7}{x-1}$ ✓

2

$9(x-1) = x+7$

$9x-9 = x+7$

$8x = 16$

$x = 2$. ✓

bi) $f(x) = -xe^{2x}$

$u = -x$

$u' = -1$

$v = e^{2x}$

$v' = 2e^{2x}$

$-e^{2x} - 2xe^{2x}$

$f'(x) = -e^{2x}(1+2x)$

let $f'(x) = 0$ to find stat points

$-e^{2x} \neq 0$ $\quad 1+2x = 0$

$2x = -1$

$x = -\frac{1}{2}$ ✓

$\frac{3}{4}(e^{2x-\frac{3}{2}})$

$= \frac{3}{4}(e^{-\frac{3}{2}})$

x	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$
$\frac{dy}{dx}$	0.11	0	-0.30

✓

8

∴ maximum point at $(-\frac{1}{2}, \frac{1}{2e})$

$(-\frac{1}{2}, \frac{1}{2e})$

Question 2 part 2

bii) $f''(x) = -2e^{2x}(1+2x) - 2e^{2x}$

$u = -e^{2x}$

$u' = -2e^{2x}$

$v = 1+2x$

$v' = 2$

$= -2e^{2x}(1+2x+1)$

$= -2e^{2x}(2x+2)$

let $f''(x) = 0$ to find possible point of inflex.

$-2e^{2x} \neq 0$ $\quad 2x = -2$

$x = -1$ ✓

Test for concavity change

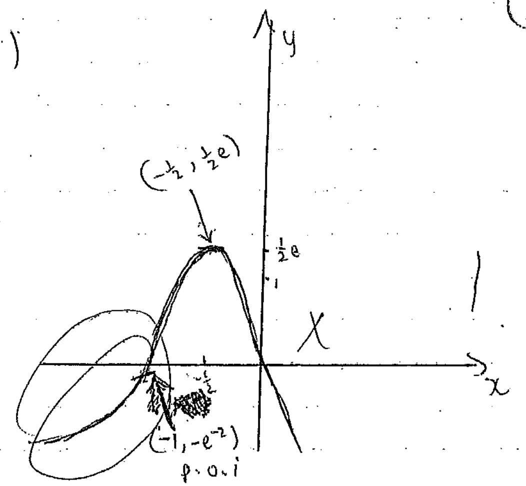
x	$-\frac{3}{2}$	-1	$-\frac{3}{4}$
$f''(x)$	0.07	0	-0.2

2

∴ due to concavity change $(-1, -e^{-2})$ is a p.o.i.

$(-1, \frac{1}{e^2})$

iii)



Question ② part ④

d) $\frac{dy}{dx} = \ln x + 1$

$U = x$

$U' = 1$

$V = \ln x$

$V' = \frac{1}{x}$

$\left[x\ln x - 1\right]_1^2$

$= (2\ln 2 - 1) - (\ln 1 - 1)$

$= 2\ln 2 - 1 + 1$

$= 2\ln 2$

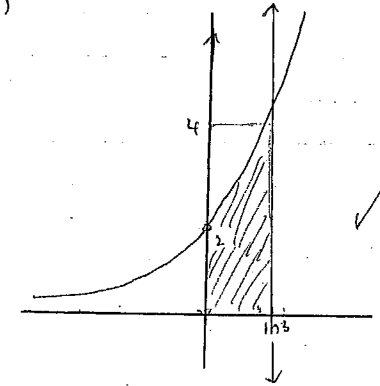
$\left[x\ln x\right]_1^2 - \left[x\right]_1^2$

$\left[2\ln 2 - \ln 1\right] - \left[2 - 1\right]$

$= 2\ln 2 - 1$

Q2 part ③

c i)



ii) $\pi \int_0^{\ln 3} (e^x + 1)^2$

$= \pi \int_0^{\ln 3} e^{2x} + 2e^x + 1 \, dx$

$= \pi \left[\frac{e^{2x}}{2} + 2e^x + x\right]_0^{\ln 3}$ 2

$= \pi \left[\left(\frac{e^{2\ln 3}}{2} + 2e^{\ln 3} + \ln 3\right) - \left(\frac{1}{2} + 2\right)\right] = \pi\left\{\left[\frac{e^{\ln 3^2}}{2} + 2e^{\ln 3} + \ln 3\right]\left[-2\tfrac{1}{2}\right]\right\}$

$= \pi\left[\frac{e^{2\ln 3}}{2} + 6 + \ln 3 - \frac{1}{2} - 2\right] = \pi\left\{\frac{9}{2} + 2\times 3 + \ln 3 - 2\tfrac{1}{2}\right\}$

~~$= \pi\left[8 + \ln 3\right] u^3$~~

$= \pi\left[8 + \ln 3\right]$

$= \pi\left[\frac{e^{2\ln 3}}{2} + \frac{7}{2} + \ln 3\right]$

$= \frac{1}{2}\pi\left[e^{2\ln 3} + 2\ln 3 + 7\right] u^3$

$=$

$= ?$