

EXT 1: DERIVATIVES / INTEGRALS OF LOGS & EXPONENTIALS
& INTEGRATION BY SUBSTITUTION:
MATHEMATICS C

DIFFERENTIATION AND INTEGRATION MISCELLANEOUS

Instructions

- (1) Draw up 3 columns and label them
 $f(x)$ $f'(x)$ $\int f(x)dx$
- (2) Copy the question out under an $f(x)$ column and number the question.
- (3) Under the $f'(x)$ column, differentiate $f(x)$.
- (4) Under the $\int f(x)dx$ column, integrate $f(x)$.
- (5) Show all necessary working.
- (6) Rule a line across at the end of each question.

Questions (These refer to $f(x)$)

- | | | |
|-------------------------------|--------------------------------------|------------------------------|
| 1. x | 2. 7 | 3. $\sqrt[5]{x^6}$ |
| 4. \sqrt{x} | 5. 0 | 6. $5x^{-1/5}$ |
| 7. $6x^2 - 7x + 3$ | 8. $\frac{8}{x^2}$ | 9. $\frac{1}{x}$ |
| 10. $\frac{1}{3x}$ | 11. e^x | 12. e^{-x} |
| 13. $(2x+7)^7$ | 14. $\frac{1}{4-5x}$ | 15. $\frac{1}{(7x+5)^4}$ |
| 16. $\frac{1}{9x-2}$ | 17. $\frac{1}{\sqrt{7x-3}}$ | 18. e^{4x+3} |
| 19. $\sqrt{4x+3}$ | 20. $\frac{5x^4}{x^5+1}$ | 21. $3x^2(x^3+1)^6$ |
| 22. $\frac{e^{2x}}{e^{2x}+3}$ | 23. $(e^x - e^{-x})(e^x + e^{-x})^7$ | 24. $\frac{x+1}{(x^2+2x)^3}$ |

Miscellaneous Differentiation

Differentiate:

- | | |
|--------------------------------------|-----------------------------------|
| 1. $2e^{-x^2}$ | 2. $\log_e(2x^2 + 5)$ |
| 3. $(3x^2 + 2x)^7$ | 4. $(3x^2 + 1)(4x + 3)^5$ |
| 5. $x^2 - 2\sqrt{x}$ | 6. $\log_e 2\sqrt{x}$ |
| 7. ex^3 | 8. $\frac{x^2}{x-2}$ |
| 9. $\log_e e^{2x^2}$ | 10. $\ln[x(x+1)]$ |
| 11. $e^{x^3-2x^2}$ | 12. $\frac{x^2 + 3x - 4}{x}$ |
| 13. $\log_e \frac{1-x^2}{1+x^2}$ | 14. $\frac{1}{\sqrt{x-2}}$ |
| 15. $\left(x + \frac{1}{x}\right)^3$ | 16. $(x^2 - 1)^{3/2}(x^2 + 3x)^4$ |
| 17. $x^2 e^x$ | 18. $x^n + x^{2n}$ |
| 19. $(\sqrt{x} + 2)^9$ | 20. $\frac{\sqrt{1-x}}{1+x}$ |
| 21. $\ln\sqrt{x-1}$ | 22. $x^2 \log_e x$ |
| 23. $\sqrt{\frac{1+x}{1-x}}$ | 24. $(1+x)^3 e^{2x}$ |
| 25. $(x^3 - 1)\log_e x$ | 26. $\frac{e^{-2x} - e^x}{e^x}$ |
| 27. $\log_e \sqrt{\frac{1+x}{1-x}}$ | 28. 2^x |

Miscellaneous Integration

1. (i) Find the derivative of $\log_e(x^2 - 5x + 7)$ and hence evaluate

$$\int_3^2 \frac{2x-5}{x^2-5x+7} dx$$

- (ii) Obtain $\frac{d}{dx}(e^x \log_e x)$ and hence evaluate $\int_1^2 e^x \left(\frac{1}{x} + \log_e x\right) dx$ giving the result in irrational form.

2. (i) Differentiate $\log_e(\log_e x)$, and evaluate $\int_e^{e^2} \frac{dx}{x \log_e x}$.
- (ii) Find $\frac{d}{dx}(\log_e x)^2$, and hence evaluate $\int_1^e \frac{\log_e x}{x} dx$.
3. (i) Find $\frac{d}{dx}(e^{x^2})$ and hence evaluate $\int_0^1 xe^{x^2} dx$, giving the result correct to 2 decimal places.
- (ii) Obtain $\frac{d}{dx}(xe^x - e^x)$ and hence calculate $\int_0^1 xe^x dx$.
answer in exact form.
4. Find $\int \frac{x^2 dx}{\sqrt{x^3 - 1}}$
- (a) Using the substitution $u = x^3 - 1$,
- (b) Using the substitution $u = \sqrt{x^3 - 1}$.
5. Evaluate $\int_e^{e^2} \frac{dx}{x \ln x}$ using the substitution $u = \ln x$.
6. Shade the area represented by the integral $\int_0^1 \sqrt{1 - x^2} dx$ and hence evaluate the integral.
7. If $\int_{\frac{1}{a}}^a \frac{1}{t^2} dt = \frac{1}{\sqrt{2}}$ and $a > 0$ find a .
8. Show that $\frac{x+5}{x+3} = 1 + \frac{2}{x+3}$ and hence evaluate $\int_0^1 \frac{x+5}{x+3} dx$ exactly.
9. Use the standard integral $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) + c$ to evaluate
- $$\int_{-1}^1 \frac{3}{\sqrt{2x^2 + 2}} dx$$
10. Differentiate $x^3 \ln x$ and hence find an indefinite integral for $x^2 \ln x$

