INTEGRATION – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics. Syllabus reference: 11.1 – 11.4.

1. Find the primitives of:

(i) $\sqrt[3]{x^5}$ (ii) $\dfrac{5}{x^3}$

(iii) $(1-x)^8$ (iv) $\sqrt{3x-1}$

2. Find the area bounded by the curve $y = 3x - x^2$ and the x–axis.

3. Complete the following table:

x	0	1	2	3	4
$f(x) = \dfrac{1}{x+1}$					

Hence evaluate $\displaystyle\int_0^4 \frac{dx}{x+1}$ using 5 function values of Simpson's Rule.

4. Find the area enclosed between the parabola $y = x^2 + 2x$ and the straight line $y = x$.

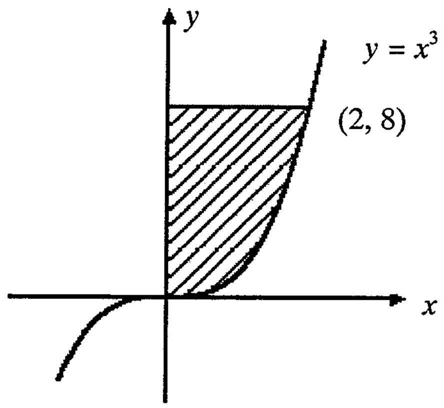
5. If $f''(x) = 6x - 8$ and $f'(0) = 6$, $f(1) = 1$, find $f(x)$.

6. Evaluate:

(i) $\displaystyle\int_1^2 2x\left(x^2+3\right)dx$

(ii) $\displaystyle\int_{-1}^1 \sqrt{5x+8}\,dx$

7.



Find the area of the shaded region.

8. The area bounded by the parabola $y = 9 - x^2$ and the x–axis is rotated about the x–axis. Find the volume generated.

9. If $y = \sqrt{1-4x^2}$,

(a) find $\dfrac{dy}{dx}$.

(b) Hence evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{x\,dx}{\sqrt{1-4x^2}}$.

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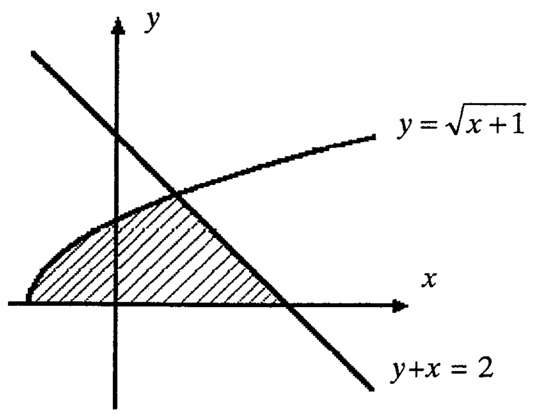


Calculate the area of the shaded region.

Integration Worksheet.

Angelina.

1 (i) $\int x^{\frac{5}{3}}\,dx = x^{\frac{8}{3}} \cdot \frac{3}{8} + c$ ✓

$= \frac{3\sqrt[3]{x^8}}{8} + c$ ✓

(ii) $\int 5x^{-3}\,dx = \frac{5x^{-2}}{-2} + c$

$= \frac{-5}{2x^2} + c$ ✓

(iii) $\int (1-x)^8\,dx = \frac{(1-x)^9}{-9} + c$

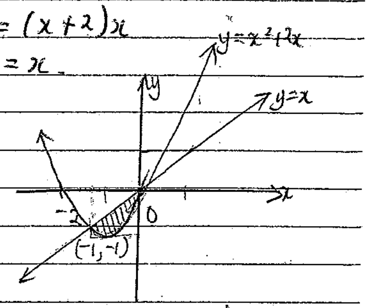
$= -\frac{(1-x)^9}{9} + c$ ✓

(iv) $\int (3x-1)^{\frac{1}{2}}\,dx = \frac{(3x-1)^{\frac{3}{2}}}{\frac{9}{2}} + c$

$= \frac{2\sqrt{(3x-1)^3}}{9} + c$ ✓

4. $y = (x+2)x$

$y = x$



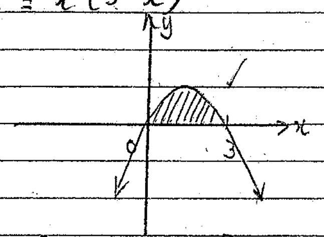
$A = \left|\int_{-1}^{0} x^2+2x-x\,dx\right|$ ✓

$= \left|\int_{-1}^{0} x^2+x\,dx\right|$

$= \left|\left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-1}^{0}\right|$ ✓

$= \left|\left(0 - \frac{1}{6}\right)\right| = \frac{1}{6}$ units^2

2. $y = x(3-x)$



✓

$A = \int_0^3 3x - x^2\,dx = \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$ ✓

$= 4\frac{1}{2}$ units^2 ✓

5. $f''(x) = 6x - 8$

$f'(x) = 3x^2 - 8x + C$ ✓

$f'(0) = 6 = C$

$\therefore f'(x) = 3x^2 - 8x + 6$ ✓

$f(x) = x^3 - 4x^2 + 6x + c$

$f(1) = 1 = 1 - 4 + 6 + c$

$\therefore c = -2$ ✓

$\therefore f(x) = x^3 - 4x^2 + 6x - 2$ ✓

6. (i) $\int_1^2 \underbrace{2x(x^2+3)}_{\text{Quicker to simplify}}\,dx$. $\int_1^2 2x^3+6x\,d$

For x^2+3, $\frac{d}{dx} = 2x$.

