

TESTS FOR QUADRILATERALS – WORKSHEET

COURSE/LEVEL

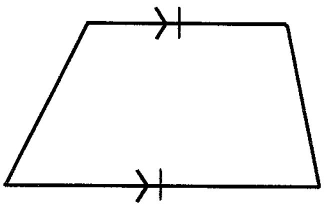
NSW Secondary High School Year 11 Preliminary Mathematics.

TOPIC

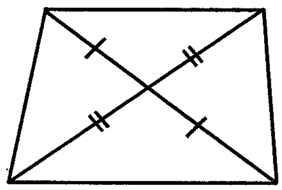
Plane Geometry: Tests for Quadrilaterals. (Syllabus Ref: 2.2)

1. Identify the type of quadrilaterals drawn below and state the test used. (Ignore the shape of the drawings.)

(a)



(b)



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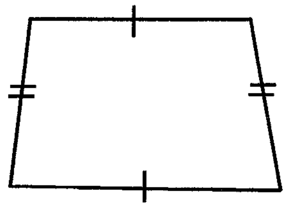

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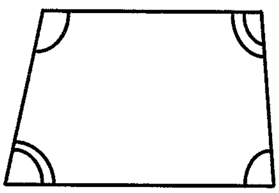

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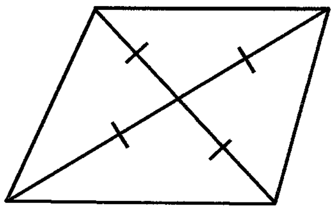

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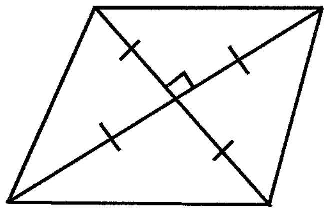


2. Draw diagrams to <u>disprove</u> the following statements that a quadrilateral is a parallelogram if

   (i) one pair of sides are parallel

   (ii) one pair of opposite sides are equal

   (iii) two pairs of sides are equal

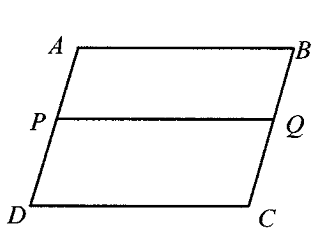
3. Provide a counterexample to <u>disprove</u> the following. (A counterexample is an example that disproves a general statement.)

   (i) A quadrilateral is a rhombus if each diagonal bisects the vertex angles through which it passes.

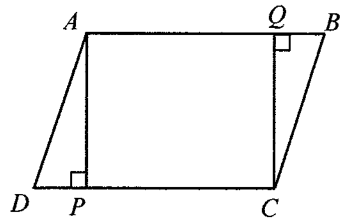
   (ii) A parallelogram is a square if the diagonals bisect each other at right angles.

   (iii) A quadrilateral is a parallelogram if a pair of cointerior angles are supplementary.

4. $ABCD$ is a parallelogram. P is the midpoint of AD, Q is the midpoint of BC. Show that $ABQP$ is a parallelogram.



5. $ABCD$ is a parallelogram.
$\angle APD = 90^0$ and $\angle CQB = 90^0$.
Show that $AQCP$ is a rectangle.

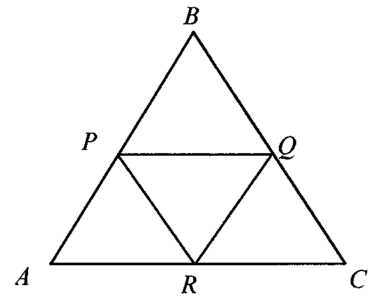


6. ΔABC is an equilateral triangle. P, Q and R are midpoints of sides AB, BC and CA respectively.

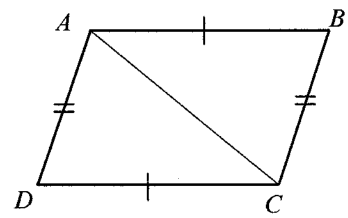
(i) Show that ΔPQR is an equilateral triangle.

(ii) How many parallelograms are there in this figure?

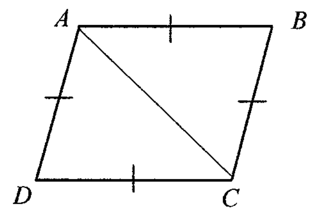
(iii) Show that each parallelogram is a rhombus.



