# MARCELLIN COLLEGE RANDWICK

## YEAR 12
## MATHEMATICS

## HSC ASSESSMENT TASK 2
## 2014

**STUDENT NAME:** _____ **MARK** /70

**TEACHER:** _____

**TIME ALLOWED:** 90 minutes
**WEIGHTING:** 25 %

**Directions:**

- Answer multiple choice questions on the page provided.
- Use a new sheet for each question.
- Show all necessary working. Where more than one mark is allocated to a question, full marks may not be awarded for answers only.
- Marks may not be awarded for careless or badly arranged work.
- Calculators may be used



### STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

**Section I**
**Start each question on a new sheet of paper**

**Question 1 (13 marks)**

**Marks**

a) Evaluate

$$\sqrt{\frac{3.15 - 0.46}{2.57^2}}$$

correct to 3 significant figures — 2

b) Factorise $4x^2 + 11x - 3$ — 2

c) Solve $|3x - 4| \leq 1$ — 2

d) Solve $2\cos x = 1$ for $0° \leq x \leq 360°$ — 2

e) Differentiate (leaving your answer in simplest form)

i) $\frac{x+1}{x-2}$ — 3

ii) $\frac{1}{\sqrt[3]{x^2}}$ — 2

**Question 2 (10 marks)**
Start a new page

**Marks**

a) i) Find — 2

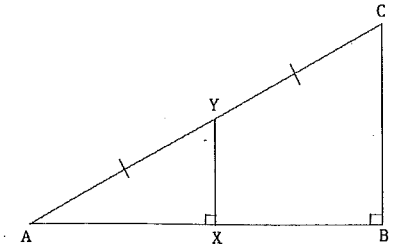
$$\lim_{x \to 0} \frac{x^2 + 6x}{x}$$

ii) Hence or otherwise, sketch the graph of — 2

$$y = \frac{x^2 + 6x}{x}$$

clearly showing any points of discontinuity.

b) For the diagram below




i) Prove that $\Delta AXY \parallel\parallel \Delta ABC$ — 3

ii) What is the ratio of AX to AB (show all working) — 2

iii) If AB = 5cm, calculate the length of AX — 1

**Question 3 (17 marks)**
Start a new page

**Marks**

a) Consider the function defined by $y = -2x^3 + 6x^2 - 3$

i) Find the coordinates of any stationary points of the curve — 3

ii) Find the coordinates of any points of inflexion — 2

iii) Sketch the curve showing all important features in the domain $-2 \leq x \leq 4$ — 2

iv) Find the maximum value of y over the domain $-2 \leq x \leq 4$ — 1

b) Find the equation of the normal to the curve $y = 3 - x - x^2$ at the point where the curve crosses the $y-$ axis — 3

c) A piece of wire of length 30cm is cut into two squares. Each section is then bent into shape of a square.

i) Prove that the total area of the two squares is $A = \frac{x^2}{16} + \frac{(30-x)^2}{16}$ — 2

ii) Find the smallest possible value of the area of the two squares. — 4

**Question 4 (15 marks)**
Start a new page

**Marks**

a) Find

i) $\int (2x-1)^3\, dx$ — 2

ii) $\int_4^9 \frac{1}{\sqrt{x}}\, dx$ — 2

b) Evaluate — 3

$$\int_0^1 \frac{dx}{x+1}$$

using the Trapezoidal rule with 5 function values (to 2 dec pl)

c) Find the area of the region bounded by the $x-$ axis, the curve $y = \sqrt{x}$ and the line $y = 6 - x$ — 4

d) Find the volume of the solid formed when the curve $y = x^2 + 1$ is rotated about the $y-$ axis from $y = 1$ to $y = 3$ (correct to 2 dec. pl) — 4

**Question 5 (10 marks)**
Start a new page

**Marks**

a) How many terms are in the series 2

$$\sum_{n=5}^{63} (3n + 6)$$

b) Calculate 2

$$\sum_{r=1}^{5} (3r - 5)$$

c) The 3rd term of an arithmetic series is 13 and the 8th term is 28.

i) Find the common difference 1

ii) Find the 20th term of the series 2

d) For what value of the n is the sum of the arithmetic series 3

$$6 + 11 + 16 + \cdots$$

equal to 234?

