

2 UNIT - INTEGRATION - WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Extension Mathematics. Syllabus reference: 11.1 - 11.5

1. Find the primitives of:

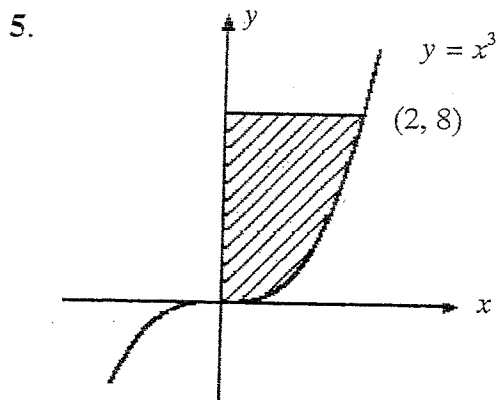
(i) $\sqrt[3]{x^5}$ (i) $\frac{5}{x^3}$

(iii) $(1-x)^8$ (iv) $\sqrt{3x-1}$

2. Find the area bounded by the curve $y = 3x - x^2$ and the x -axis.

3. Complete the following table:

x	0	1	2	3	4
$f(x) = \frac{1}{x+1}$					

Hence evaluate $\int_0^4 \frac{dx}{x+1}$ using 5 function values of Simpson's Rule.4. Find the area enclosed between the parabola $y = x^2 + 2x$ and the straight line $y = x$.

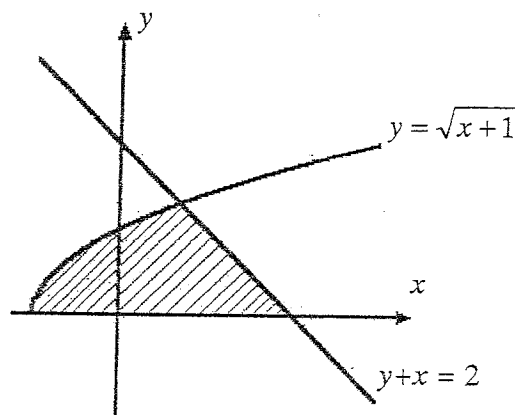
Find the area of the shaded region.

6. If $f''(x) = 6x - 8$ and $f'(0) = 6$, $f(1) = 1$, find $f(x)$.7. The area bounded by the parabola $y = 9 - x^2$ and the x -axis is rotated about the x -axis. Find the volume generated.†8. If $y = \sqrt{1 - 4x^2}$,

(a) find $\frac{dy}{dx}$.

(b) Hence evaluate $\int_0^{\frac{1}{2}} \frac{x dx}{\sqrt{1 - 4x^2}}$.

†9.

Calculate the area of the shaded region.
(Answer to 1 decimal place).

Q1

$$(i) y' = x^{\frac{5}{3}}$$

$$y = \frac{3x^{\frac{8}{3}}}{8} + C \checkmark$$

Q3

x	0	1	2	3	4
$f(x) = \frac{1}{x+1}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$$(ii) -y' = \frac{5}{x^3}$$

$$= 5(x)^{-3}$$

$$y = -\frac{5}{2}(x)^{-2} + C \checkmark$$

$$\int \frac{dx}{x+1}$$

$$A = \frac{1}{3} \left[\left(1 + \frac{1}{5}\right) + 4\left(\frac{1}{2} + \frac{1}{4}\right) + 2\left(\frac{1}{3}\right) \right]$$

