

Mathematics C
Indefinite Integration Involving Logarithmic Functions

Syllabus References: 12-1(c), 12-2(c)

Suggested Textbook Exercises:

12-1/15,29,55,67

12-2/5,13,17,31

Review: Differentiation of Logarithmic Functions	Indefinite Integration Involving Logarithmic Functions
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + c$
$\frac{d}{dx}\{\ln(ax+b)\} = \frac{a}{ax+b}$	$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln ax+b + c \quad (a \neq 0)$
$\frac{d}{dx}\{\ln f(x)\} = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$

Examples

1. Find the following indefinite integrals:

(a) $\int \left(2 + \frac{1}{x}\right) dx$	(b) $\int \frac{3x^4 + 4x}{x^2} dx$
(c) $\int \left(4e^x - \frac{5}{3x}\right) dx$	(d) $\int \frac{3 - \sqrt{x}}{x} dx$
(e) $\int \frac{1}{3x-2} dx$	(f) $\int \frac{2}{4-x} dx$

2. Find the following indefinite integrals:

(a) $\int \frac{2x}{x^2+5} dx$	(b) $\int \frac{6m+3}{m^2+m-5} dm$
(c) $\int \frac{x dx}{3-2x^2}$	(d) $\int \frac{e^x}{1+e^x} dx$
(e) $\int \frac{e^{3x} + e^{-3x}}{e^{3x} - e^{-3x}} dx$	(f) $\int \frac{\sqrt{t}}{t\sqrt{t}+5} dt$
(g) $\int \frac{1}{x \ln x} dx$	(h) $\int \frac{x dx}{(2x^2-3)^3}$

3. Show that $\frac{1}{(x+2)(x+3)} = \frac{1}{x+2} - \frac{1}{x+3}$ and hence show that

$$\int \frac{dx}{(x+2)(x+3)} = \ln \left| \frac{x+2}{x+3} \right| + c.$$

4. Find $\frac{d}{dx}(x \ln x - x)$ and hence find $\int \ln x \, dx$.

5. The gradient of the tangent at any point (x, y) on the curve $y = f(x)$ is given by $f'(x) = \left(2x + \frac{1}{x^2}\right)^2$. If the curve passes through $(1, -1)$, find the equation of the curve.

6. Given $f'(x) = -5f(x)$ and $f(0) = 8$, find $f(x)$.

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$$\begin{aligned} 1, a) \quad & \int \left(2 + \frac{1}{x}\right) dx \\ & = \underline{2x + \ln|x| + C} \quad \checkmark \end{aligned}$$

$$\begin{aligned} b) \quad & \int \frac{3x^4 + 4x}{x^2} dx \\ & = \int 3x^2 + \frac{4}{x} dx \\ & = \underline{x^3 + 4\ln|x| + C} \quad \checkmark \end{aligned}$$

$$\begin{aligned} c) \quad & \int \left(4e^x - \frac{5}{3x}\right) dx \\ & = \underline{4e^x - \frac{5}{3}\ln|x| + C} \quad \checkmark \end{aligned}$$

$$\begin{aligned} d) \quad & \int \frac{3 - \sqrt{x}}{x} dx \\ & = \int \frac{3}{x} - x^{\frac{1}{2}-1} dx \\ & = \int \frac{3}{x} - x^{-\frac{1}{2}} dx \\ & = \underline{3\ln|x| - 2\sqrt{x} + C} \quad \checkmark \end{aligned}$$

$$\begin{aligned} e) \quad & \int \frac{1}{3x-2} dx \\ & = \underline{\frac{1}{3}\ln|3x-2| + C} \quad \checkmark \end{aligned}$$

$$\begin{aligned} f) \quad & \int \frac{2}{4-x} dx \\ & = \underline{2\ln|4-x| + C} \quad \checkmark \end{aligned}$$

