J.M.J.

MARCELLIN COLLEGE RANDWICK

**YEAR 12 HSC**

**ASSESSMENT TASK # 2**

**EXTENSION ONE**

**MATHEMATICS**

**2007**

**Weighting: 20% of H.S.C. Assessment Mark.**

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| **STUDENT NAME:**____________________ | **MARK:** | | **/ 28** |
| | **PERCENTAGE:** | | **%** |
| | **RANK ON THIS TASK:** | | **/ 23** |

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**Time Allowed:** **50 minutes.**

**Directions:**

- Answer all questions on separate lined paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

**Outcomes examined:**

PE3 – Solves problems involving permutations and combinations.

HE3 – Uses a variety of strategies to investigate Mathematical models of situations involving binomial probability.

Question One ( 4 marks )

Write the coefficient of:

a) $x^5$ in the expansion of $(5x - 2)^{11}$ 2

b) $x$ in the expansion of $(x + \frac{3}{x})^4\ (x-2)^5$ 2

Question Two ( 7 marks )

a) Show $\dfrac{{}^7C_k}{{}^7C_{k-1}} = \dfrac{8-k}{k}$ 2

b) Determine the greatest coefficient in the expansion of $(3x - 7)^7$ 5

Question Three ( 4 marks )

a) Write the expansion of $(1 + x)^n$ 1

b) Show that $n\,[\,3^{n-1} - 1\,] = \sum_{r=0}^{n-2} {}^nC_{r+2}\, 2^{r+1}\,(r+2)$ 3

Question Four ( 2 marks )

4 men and 4 women are to be seated randomly around a round table.
What is the probability that the men and women will alternate?

Question Five ( 3 marks )

From a group of 7 men and 5 women a team of six is to be formed.
Given each person is equally likely to be selected, what is the probability that the selected team contains at least 4 men?

Question Six ( 3 marks )

Consider the letters of the word TEMPERATURE.

a) In how many different ways can the letters be arranged? 1

b) What is the probability that the word formed begins with a P and that the two R's are next to each other? 2

Question Seven ( 5 marks )

PIN numbers are 4 digit numbers using any of the digits from 0 to 9. However, they must start with a non-zero digit and digits can be repeated.

a) What is the probability that a particular PIN number has at least one 9 among its digits? 3

b) 5 people are randomly given PIN numbers. What is the probability that exactly 3 of them have at least one 9 as one of the digits of their PINnumbers? ( Answer correct to 2 decimal places ) 2

END OF ASSESSMENT TASK

# SOLUTIONS TO YR 12

# EXTENSION I MATHS

# TASK 2 2007

## Question One

a) $T_{k+1} = {}^{11}C_k\, 5^{11-k} (-2)^k x^{11-k}$ ← 1 mark

Coeff. of $x^5$ when $k = 6$

$\therefore C_5 = {}^{11}C_6\, 5^5 (-2)^5$

$= -46\ 200\ 000$ ← 1 mark

b) $(x + \frac{3}{x})^4 (x-2)^5$

$= \left(\frac{x^2+3}{x}\right)^4 (x-2)^5$

$= \frac{1}{x^4} (x^2+3)^4 (x-2)^5$

1 mark for correct general terms

$T_{k+1} = {}^4C_k\, x^{8-2k} 3^k$

$T_{k+1} = {}^5C_k\, x^{5-k} (-2)^k$

when $k=4$ $T_5 = {}^4C_4\, 3^4$ when $k=0$ $T_1 = {}^5C_0\, x^5$

when $k=3$ $T_4 = {}^4C_3\, 3^3 x^2$ when $k=2$ $T_3 = {}^5C_2 (-2)^2 x^3$

when $k=2$ $T_3 = {}^4C_2\, 3^2 x^4$ when $k=4$ $T_5 = {}^5C_4 (-2)^4 x$

Now coeff. of $x^5$ in exp. of $(x^2+3)^4(x-2)^5$ will be the same as coeff of $x$ in exp. of $\frac{1}{x^4}(x^2+3)^4(x-2)^5$

ie. Coeff. of $x$ $= 3^4 + {}^4C_3\, 3^3\, {}^5C_2 (-2)^2 + {}^4C_2\, 3^2\, {}^5C_4 (-2)^4$