END OF SECTION 1

## Section II – Multiple Choice (5 marks)
**Attempt all questions**

Use the Multiple Choice Answer Sheet for Questions 6 to 10

6. What is $\frac{1+\sqrt{3}}{5-2\sqrt{3}}$ as a fraction with rational denominator?

(A) $\frac{11+7\sqrt{3}}{13}$ (B) $\frac{-5+\sqrt{3}}{13}$ (C) $\frac{11-7\sqrt{3}}{7}$ (D) $\frac{-5-\sqrt{3}}{7}$

7. What is the solution to the equation $\frac{x+4}{3}=\frac{x}{2}-2$?

(A) $x=2$ (B) $x=5$ (C) $x=20$ (D) $x=6$

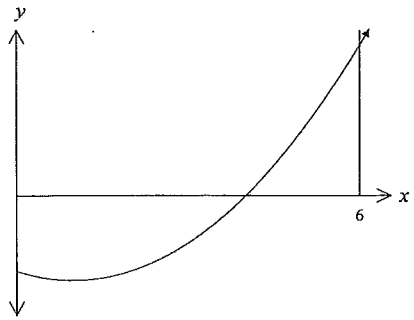
8. The line $6x-ky=2$ passes through the point $(3,2)$. What is the value of $k$?

(A) $\frac{10}{3}$ (B) 8 (C) $-8$ (D) $-\frac{10}{3}$

9. What are the coordinates of the focus of the parabola $x^2=2(y-1)$?

(A) $(0,\frac{1}{2})$ (B) $(0,\frac{3}{2})$ (C) $(\frac{1}{2},0)$ (D) $(\frac{3}{2},0)$

10. The diagram below shows the graph of $y = x^2 - 2x - 8$.




What is the correct expression for the area bounded by the $x$-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

(A) $A = \int_0^5 x^2 - 2x - 8dx + \left|\int_5^6 x^2 - 2x - 8dx\right|$ (B) $A = \int_0^4 x^2 - 2x - 8dx + \left|\int_4^6 x^2 - 2x - 8dx\right|$

(C) $A = \left|\int_0^5 x^2 - 2x - 8dx\right| + \int_5^6 x^2 - 2x - 8dx$ (D) $A = \left|\int_0^4 x^2 - 2x - 8dx\right| + \int_4^6 x^2 - 2x - 8dx$

END OF PAPER

Question 1 — 13/13 — Pg 1

a) $= \sqrt{\frac{2.69}{6.6049}}$

$= 0.638$ 3 sig. figs. ✓2

b) $4x^2 + 11x - 3$

$p = 4 \times -3 \quad s = 11$

$= 12x, -1$

$4x^2 + 12x - x - 3$

$4x(x+3) - 1(x+3)$

$(4x-1)(x+3)$ ✓2

c) $|3x - 4| \leq 1$

$3x - 4 \leq 1$

$3x \leq 5$

$x \leq \frac{5}{3}$

$-|3x - 4| \leq 1$

$-3x + 4 \leq 1$

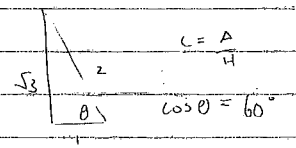
$-3x \leq -3$

$x \geq 1$

$1 \leq x \leq \frac{5}{3}$ ✓2

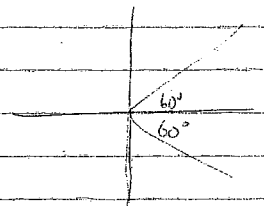
d) $2\cos x = 1$

$\cos x = \frac{1}{2}$ Well done!




$\cos\theta = \frac{A}{H}$

$\cos\theta = 60°$




$x = 60°, 300°$ ✓2

e) i) $V = x - 2$

$v' = 1$

$y = x + 1$

$u' = 1$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{(x-2) - (x+1)}{(x-2)^2}$