7. Use the above diagram to prove that a quadrilateral is a parallelogram if both pairs of opposite sides are equal. (Hint: first prove that $\Delta ABC \equiv \Delta ADC$. Then show that alternate angles are equal.)



8. Use the above diagram to prove that a quadrilateral is a rhombus if all sides are equal. (You need to prove that the quadrilateral $ABCD$, which has four equal sides, is a parallelogram.)



9. Draw a quadrilateral and join the midpoints of the adjacent sides.

(a) What figure results when the quadrilateral is

(i) a square?
(ii) a rectangle?
(iii) a rhombus?
(iv) a parallelogram?

(b) What general statement can you make that applies to the figure formed by joining the midpoints of any quadrilateral? Try to prove this for any quadrilateral.

Tests for Quadrilaterals - Worksheet

① a) parallelogram (1 pair of opp sides eq and parallel) ✓

b) parallelogram (diagonals bisect eachother)

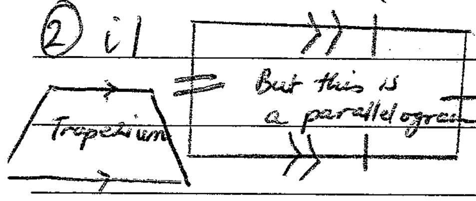
c) parallelogram (both pairs of opp sides are eq) ✓

d) parallelogram (both pairs of opp ∠s are eq) ✓

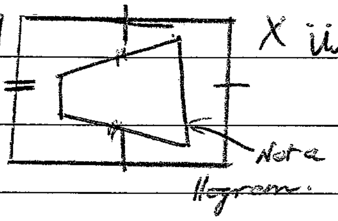
e) parallelogram (diagonals bisect each other)

f) rhombus (diagonals bisect eachother at right ∠s) ✓

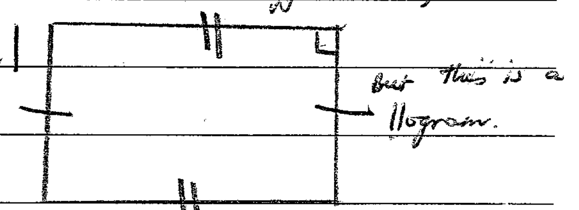
② i|



ii|



X iii|



③ i| A quadrilateral is a ~~rectangle~~ parallelogram if each diagonal bisects the vertex angles through which it passes ✓

ii| A parallelogram is a rhombus if the diagonals bisect eachother at right angles ✓

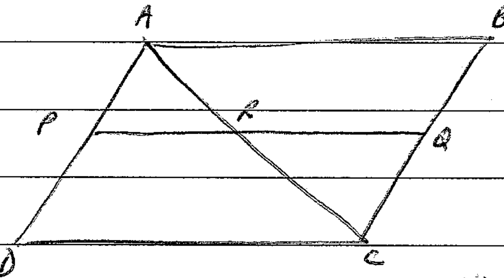
iii| A quadrilateral is a rectangle if a pair of cointerior angles are supp ✓

④ AP is half of AD

BQ is a half of BC

as AD = BC X

AP = BQ



Join AC & show that Δ APR ||| Δ ADC

Similarly, as AD // BC

AP // BQ

∴ ∠APR = ∠ADC (

∴ PR // DC (

& DC // AB ∴ ABQP is a llogram.

∴ ABQP is a parm (1 pair of opp sides is eq and parallel)

⑤ $A\hat{D}C = D\hat{P}C - A\hat{P}D$

$= 180 - 90$ (str ∠ = 180°)

∴ $A\hat{P}C = 90°$ X

Just show that ∠PAQ = ∠PCQ = 90°

and also AP ≠ AQ.

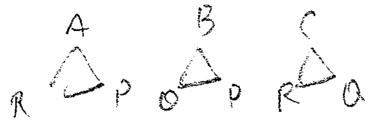
∴ AQCP is a rectangle as it has a right angle

~~) ii| in Δs PBQ, QCR & RAP~~

~~1. AP = QC = BQ (given) (halfs~~

~~2. AR = CR = BP (given)~~

3. $\hat{A} = \hat{?} = \hat{B}$ = ... (angles of a ~~riat~~ equil Δ)