$$= \frac{28}{45} U^2 \checkmark$$

$$(iii) y' = (1-x)^8$$

$$y = \frac{(1-x)^9}{9} + C \checkmark$$

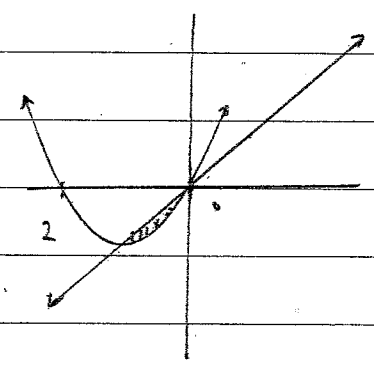
Q4

$$(iv) y' = \sqrt{3x-1}$$

$$= (3x-1)^{\frac{1}{2}}$$

$$y = \frac{2 \cdot \frac{2}{3} (3x-1)^{\frac{3}{2}}}{3 \times 3} + C$$

$$= \frac{2}{9} (3x-1)^{\frac{3}{2}} + C$$



$$y = x^2 + 2x$$

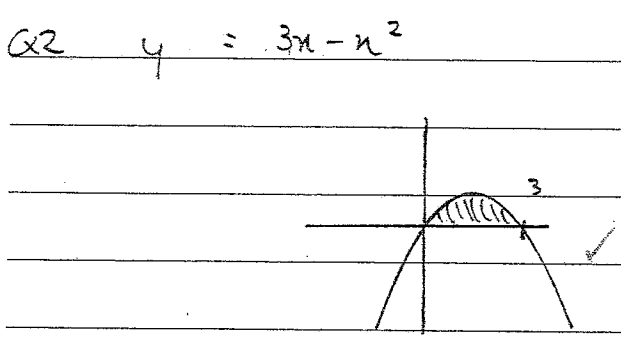
$$y = x$$

$$x = x^2 + 2x$$

$$0 = x^2 + x$$

$$= x(x+1) \checkmark$$

$$(-1, -1)$$



$$\int_0^3 3x - x^2 dx$$

$$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3 \right]_0^3 \checkmark$$

$$= \left(\frac{13\frac{1}{2}}{6} - 9 \right) - 0$$

$$= 4\frac{1}{2} U^2$$

$$\int_{-1}^0 -(x^2 + 2x) + (x) dx$$

$$= - \int_{-1}^0 x^2 + x dx$$

$$= - \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_{-1}^0 \checkmark$$

$$= \left(-\frac{1}{3} + \frac{1}{2} \right) = \frac{1}{6} U^2$$

Q5

-2-

$$A_1 = \int_0^2 x^3 dx \quad \therefore \text{shaded area} = \text{area rectangle} - A_1$$

$$= \left[\frac{1}{4} x^4 \right]_0^2 = 16 - 4$$

$$= 12 \checkmark$$

$$Q6 \quad f''(x) = 6x - 8$$

$$f'(0) = 6$$

$$f(1) = 1$$

$$f'(x) = 3x^2 - 8x + c \checkmark$$

$$f'(0) = 3 \cdot 0^2 - 8 \cdot 0 + c$$

$$f'(0) = 6$$

$$c = 6 \checkmark$$

$$f'(x) = 3x^2 - 8x + 6 \checkmark$$

$$f(x) = x^3 - 4x^2 + 6x + c \checkmark$$

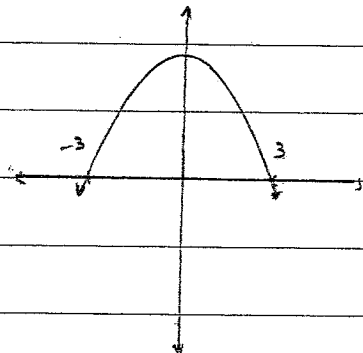
$$f(1) = 1$$

$$f(1) = (1)^3 - 4(1)^2 + 6(1) + c$$

$$c = -3 \checkmark$$

$$f(x) = x^3 - 4x^2 + 6x - 3 \checkmark$$

Q7



$$A = \int_{-3}^3 (9 - x^2) dx \checkmark$$

$$2 \left[9x - \frac{1}{3}x^3 \right]_0^3$$

$$\int_{-1}^{0.7} \sqrt{x+1} \, dx$$

$$= 18 \times 2 \checkmark$$

$$= \left[\frac{2}{3} (x+1)^{3/2} \right]_{-1}^{0.7} \checkmark$$

$$= 36 \, u^2 \checkmark$$

Q8 (a) $y = \sqrt{1-4x^2}$

$$= 1.477... - 0 \checkmark$$

$$y = (1-4x^2)^{1/2}$$

$$= 1.478 \text{ (3 dec. plc.)}$$

$$y' = \frac{1}{2} \times -8x(1-4x^2)^{-1/2}$$

$$\int_{0.7}^2 2-x \, dx$$

$$= -4x(1-4x^2)^{-1/2}$$

0.7

(b) $\int_0^{1/2} \frac{x}{\sqrt{1-4x^2}} \, dx \checkmark$

$$= \int [2-x]$$

$$= \left[2x - \frac{1}{2}x^2 \right]_{0.7}^2$$

$$= -\frac{1}{4} \int \frac{4x}{\sqrt{1-4x^2}} \, dx$$

$$= 2 - 1.155$$

$$= 0.845$$

$$= -\frac{1}{4} \left[\sqrt{1-4x^2} \right]_0^{1/2}$$

$$A_1 + A_2 = \text{Total Area}$$

$$= 2.323 \, u^2$$

$$= \left[0 - \left(-\frac{1}{4} \right) \right]$$

$$= \frac{1}{4} \, u^2 \checkmark$$

Q9 $y = \sqrt{x+1}$

$$y = 2-x$$

$$2-x = \sqrt{x+1}$$

$$(2-x)^2 = x+1 \checkmark$$

$$x^2 - 4x + 4 = x+1$$

$$x^2 - 5x + 3 = 0$$

$$x = \frac{5 \pm \sqrt{25 - (4 \cdot 1 \cdot 3)}}{2}$$

$$= \frac{5 \pm \sqrt{13}}{2} \checkmark$$