$= \frac{x - 2 - x - 1}{x^2 - 4x + 4}$

$= \frac{-3}{x^2 - 4x + 4}$ ✓3

ii) $y = \frac{1}{\sqrt[3]{x^2}}$

$= \frac{1}{x^{\frac{2}{3}}}$

$= x^{-\frac{2}{3}}$

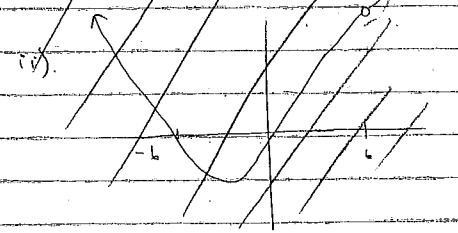
$y' = -\frac{2}{3}x^{-\frac{5}{3}}$

$= \frac{-2}{3\sqrt[3]{x^5}}$ ✓2

a) i) $\lim_{x \to 0} \frac{x^2 + 6x}{x}$

$\lim_{x \to 0} \frac{x(x+6)}{x} = 6$

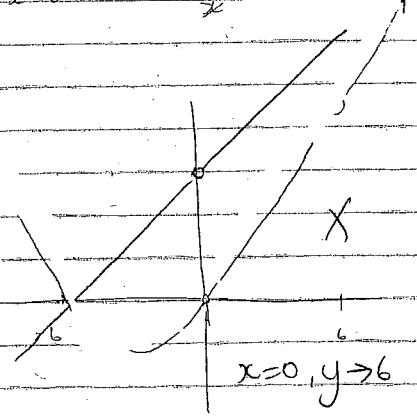
iv)



a) i). $\lim_{x \to 0} \frac{x^2 + 6x}{x}$

$\lim_{x \to 0} \frac{x(x+6)}{x} = 6$ ✓✓

ii).




b) i). $\angle CAB$ is common

$\angle YXA = \angle CBA$ (given)

$\angle XYA = \angle BCA$ (ALE $\angle$'S $CB \parallel YX$)

$\therefore \Delta AXY \,|||\, \Delta ABC$ (3 pairs of corresponding angles are equal.) ✓✓

ii) using ||| $\Delta$'s

$AY = YC$

$AY + YC = AC$

$\frac{AY}{AC} = \frac{1}{2}$

$\frac{AY}{AC} = \frac{AY}{AB}$ (corresponding sides on ||| $\Delta$'s)

$\therefore \frac{AY}{AB} = \frac{1}{2}$

$AX : AB = 1 : 2$ ✓✓

iii) $\frac{x}{5} = \frac{1}{2}$

$x = 2\frac{1}{2}$ ✓

$= 2.5$ cm



). i). $y = -2x^3 + 6x^2 - 3$

$y' = -6x^2 + 12x$

SP. $y' = 0$

$0 = -6x^2 + 12x$

$= -6x(x - 2)$

$x = 0$

$x = 2$

when $x = 0$

$y = -2(0)^3 + 6(0)^2 - 3$

$= -3$

when $x = 2$

$y = -2(2)^3 + 6(2)^2 - 3$

$= 5$

$\therefore$ SP at $(0, -3)$ and $(2, 5)$

Nature:

$y'' = -12x + 12$

when $x = 0$

$y'' = -12(0) + 12$

$= 12$

$> 0$

$\therefore$ con $\cup$ min @ $(0, -3)$

when $x = 2$

$y'' = -12(2) + 12$

$= -12$

$< 0$ ✓✓

$\therefore$ con $\cap$, max @ $(2, 5)$

ii). POI $y'' = 0$

$0 = -12x + 12$

$-12 = -12x$

$1 = x$

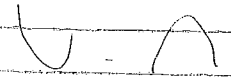
$y = -2(1)^3 + 6(1)^2 - 3$

$= 1$

Possible POI @ $(1, 1)$ ✓✓

Test

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y''$ | 12 | 0 | -12 |



$\therefore$ concavity change

$\therefore (1, 1)$ is POI

End points

$x = 4$

$y = -2(4)^3 + 6(4)^2 - 3$

$= -35 \quad (4, -35)$

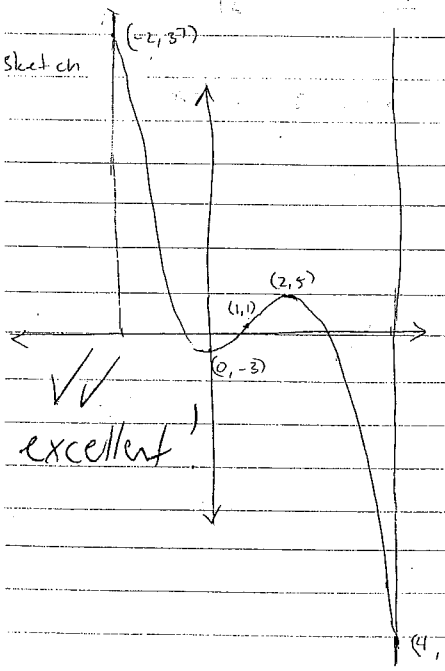
$x = -2$

$y = -2(-2)^3 + 6(-2)^2 - 3$

$= 37$

$(-2, 37)$

sketch




✓✓ excellent!