⑥ i| As $BA = BC = AC$ (eq sides of equil $\triangle$)

and P, Q, R are midpoints, respectively

$BP = PA = AR = RC = CQ = QB$

in $\triangle$s ARP & BQP & CRQ

1. $BP = AP = CQ$ (as above)
2. $AR = BQ = CR$ (as above)
3. $\hat{A} = \hat{B} = \hat{C} = 60°$ (eq $\angle$s of equil $\triangle$)

$\therefore \triangle ARP \equiv \triangle BQP \equiv \triangle CRQ$ (SAS)

$\therefore PQ = QR = PR$ (corres sides, cong $\triangle$s)

$\therefore PQR$ is an equil $\triangle$ (all sides eq)

ii| 3

iii| $PQ = PR = QR$ (as above)

$\therefore BQRP$ is a rhombus (1 pair of adj sides eq)

similarly

$PQRA$ is a rhombus

$PQCR$ is a rhombus

7) in $\triangle$s ABC & ADC

1. AC is common
2. $AB = DC$ (given)
3. $AD = BC$ (given)

$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

$\therefore D\hat{A}C = A\hat{C}B$ (~~alt ∠s e~~ corres $\angle$'s, cong $\triangle$s)

$\therefore AD \parallel BC$ (alt $\angle$s using transversal AC are eq)

$\therefore ABCD$ is a parm as 1 pair of opp sides is eq and parallel

8) in $\triangle$s ABC & ADC

1. AC is common
2. $AB = DC$ (given)
3. $AD = BC$ (given)

$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

$\therefore D\hat{A}C = B\hat{C}A$ (corres $\angle$'s, cong $\triangle$s)

$\therefore AD \parallel BC$ (alt $\angle$s eq)

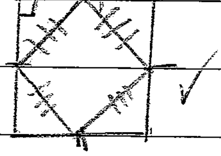
$\therefore ABCD$ is a parm (1 pair of opp sides eq and parallel)

however as adj sides are eq

$ABCD$ is a parm with adj sides eq

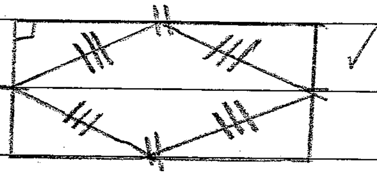
$\therefore ABCD$ is a rhombus

9 a) i)



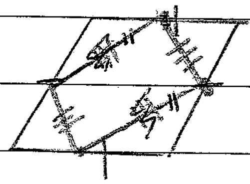
rhombus

ii)



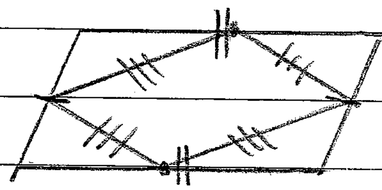
rhombus

iii)



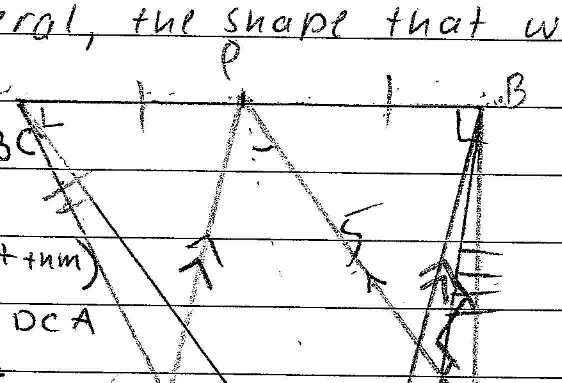
~~rhombus~~ parallelogram.

iv)



rhombus

b) "when joining the midpoints of the adj sides of any quadrilateral, the shape that will form is a ~~rhombus~~ parallelogram"



In Δs PBQ & ABC

$\frac{BP}{BA} = \frac{BQ}{BC}$

∴ PQ // AC (eq int thm)

In Δs DRS & DCA

$\frac{DS}{DA} = \frac{DR}{DC}$

∴ AC // SR (eq int thm)

∴ PQ // SR

In Δs APS & ABD

$\frac{AD}{AB} = \frac{AS}{AD}$

∴ SP // PB (eq int thm) (contd ↗)

In Δs RCQ & DCB

$\frac{RC}{DC} = \frac{QC}{BC}$

∴ DB // RQ (eq int thm)

∴ PS // QR

∴ PQDS is a parm (both pairs of opp sides are parallal)

∴ the above statement is false as PQRS is not a rhombus.

however as PQRS is a parallelogram it can be said "When joining the midpoints of the adj sides of any quad, the shape that will form is a ~~rh~~ parallelogram"