This is Ext. I work ✓ $\int_1^2 2x(x^2+3)\,dx$

$= \left[\frac{(x^2+3)^2}{2}\right]_1^2$

$= \left(\frac{49}{2} - \frac{16}{2}\right)$

$= 16\frac{1}{2}$ ✓

3.

x	0	1	2	3	4
$f(x)$	1.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$\int_0^4 \frac{1}{x+1}\,dx \doteq \frac{1}{3}\left(1 + \frac{1}{5} + 4\left(\frac{1}{2} + \frac{1}{4}\right) + 2\left(\frac{1}{3}\right)\right)$

$\doteq 1\frac{28}{45}$ ✓

(ii) $\int_{-1}^{1} (5x+8)^{\frac{1}{2}}\,dx = \left[\frac{(5x+8)^{\frac{3}{2}}}{\frac{15}{2}}\right]_{-1}^{1}$

$= \left[\frac{2(5x+8)^{\frac{3}{2}}}{15}\right]_{-1}^{1}$ ✓

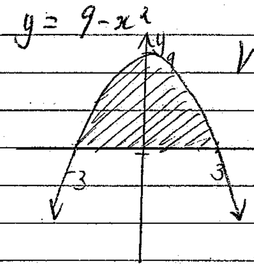
$= \left(\frac{2}{15} \cdot 13\sqrt{13}\right) - \left(\frac{2}{15} \cdot 3\sqrt{3}\right)$ ✓

$= 5.56$ to 2dp ✓

7. $A = \int_0^8 y^{\frac{1}{3}}\,dy = \left[\frac{3}{4}y^{\frac{4}{3}}\right]_0^8$ ✓

$= 12$ units^2 ✓

8. $y = 9 - x^2$



$V = \pi\int y^2\,dx$

$= \pi\int_{-3}^{3} 81 - 18x^2 + x^4\,dx$ ✓

$= \pi\left[81x - 6x^3 + \frac{1}{5}x^5\right]_{-3}^{3}$ ✓

$= \pi\left(129\frac{3}{5} + 129\frac{3}{5}\right)$

$= 259\frac{1}{5}\pi$ units^3 ✓

9. $y = (1-4x^2)^{\frac{1}{2}}$

(a) $y' = \frac{1}{2} \cdot -8x(1-4x^2)^{-\frac{1}{2}}$

$= \frac{-4x}{\sqrt{1-4x^2}}$ ✓

(b) $\frac{1}{4}\int_0^{\frac{1}{2}} \frac{-4x}{\sqrt{1-4x^2}}\,dx$.

$\int \frac{-4x}{\sqrt{1-4x^2}} = \sqrt{1-4x^2}$

$-\frac{1}{4}\int \frac{-4x}{\sqrt{1-4x^2}} = -\frac{1}{4}\sqrt{1-4x^2}$ ✓

$\therefore \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-4x^2}}\,dx = \left[-\frac{1}{4}\sqrt{1-4x^2}\right]_0^{\frac{1}{2}}$

$= \left(-\frac{1}{4}\sqrt{0} - \left(-\frac{1}{4}\right)\right)$

$= \frac{1}{4}$ ✓

10. $y = \sqrt{x+1}$

$x = 2 - y$

$\therefore y = \sqrt{3-y}$

$y^2 = 3 - y$

$y^2 + y - 3 = 0$

$y = \frac{-1 \pm \sqrt{13}}{2}$.

$-x + 2 = \sqrt{x+1}$

$x^2 - 4x + 4 = x + 1$

$x^2 - 5x + 3 = 0$ ✓

$x = \frac{5 \pm \sqrt{13}}{2}$. ✓

$\therefore A_I = \int_0^{\frac{5-\sqrt{13}}{2}} (x+1)^{\frac{1}{2}}\,dx$ ✓

$= \left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_0^{\frac{5-\sqrt{13}}{2}}$

$= 1.47$ (to 2dp) ✓

$A_{II} = \int_{\frac{5-\sqrt{13}}{2}}^{2} -x+2\,dx$

$= \left[-\frac{1}{2}x^2 + 2x\right]_{\frac{5-\sqrt{13}}{2}}^{2}$

$= 0.85$ to 2dp

$\therefore A_{total} = A_I + A_{II}$

$= 2.32$ units^2 (to 2dp)