max value = 37. ✓

b). $x = 0$

$y = 3$

∴ tangent at $(0, 3)$

$y = -x^2 - x + 3$

$y' = -2x - 1$

when $x = 0$

$y' = -2(0) - 1 \quad 0 - 1$

$= -3$ (-1)

$m = -3$

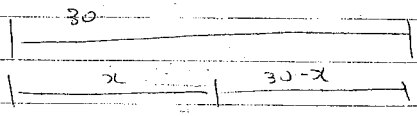
$y - y_1 = m(x - x_1)$

$y - 3 = -3(x - 0)$

$y - 3 = -3x$ (2)

$3x + y - 3 = 0$






$A = \frac{x^2}{16} + \frac{(30-x)^2}{16}$ ✓✓

ii). $A = \frac{x^2}{16} + \frac{(30-x)^2}{16}$

$A' = \frac{2x}{16} + \frac{2(30-x)(-1)}{16}$ (1)

$0 = \frac{1}{8}x - \frac{1}{8}(30-x)$

$2x - 30 = 0$

$2x = 30$

$x = 15$

$A'' = \frac{1}{8} - \left(-\frac{1}{8}\right)$

$= \frac{1}{4} > 0$ ∪

∴ Minimum occurs.

$A = \frac{15^2}{16} + \frac{15^2}{16}$

$= \frac{450}{16}$

$= 28\frac{1}{8}$ sq. cm.

a) i).

$$\int (2x-1)^3 \, dx$$

$$= \frac{(2x-1)^4}{4(2)} + C$$

$$= \frac{(2x-1)^4}{8} + C \quad \checkmark\checkmark$$

ii). $\int_4^9 \frac{1}{\sqrt{x}} \, dx$

$$\int_4^9 \frac{1}{x^{\frac{1}{2}}} \, dx$$

$$\int_4^9 x^{-\frac{1}{2}} \, dx$$

$$= \left[ \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^9$$

$$= \left[ 2 \cdot x^{\frac{1}{2}} \right]_4^9$$

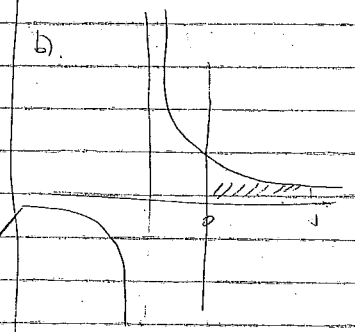
$$= \left[ 2\sqrt{x} \right]_4^9$$

$$= \left[ 2\sqrt{9} \right] - \left[ 2\sqrt{4} \right]$$

$$= [6 - 4] \quad \checkmark$$

$$= 2$$

b).



$h = \frac{1-0}{5}$

$= \frac{1}{5}$

| $x$ | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|---|
| $y$ | $\frac{5}{6}$ | $\frac{5}{7}$ | $\frac{5}{8}$ | $\frac{5}{9}$ | $\frac{1}{2}$ |

$$A \doteq \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 \cdots + y_{n-1})\right]$$

$$= \frac{\frac{1}{5}}{2}\left[\left[\frac{5}{6} + \frac{1}{2}\right] + 2\left[\frac{5}{7} + \frac{5}{8} + \frac{5}{9}\right]\right]$$

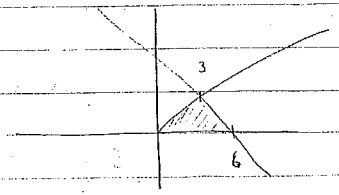
$$= \frac{1}{10}\left[\left[1\frac{1}{3}\right] + 2\left[1\frac{45}{504}\right]\right]$$

$\doteq 0.51$ units$^2$ (2 dp). (2)

$$h = \frac{b-a}{n} = \frac{1-0}{4}$$

Question 4

c).




