## Exercise 4A

Perform the indicated operations and express the answers in the form $a + ib$:

1. $(3 + 2i) + (2 - 3i)$
2. $(5 - 2i) - (3 - 2i)$
3. $(-3 - 4i) - (12 - 5i)$
4. $(3 + 2i) - (3 - 2i)$
5. $(3 + 4i)(2 + i)$
6. $(5 - i)(3 - 4i)$
7. $3i(2 - i)$
8. $(4 - 3i)^2$
9. $(1 + i)^2$
10. $(1 - 3i)^{-2}$
11. $i(2 + i)(2 - i)$
12. $(-4i)(2i)$
13. $\frac{1 + i}{1 - i}$
14. $\frac{3 + 2i}{5 + 2i}$
15. $\frac{3 - 2i}{5i}$
16. $\frac{1 + 2i}{i^3}$

Find $x$ and $y$ in each of the following:

17. $3x + 2iy = 12 + 5i$
18. $(2 - 3i) + (x + 2iy) = 5 - 4i$
19. $(x - iy)^2 = 2i$
20. $(x + iy)(3 + 4i) = 2 - 5i$
21. Expand: $(2 + i)^3$ and answer in the form $a + ib$.
22. If $x + iy = 5(\cos 60° - i\sin 60°)$, find $x$ and $y$ in the surd form and hence express (a) $(x + iy)^2$ (b) $\frac{1}{x - iy}$ in the form $a + ib$
23. If $z = 2 + i$, evaluate:
    (a) $3z + 4$ (b) $z^2 - 2z + 3$ (c) $\frac{2z - 1}{2z + 1}$ (d) $(z - 1)(z^2 + z + 1)$
24. If $z = x + iy$, express each of the following in the form $a + ib$:
    (a) $\bar{z}$ (b) $\frac{1}{z}$ (c) $\frac{z + 1}{z - 1}$ (d) $z^2 - 1$
25. Solve the following equations for $z$; express answers in the form $a + ib$:
    (a) $(1 + i)z = 2 - i$
    (b) $\frac{2z}{2 + i} + 3 - 2i = (1 - i)z$
    (c) $\frac{2}{z} = 1 + i + \frac{3}{1 - i}$
    (d) $\frac{z + 3}{z - 1} = 2 - 3i$

26. Solve the following equations for $z$; express answers in the form $a + ib$:

(a) $z^2 + z + 1 = 0$ (b) $z^2 - 2z + 4 = 0$ (c) $2z^2 - 3z + 2 = 0$ (d) $z + \frac{1}{z} = 2$

27. Find the quadratic equations with roots given below:

(a) $i, -i$ (b) $1 + i, 1 - i$ (c) $2 + 3i, 2 - 3i$

(d) $3 + i, 1 + 3i$ (e) $2 + i, \frac{1}{2 + i}$

28. Solve the following pairs of equations for $z$ and $w$ where $z$ and $w$ are complex numbers. Express answers in the form $a + ib$.

(a) $z + iw = 2 + 3i$
$z - iw = 2 - 3i$

(b) $2z + w = 1 + i$
$z - w = 1 - i$

(c) $(2 + i)z + (2 - i)w = 1$
$(2 - i)z + (2 + i)w = 2$

(d) $z + (1 - i)w = 2i$
$w + (1 - i)z = 1$

29. Given $z = 2 + i$, evaluate the following in the form $a + ib$:

(a) $\frac{1}{z}$ (b) $z^2$ (c) $\frac{1}{z^2}$ (d) $z^2 + \frac{1}{z^2}$ (e) $z^3$ (f) $z^4$

(Hint: $z^3 = z^2 . z$ and $z^4 = z^2 . z^2$)

30. What is the fallacy in the following:

$$\sqrt{-3} \cdot \sqrt{-12} = \sqrt{(-3) \cdot (-12)} = 6 \ ?$$

What is the correct answer?

31. Prove the associative law for multiplication of complex numbers:

$(z_1 \cdot z_2)z_3 = z_1(z_2 \cdot z_3)$ (Hint: Let $z_1 = x_1 + iy_1$ etc.)

32. Prove the commutative law for multiplication: $z_1z_2 = z_2z_1$

# CHAPTER 4 COMPLEX NUMBERS

## Exercise 4A

1. $5 - i$ 2. $2$ 3. $-15 + i$

4. $4i$ 5. $2 + 11i$ 6. $11 - 23i$

7. $3 + 6i$ 8. $7 - 24i$ 9. $2i$

10. $-\frac{2}{25} + \frac{3}{50}i$ 11. $5i$ 12. $8$

13. $i$ 14. $\frac{19 + 4i}{29}$ 15. $-\frac{2}{5} - \frac{3}{5}i$

16. $-2 + i$ 17. $x = 4$, $y = \frac{5}{2}$ 18. $x = 3$, $y = -\frac{1}{2}$

19. $(x = 1, y = -1)$ or $(x = -1, y = 1)$ 20. $x = -\frac{14}{25}$, $y = -\frac{23}{25}$

21. $2 + 11i$ 22. $x = \frac{5}{2}$, $y = -5\frac{\sqrt{3}}{2}$ (a) $-\frac{25}{2} - \frac{25}{2}\sqrt{3}\,i$

(b) $\frac{1}{10} - \frac{\sqrt{3}}{10}i$ 23. (a) $10 + 3i$ (b) $2 + 2i$

(c) $\frac{19 + 4i}{29}$ (d) $1 + 11i$ 24. (a) $x - iy$

(b) $\frac{x - iy}{x^2 + y^2}$ (c) $\frac{x^2 + y^2 - 1 - 2yi}{(x - 1)^2 + y^2}$ (d) $x^2 - y^2 - 1 + 2xyi$

25. (a) $\frac{1}{2} - \frac{3}{2}i$ (b) $\frac{9}{2} + \frac{7}{2}i$ (c) $\frac{2}{5} - \frac{2}{5}i$

25. (d) $\frac{7}{5} + \frac{6}{5}i$ 26. (a) $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ (b) $1 \pm \sqrt{3}\,i$

(c) $\frac{3}{4} \pm \frac{\sqrt{7}\,i}{4}$ (d) $1$ 27. (a) $x^2 + 1 = 0$

(b) $x^2 - 2x + 2 = 0$ (c) $x^2 - 4x + 13 = 0$ (d) $x^2 - (4 + 4i)x + 10i = 0$

(e) $5x^2 - (12 + 4i)x + 5 = 0$ 28. (a) $z = 2, w = 3$

(b) $z = \frac{2}{3}, w = -\frac{1}{3} + i$ (c) $z = \frac{3}{8} + \frac{i}{4}, w = \frac{3}{8} - \frac{i}{4}$ (d) $z = 1 + i, w = -i$

29. (a) $\frac{2}{5} - \frac{1}{5}i$ (b) $3 + 4i$ (c) $\frac{3}{25} - \frac{4}{25}i$ (d) $\frac{78}{25} + \frac{96}{25}i$

(e) $2 + 11i$ (f) $-7 + 24i$

30. $-6$, Rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ is not defined for the imaginary numbers.