Waverley College
Year 11 2U Mathematics Assessment
Term 3, 2012

TIME ALLOWED: 50 MINUTES

STUDENT NUMBER:

INSTRUCTIONS:
Attempt all questions
Calculators may be used
Write in blue or black pen or pencil only
Show all necessary working
You may use your single sided A4 summary sheet

START EACH QUESTION ON A NEW PIECE OF PAPER
WRITE YOUR STUDENT NUMBER ON EACH SECTION

Question 1	/5
Question 2	/12
Question 3	/18
TOTAL	/35
%	/

In this exam the following outcomes will be assessed:

P2 – Provides reasoning to support conclusions which are appropriate to the context.
P4 – Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
P5 - Understands the concept of a function and the relationship between a function and its graph.

Question 1 (5 marks)

a) Sketch the region $y > x^2 - 1$, showing all relevant information. 2

b) Draw a neat sketch showing the region on the number plane where the inequalities $x^2 + y^2 \le 9$ and $x + y > 3$ hold simultaneously. (Showing all relevant information) 3

Question 2 (12 marks)   (START ON A NEW PIECE OF PAPER)

(a) Find the value of t if,

i) $\cos 75° = \sin t°$ 1

ii) $\cot(2t + 5)° = \tan(3t - 15)°$ 2

(b) If $\sin A = 0.6$, find the exact ratios for

i) $\cos A$ 1
ii) $\cot A$ 1

(c) From a small boat out at sea, the angle of elevation of the top of a vertical cliff is $5°$. The height of the cliff is 250 metres.

(i) Draw a neat sketch to represent the information given. 1

(ii) Find the distance from the boat to the base of the cliff. (nearest metre) 2

(d) Find the exact value of

i) $\tan 330°$ 1

ii) $\sin(-135)°$ 1

iii) Hence, or otherwise, evaluate $\tan 330° \times \sin(-135)°$

Leaving your answer in exact form with a rational denominator. 2

Question 3 (18 marks)   (START ON A NEW PIECE OF PAPER)

a) Three mobile phone towers called P, Q and R are located on top of hills.

Tower P is 12km away from tower Q, which in turn is 14km away from tower R.
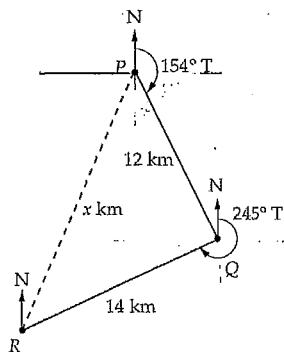
The bearing of Q from P is $154°T$ and the bearing of R from Q is $245°T$.

i) Find $\angle PQR$ 1
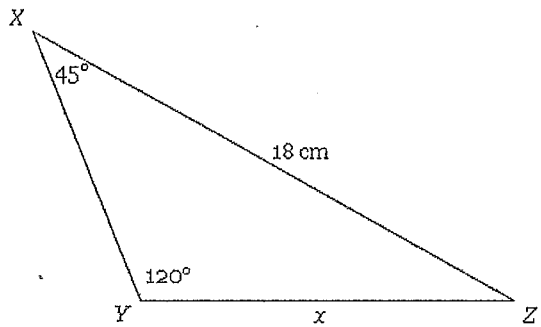
ii) How far is it from tower P to tower R? (1 dec pl)   2

b) In the diagram $\triangle XYZ$ find the value of x in exact form with a rational denominator.   3

c) Simplify $\sec^2 \theta(1 - \cos^2 \theta)$ 3

d) Solve the following trigonometric equations for $0° \le x \le 360°$

i) $\tan x = 0.2468$ 2

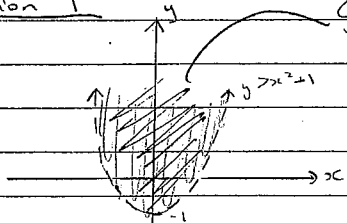ii) $2 \sin 2x + 1 = 0$ 4
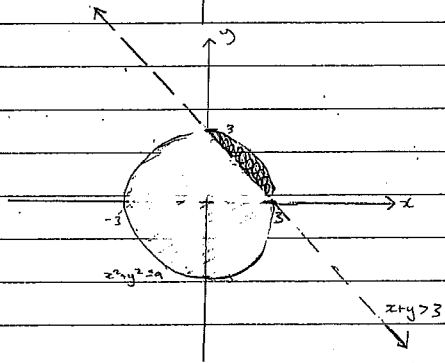
e) Prove that $\dfrac{1}{1 + \sin \theta} + \dfrac{1}{1 - \sin \theta} = 2\sec^2 \theta$.   3

END OF EXAMINATION

Question 1

1) a)

shade
$y > x^2 - 1$

b)