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$$2. a) \int \frac{2x}{x^2+5} dx$$

$$= \underline{\underline{\ln|x^2+5| + C}} \checkmark$$

$$g) \int \frac{1}{x \ln x} dx$$

$$= \int \frac{1/x}{\ln x} dx \checkmark$$

$$= \underline{\underline{\ln|\ln|x|| + C}} \checkmark$$

$$b) \int \frac{6m+3}{m^2+m-5} dx$$

$$= 3 \int \frac{2m+1}{m^2+m-5} dx$$

$$= 3 \underline{\underline{\ln|m^2+m-5| + C}} \checkmark$$

$$h) \int \frac{x}{(2x^2-3)^3} dx$$

$$= \int x (2x^2-3)^{-3} dx$$

$$c) \int \frac{x}{3-2x^2} dx$$

$$= \frac{-1}{4} \int \frac{-4x}{3-2x^2} dx$$

$$= \underline{\underline{-\frac{1}{4} \ln|3-2x^2| + C}} \checkmark$$

let $2x^2-3 = u$
 $du = 4x \cdot dx \checkmark$

$$\therefore = \frac{1}{4} \int 4x (2x^2-3)^{-3} dx$$

$$= \frac{1}{4} \int u^{-3} du$$

$$d) \int \frac{e^x}{1+e^x} dx$$

$$= \underline{\underline{\ln|1+e^x| + C}} \checkmark$$

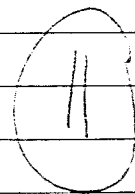
$$= \frac{1}{4} \cdot \frac{-1}{2u^2} + C$$

$$= \underline{\underline{-\frac{1}{8(2x^2-3)^2} + C}} \checkmark$$

$$e) \int \frac{e^{3x} + e^{-3x}}{e^{3x} - e^{-3x}} dx$$

$$= \frac{1}{3} \int \frac{3e^{3x} + 3e^{-3x}}{e^{3x} - e^{-3x}} dx$$

$$= \underline{\underline{\frac{1}{3} \ln|e^{3x} - e^{-3x}| + C}} \checkmark$$



$$f) \int \frac{\sqrt{t}}{t\sqrt{t}+5} dt$$

$$= \int \frac{t^{1/2}}{t^{3/2}+5} dt \checkmark$$

$$= \frac{2}{3} \int \frac{3/2 t^{1/2}}{t^{3/2}+5} dt$$

$$= \underline{\underline{\frac{2}{3} \ln|t\sqrt{t}+5| + C}} \checkmark$$

$$3, \quad \text{RHS} = \frac{1}{(x+2)} - \frac{1}{(x+3)}$$

$$= \frac{x+3 - x-2}{(x+2)(x+3)}$$

$$= \frac{1}{(x+2)(x+3)} = \underline{\underline{\text{LHS}}}$$

$$\begin{aligned} \int \frac{dx}{(x+2)(x+3)} dx &= \int \frac{1}{x+2} - \frac{1}{x+3} dx \\ &= \ln|x+2| - \ln|x+3| + C \\ &= \underline{\underline{\ln \left| \frac{x+2}{x+3} \right| + C}} \quad \checkmark \end{aligned}$$

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$$4, \quad \frac{d}{dx} (x \ln x - x)$$

$$= x \frac{1}{x} + \ln x - 1$$

$$= \underline{\underline{\ln x}} \quad \checkmark$$

$$\begin{aligned} \int \ln x dx &= \underline{\underline{x \ln x - x + C}} \quad \checkmark \end{aligned}$$

$$5, \quad f'(x) = 4x^2 + 4\frac{1}{x} + \frac{1}{x^4}$$

$$= 4x^2 + 4x^{-1} + x^{-4} \quad \checkmark$$

$$f(x) = \frac{4}{3}x^3 + 4\ln x - \frac{1}{3x^3} + C$$

$$\text{Put } x=1, \quad f(1) = -1$$

$$\therefore -1 = \frac{4}{3} + 4(0) - \frac{1}{3} + C$$

$$C = -2$$

$$\therefore f(x) = \underline{\underline{\frac{4}{3}x^3 + 4\ln x - \frac{1}{3x^3} - 2}} \quad \checkmark$$

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$$f'(x) = -5f(x)$$

$$\frac{f'(x)}{f(x)} = -5$$

$$\therefore \int \frac{f'(x)}{f(x)} = -5 \int dx$$

$$\ln|f(x)| = -5x + C$$

when $x=0, f(x)=8$.

$$\Rightarrow \ln 8 = C$$

$$\therefore C = \ln 8$$

$$\therefore \underline{\underline{f(x) = e^{-5x + \ln 8}}}$$