$y = 6 - x$

$0 = 6 - x$

$x = 6$

PO − int

$\sqrt{x} = 6 - x$

$x = (6-x)^2$

$= 36 - 12x + x^2$

$= 36 - 12x$

$12x = 36$

$x = 3$ 4

$$A_1 = \int_0^3 \sqrt{x} \, dx$$

$$= \int_0^3 x^{\frac{1}{2}} \, dx$$

$$= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3$$

$$= \left[ \frac{2\sqrt{x^3}}{3} \right]_0^3$$

$$= \left[ \frac{2\sqrt{27}}{3} \right] - 0$$

$= 3.464$ units$^2$

$$A_2 = \int_3^6 6 - x \, dx$$

$$= \left[ 6x - \frac{x^2}{2} \right]_3^6$$

$$= \left[ \left[ 6(6) - \frac{(6)^2}{2} \right] - \left[ 6(3) - \frac{(3)^2}{2} \right] \right]$$

$= 18 - 13.5$

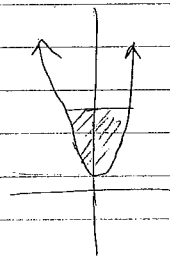
$= 4.5$ units$^2$

$A = A_1 + A_2$

$= 3.464 + 4.5$ error carried.

$\doteq 7.96$ units$^2$ (2 dp) (8)

d).



$y = x^2 + 1$

$y - 1 = x^2$

$$V = \pi \int_a^b x^2 \, dy$$

$$= \pi \int_1^3 y - 1 \, dy$$

$$= \pi \left[ \frac{y^2}{2} - y \right]_1^3$$

## Question 4

$= \pi\left[\left[\frac{(3)^2}{2} - 3\right] - \left[\frac{1^2}{2} - 1\right]\right]$

$= \pi\left[1.5 - (-0.5)\right]$

$= \pi\left[2\right]$

$= 2\pi$ units$^3$ ✓✓✓✓

$= 6.28$ units$^3$ (2 dp).



## Question 5

a) $n = 59$ terms ✓✓

b) $\sum_{r=1}^{5} (3r-5) = \underset{(3(5)-5)}{(3(1)-5) + (3(2)-5) + (3(3)-5) + (3(4)-5) +}$

$= -2 + 1 + 4 + 7 + 10$

$= 20$ ✓✓

c) i) $T_n = a + (n-1)d$

$T_3 = 13$

$13 = a + (3-1)d$

$= a + 2d$

$T_8 = 28$

$28 = a + (8-1)d$

$= a + 7d$

$13 = a + 2d$

$28 = a + 7d$ −

$-15 = -5d$

$3 = d$

$\boxed{d = 3}$ ✓

ii) $13 = a + 6$ T20

$7 = a$

$a = 7$

$d = 3$

$a = 7$

$n = 20$

T20 not S20.

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(2(7) + (20-1)3)$ (1)

$= 710$

d) $d = 11 - 6$

$= 5$

$a = 6$

$n = ?$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$234 = \frac{n}{2}(2(6) + (n-1)5)$

$468 = n(12 + 5n - 5)$

$= n(5n + 7)$

$= 5n^2 + 7n - 468$

$p = \frac{5}{1} \times \frac{468}{11}$ $S = 7$

$5 \quad 1239$

1 3 3

1 1 1 1
5 3 4 13 3

Question 5

1 1 1 1 1
5 3 22 13 3

52, 45

n =

= $5n^2 - 45n + 52n - 468$

= $5n(n-9) + 52(n-9)$

= $n = 9$

$n \neq -10.4$ (negative and not integer)

$\therefore n = 9$ ✓✓✓

Name

YEAR 12
MATHEMATICS
HSC
TASK 2

MULTIPLE CHOICE ANSWER SHEET

Section II (5 marks)
Allow about 5 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

5. ● A ○ B ○ C ○ D

7. ○ A ○ B ● C ○ D

8. ○ A ● B ○ C ○ D

9. ○ A ● B ○ C ○ D

10. ○ A ○ B ○ C ● D

5

Name_____

**YEAR 12**
**MATHEMATICS**
**HSC**
**TASK 2**

**MULTIPLE CHOICE ANSWER SHEET**

**Section II (5 marks)**
**Allow about 5 minutes for this section**
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

6. ● A ○ B ○ C ○ D ✓

7. ○ A ○ B ● C ○ D ✓

8. ○ A ● B ○ C ○ D ✓

9. ○ A ● B ○ C ○ D ✓

10. ○ A ○ B ○ C ● D ✓