$x^2 + y^2 \leq 9$

$x + y > 3$

$y > x + 3$

Test $(0,0)$: $x^2 + y^2 \leq 9$

$0 + 0 \leq 9$ ✓

$x + y > 3$

2

Question 2

i) $\cos 75° = \sin t$

$\therefore t = 15°$ ✓

ii) $\cot(2t+5)° = \tan(3t-15)°$

$t = 20°$

$\cot(2t+5) = \tan(3t-15)$

$\dfrac{1}{2t+5} = 3t-15$

$1 = (3t-15)(2t+5)$

$1 = 6t^2 + 15t - 30t - 75$

$6t^2 - 15t - 76 = 0$

b) i) $\sin A = 0.6$

$A = \sin^{-1}(0.6)$

$A = 36° 52'$

ii) $\cos A = \cos(36° 52')$

$\cos A = 0.8 = \dfrac{4}{5}$ ✓

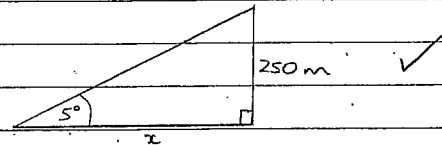ii) $\cot A = \cot(36° 52')$

$\tan A = 0.75$

$= \dfrac{3}{4}$

$\therefore \cot A = \dfrac{4}{3}$ ✓

c) i)

$250 m$ ✓

$5°$

x

ii) $\tan 5° = \dfrac{250}{x}$ ✓

$x = \dfrac{250}{\tan 5°}$

$x = 2858 m$

d) i) $\tan 330° = -\tan 30°$ ✓

$= \dfrac{-1}{\sqrt{3}}$

ii) $\sin(-135°) = \sin 45°$ ✓

$= -\dfrac{1}{\sqrt{2}}$

iii) $\tan 330° \times \sin(-135°) = \dfrac{-1}{\sqrt{3}} \times \dfrac{-1}{\sqrt{2}}$

$= \dfrac{1}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}}$

$= \dfrac{\sqrt{6}}{6}$

3) a) i) $180 - 154 = 26°$

$90 - 26 = 64°$

$\therefore 270 - 245° =$

$\therefore 25 + 64 = 89°$ ✓

$\therefore \angle PQR = 89°$

ii) $c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 12^2 + 19^2 - 2 \times 12 \times 14 \times \cos 89°$

$c^2 = 334.14$ ✓✓✓

$c = 18.3 km$ ✓✓

b) $\dfrac{x}{\sin 45°} = \dfrac{18}{\sin 120°}$

$\therefore x = \dfrac{18 \sin 45°}{\sin 120°}$

$(x = \dfrac{\frac{18}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$

$= \dfrac{18}{\sqrt{2}} \times \dfrac{2}{\sqrt{3}}$

$= \dfrac{36}{\sqrt{6}}$

$= \dfrac{36\sqrt{6}}{6}$ ✓✓✓

$= 6\sqrt{6}$

c) $\sec^2\theta(1 - \cos^2\theta)$

$= \dfrac{1}{\cos^2\theta} \times \sin^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$ ✓✓✓

$= \tan^2\theta$

d) i) * 1st and 3rd quadrants *

$\tan x = 0.2468$

$x = \tan^{-1}(0.2468)$

$x = 13°52'$

$, x = 180 + 13°52'$

$\therefore x = 13°52' / x = 193°52'$ ✓ ✓

d) ii) $2\sin 2x + 1 = 0$

$\sin 2x = -\dfrac{1}{2}$

Since $\sin 2x$: $0° \leq 2x \leq 720°$

$-\dfrac{1}{2} = \sin 30°$

since negative : 3rd & 4th quadrants ✓

$2x = 180 + 30°, \quad 360 - 30, \quad 360 + (180 + 30), \quad 360 + (360 - 30)$

$\therefore 2x = 210°, \quad 2x = 330°, \quad 2x = 570°, \quad 2x = 690°$

$\therefore x = 105°, \; x = 165°, \; x = 285°, \; x = 345°$
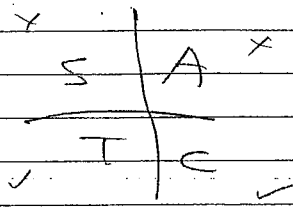
e) $\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta} = 2\sec^2\theta$

$LHS = \dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta}$

$= \dfrac{\frac{(1 - \sin\theta)}{(1+\sin\theta)(1-\sin\theta)} + \frac{(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}{}$

$= \dfrac{(1-\sin\theta) + (1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{2}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{2}{1 - \sin\theta + \sin\theta - \sin^2\theta}$

$= \dfrac{2}{1 - \sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta$ ✓✓✓

$= RHS.$