$= 8721$ ← 1 mark

## Question Two

1 mark for correct forms

a) $$\frac{{}^7C_k}{{}^7C_{k-1}} = \frac{7!}{k!(7-k)!} \times \frac{(k-1)!(8-k)!}{7!}$$

$$= \frac{7!}{k(k-1)!(7-k)!} \times \frac{(k-1)!(8-k)(7-k)!}{7!}$$

$$= \frac{8-k}{k}$$

1 mark for correctly deriving result

b) Consider the exp. of $(3x+7)^7$

Now $T_{k+1} = {}^7C_k \, 3^{7-k} \, 7^k \, x^{7-k}$

and $T_k = {}^7C_{k-1} \, 3^{8-k} \, 7^k \, x^{8-k}$

$$\therefore \frac{C_{k+1}}{C_k} = \frac{{}^7C_k \, 3^{7-k} \, 7^k}{{}^7C_{k-1} \, 3^{8-k} \, 7^k}$$

$$= \frac{8-k}{k} \cdot \frac{7}{3} \quad \text{(from part (a) above)}$$

$$= \frac{56-7k}{3k}$$

1 mark for correct to this result

Now Greatest coeff. when $\frac{C_{k+1}}{C_k} > 1$

ie $\frac{56-7k}{3k} > 1$

$\therefore k < 5\,^3/_5$ ← 1 mark

$\therefore$ Greatest coeff. when $k = 5$

continued next page

But in the exp. of $(3x-7)^7$, the coeff. when $k=5$ is negative ← 1 mark for correctly disregarding $k=5$ with reason.

$\therefore$ G.C. when $k=4$ or $k=6$

Test $k=4$

${}^7C_4 \, 3^3 \, 7^4 = 2\ 268\ 945$ ← 1 mark for checking both $k=4$ and $k=6$

Test $k=6$

${}^7C_6 \, 3^1 \, 7^6 = 2\ 470\ 629$

$\therefore$ G.C. is $2\ 470\ 629$ (ie. when $k=6$) ← 1 mark for correct greatest coeff.

## Question Three

a) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$ ← 1 mark

b) Differentiate both sides:

ie. $n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + \dots + n\,{}^nC_n x^{n-1}$ ← 1 mark

let $x = 2$

$\therefore n(3^{n-1}) = {}^nC_1 + 2^2\,{}^nC_2 + 2^3 . 3\,{}^nC_3 + n\,2^{n-1}\,{}^nC_n$ ← 1 mark

$\therefore n(3^{n-1}) = n + {}^nC_2\, 2^2 . 3 + {}^nC_3\, 2^3 . 4 + \dots + {}^nC_n\, n\, 2^{n-1}$

$$\therefore n(3^n - 1) - n = \sum_{r=0}^{n-2} {}^nC_{r+2}\, 2^{r+1} (r+2)$$

$$\therefore n\left[3^{n-1} - 1\right] = \sum_{r=0}^{n-2} {}^nC_{r+2}\, 2^{r+1} (r+2)$$

1 mark for correctly deriving

## Question Four

$3!\,4!$ ← No. of ways of placing women among the already seated men

no. of ways of seating 4 men around a circular table

1 mark

Total ways without restrictions $= 6!$

$\therefore \text{Prob}(M + W \text{ alternate}) = \frac{3!\,4!}{6!}$

$= \frac{1}{5}$ ← 1 mark

## Question Five

$P(4 \text{ men } 2 \text{ women}) = \frac{{}^7C_4 \, {}^5C_2}{{}^{12}C_6} = \frac{350}{924}$

$P(5 \text{ men } 1 \text{ woman}) = \frac{{}^7C_5 \, {}^5C_1}{{}^{12}C_6} = \frac{105}{924}$

$P(\text{all } 6 \text{ men}) = {}^7C_6 / {}^{12}C_6 = \frac{7}{924}$

2 marks (1 mark deducted for each error)

$\therefore \text{Total prob(at least 4 men)} = 462/924$

$= \frac{1}{2}$ ← 1 mark

## Question Six

a) $\frac{11!}{2!\,3!\,2!} = \frac{39916800}{24}$

$= \frac{9979200}{6}$

$= 1663200$ ← 1 mark

## Question Six continued

b) No. of ways beg. with P with 2 R's together $= \frac{9!}{2!\,3!}$ ← 1 mark

$= 30240$

$\therefore P(\quad) = \frac{30240}{1663200}$

$= \frac{1}{55}$ ← 1 mark

## Question Seven

a) Total no. of possible PIN no.s $= 9 \times 10 \times 10 \times 10$

$= 9000$ ← 1 mark

Prob.(at least 1 9) $= 1 - P(\text{no } 9\text{'s})$ ← 1 mark

$= 1 - \left(\frac{8}{9} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right)$

$= 1 - \frac{5832}{9000}$

$= \frac{44}{125}$ ← 1 mark

b) $P(\text{exactly } 3) = {}^5C_3 \left(\frac{44}{125}\right)^3 \left(\frac{81}{125}\right)^2$ ← 1 mark

$= 0.183137804$

$= 0.18 \text{ (2dp)}$ ← 1 mark for correct prob. (2dp) only if binomial prob. used.