Differentiation and Integration Miscellaneous Answers

	$f'(x)$	$\int f(x)dx$
1.	1	$\frac{x^2}{2} + k$
2.	0	$7x + k$
3.	$\frac{6}{5}x^{1/5}$	$\frac{5}{11}x^{11/5} + k$
4.	$\frac{1}{2\sqrt{x}}$	$\frac{2}{3}x^{3/2} + k$
5.	0	k
6.	$-x^{-6/5}$	$\frac{25}{4}x^{4/5} + k$
7.	$12x - 7$	$2x^3 - \frac{7}{2}x^2 + 3x + k$
8.	$\frac{-16}{x^3}$	$-8x^{-1} + k$
9.	$-\frac{1}{x^2}$	$\log_e x + k$
10.	$-\frac{1}{3x^2}$	$\frac{1}{3}\log_e x + k$
11.	e^x	$e^x + k$
12.	$-e^{-x}$	$-e^{-x} + k$
13.	$14(2x+7)^6$	$\frac{1}{16}(2x+7)^8 + k$
14.	$\frac{5}{(4-5x)^2}$	$-\frac{1}{5}\log_e 4-5x + k$
15.	$-\frac{28}{(7x+5)^5}$	$-\frac{1}{21(7x+5)^3} + k$
16.	$-\frac{9}{(9x-2)^2}$	$\frac{1}{9}\log_e 9x-2 + k$
17.	$\frac{-7}{2(7x-3)\sqrt{7x-3}}$	$\frac{2}{7}\sqrt{7x-3} + k$
18.	$4e^{4x+3}$	$\frac{1}{4}e^{4x+3} + k$
19.	$\frac{2}{\sqrt{4x+3}}$	$\frac{1}{6}(4x+3)\sqrt{4x+3} + k$

20. $\frac{20x^3 - 5x^8}{(x^5 + 1)^2}$ $\log_e |x^5 + 1| + k$
21. $6x(10x^3 + 1)(x^3 + 1)^5$ $\frac{1}{7}(x^3 + 1)^7 + k$
22. $\frac{6e^{2x}}{(e^{2x} + 3)^2}$ $\frac{1}{2} \log_e (e^{2x} + 3) + k$
23. $(e^x + e^{-x})^6 (8e^{2x} - 12 + 8e^{-2x})$ $\frac{1}{8}(e^x + e^{-x})^8 + k$
24. $\frac{-(5x^2 + 10x + 6)}{x^4(x+2)^4}$ $\frac{-1}{4x^2(x+2)^2} + k$

Miscellaneous Differentiation (Answers)

- | | |
|--|---|
| 1. $-4xe^{-x^2}$ | 2. $\frac{4x}{2x^2 + 5}$ |
| 3. $14x^6(3x+1)(3x+2)^6$ | 4. $2(4x+3)^4(42x^2+9x+10)$ |
| 5. $2x - \frac{1}{\sqrt{x}}$ | 6. $\frac{1}{2x}$ |
| 7. $3ex^2$ | 8. $\frac{x(x-4)}{(x-2)^2}$ |
| 9. $4x$ | 10. $\frac{1}{x} + \frac{1}{x+1}$ |
| 11. $x(3x-4)e^{x^2(x-2)}$ | 12. $1 + \frac{4}{x^2}$ |
| 13. $\frac{-4x}{(1-x^2)(1+x^2)}$ | 14. $\frac{1}{2(x-2)^{\frac{3}{2}}}$ |
| 15. $3\left(x + \frac{1}{x}\right)^2 \left(1 - \frac{1}{x^2}\right)$ | 16. $\sqrt{x^2-1}(x^2+3x)^3(11x^3+21x^2-8x-12)$ |
| 17. $xe^x(2+x)$ | 18. $nx^{n-1}(1+2x^n)$ |
| 19. $\frac{9}{2\sqrt{x}}(\sqrt{x}+2)^8$ | 20. $\frac{x-3}{2\sqrt{1-x}(1+x)^2}$ |
| 21. $\frac{1}{2(x-1)}$ | 22. $x(1+2\log_e x)$ |
| 23. $\frac{1}{\sqrt{(1+x)(1-x)^3}}$ | 24. $(1+x)^2 e^{2x}(5+2x)$ |
| 25. $x^2(3\log_e x + 1) - \frac{1}{x}$ | 26. $-3e^{-3x}$ |
| 27. $\frac{1}{1-x^2}$ | 28. $2^x \ln 2$ |

Miscellaneous Integration Answers

1. (i) 0 (ii) $e^x \left(\frac{1}{x} + \log_e x \right); e^2 \log_e 2$
2. (i) $\frac{1}{x \log_e x}; \log_e 2 \approx 0.6931$ (ii) $\frac{2 \log_e x}{x}; \frac{1}{2}$
3. (i) $2xe^{x^2}; \frac{1}{2}(e-1) \approx 0.86$ (ii) $xe^x; 1$
4. $\frac{2}{3}\sqrt{x^3-1} + c$
5. $\ln 2$
6. $\frac{\pi}{4}$
7. $a = \sqrt{2}$
8. $1 + 2 \ln \frac{4}{3}$
9. $\frac{3}{\sqrt{2}} \ln \left(\frac{\sqrt{2}+1}{\sqrt{2}-1} \right)$
10. $\frac{1}{3} \left(x^3 \ln x - \frac{x^3}{3} \right)$