2 UNIT TEST NUMBER 2
1996
Series and Applications - Probability.

QUESTION 1. (12 marks)   Marks

(a) (i)   How many terms are there in the series $-7, -2, 3, ..., 123$?   4   *(i)* $n = 27$

    (ii)   What is the sum of this series?   *(ii)* $S_{27} = 156$

(b) Find x if the first three terms of a geometric sequence are: x, $x+3$, $x+12$.   3 $x = 1\frac{1}{2}$

(c) The limiting sum of a G.P. is -1, and the common ratio is $(k+1)$.   5   *(i)* $-2 < k <$

    (i)   For what values of k does the G.P. have a limiting sum?   *(ii)* $T_1 = k$

    (ii)   Find the first three terms (in terms of k). $T_2 = k(k+$

                            $T_3 = k(k+$

QUESTION 2. (11 marks)

(a) When Mark was born his father decided to invest $1 000 on each of his birthdays,   6
and when Mark reached 21 years of age he would receive the total investment.
The first deposit of $1 000 was made on Mark's first birthday and the last was
made on his 21st birthday. Interest would compound at the end of each year, at
9% p.a., for any money invested.

    (i)   How much would Mark receive on his 21st birthday?   *(i)* $1000 + 1000(1.09)$

                                           $\dfrac{[(1.09)^{20} - 1]}{0.09}$

    (ii)   On Mark's 18th birthday he convinced his father to pay him the $1 000 $= \$56\,764.$
rather than deposit it into the investment. What would Mark now receive on
his 21st birthday?   *(ii)* $\$55\,469.50$

(b) A student borrows $20 000 at $1\frac{1}{2}$% per month reducible interest for her University   5
fees and pays it off in equal monthly instalments.

    (i)   Let $\$A_3$ be the amount owing after 3 months, and $\$M$ be the monthly $A_3 = 20\,913.57 -$
instalment. Write down an expression for $\$A_3$. $M(3.045)$

    (ii)   What should her instalments ($\$M$) be in order to pay off the loan at the end
of 10 years? $M = \$360.37$

QUESTION 3. (17 marks)   Marks

(a) In a large bin full of green and yellow marbles, the green and yellow marbles   **2**
     occur in the ratio of 5 to 3 respectively. If two marbles are chosen from the bin,
     find the probability that there is one of each colour. $\frac{15}{32}$

(b) From the numbers 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, a number is selected at   **2**
     random. If E is the result "an even number is selected" and G is the result "a
     number greater than 10 is selected", find $P(E \cup G)$. $\frac{3}{4}$

(c) A die whose faces are numbered 1, 2, 3, 4, 5, 6 is tossed with another die whose   **5**
     faces are numbered 1, 2, 3, 4, 6, 6.

   (i)   What is the probability that at least one 6 appears on the uppermost faces. $\frac{4}{9}$

   (ii) What is the probability that the sum of the numbers on the uppermost faces
         is 8. $\frac{5}{36}$

   (iii) What is the probability that the sum of the numbers on the uppermost faces is
         greater than 8 if a '6' appears on at least one of the dice. $\frac{2}{9}$

(d) Helen buys two tickets in a raffle in which 100 tickets are sold. Three different   **6**
     tickets are drawn for the first three prizes. With the aid of a tree diagram find the
     probability that Helen:

   (i)   wins first prize; $\frac{1}{50}$

   (ii) wins at least one prize (correct to 4 decimal places); 0.0594

   (iii) wins exactly one prize (correct to 4 decimal places). $\left[\frac{2}{100} \times \frac{98}{99} \times \frac{97}{98} + \frac{98}{100} \times \frac{2}{99} \times \frac{97}{98} + \right.$
         $\left. \frac{98}{100} \times \frac{97}{99} \times \frac{2}{98} \right] = \cancel{0.0590}$

         $\underline{0.0588}$

(e) During the Gulf war three gun posts were set up to shoot down enemy aircraft   **2**
     which flew overhead. The probability of shooting down an aircraft by the first
     gun is $\frac{1}{2}$, by the second gun is $\frac{3}{5}$ and by the third gun is $\frac{2}{3}$. Find the probability
     that an aircraft would be shot down if it flew overhead.

     $P(F) = First\ gun = \frac{1}{2}$

     $1 - P(\tilde{F}.\tilde{S}.\tilde{T})$

     $= 1 - \left(\frac{1}{2}.\frac{2}{5}.\frac{1}{3}\right)$

     $= 1 - \frac{1}{15} = \frac{14}{15}.$