ELEMENTARY TRIGONOMETRY EQUATIONS – WORKSHEET

COURSE/LEVEL
NSW Secondary High School Year 11 Preliminary Mathematics.
Syllabus reference: 5.1 – 5.2.

1. Solve the following equations for θ if $0° \le \theta < 360°$.

	I	**II**
(a)	$\sin\theta = 0$	$\tan\theta = 0$
(b)	$\sin\theta = 1$	$\tan\theta = 1$
(c)	$\cos\theta = \dfrac{1}{\sqrt{2}}$	$\sec\theta = \sqrt{2}$
(d)	$\sin\theta = -1$	$\tan\theta = -1$
(e)	$\tan\theta = -\sqrt{3}$	$\tan(\theta - 30°) = -\sqrt{3}$
(f)	$2\sin\theta = 1$	$\sqrt{3}\tan\theta - 1 = 0$
(g)	$\sin\theta = \cos\theta$	$\sin\theta = \sqrt{3}\cos\theta$
(h)	$\sin^2\theta = \dfrac{1}{2}$	$4\sin^2\theta = 1$
(i)	$\tan^2\theta - 3 = 0$	$9\sec^2\theta = 16$
(j)	$\sin\theta = \mathrm{cosec}\,\theta$	$3\tan\theta = \cot\theta$
(k)	$\tan 2\theta = \sqrt{3}$	$2\sin 2\theta - \sqrt{2} = 0$
(l)	$\sin 3\theta = \dfrac{1}{2}$	$\sin^2 2\theta = \frac{3}{4}$

2. (a) If $\sin\theta = \dfrac{3}{5}$ and $\cos\theta > 0$, find $\cos\theta$ and $\tan\theta$.

   (b) If $\sin\theta = \dfrac{-4}{5}$ and $\cos\theta < 0$, find $\cos\theta$ and $\tan\theta$.

   (c) If $\tan\theta = \dfrac{-5}{12}$ and $\sin\theta > 0$, find $\sin\theta$ and $\cos\theta$.

   (d) If $\cos\theta = \dfrac{3}{4}$ and $\tan\theta < 0$, find the exact values of $\sin\theta$, $\tan\theta$ and $\mathrm{cosec}\,\theta$.

   (e) If $\tan\theta = 3$ and $\sin\theta > 0$, find the exact value of $\sin\theta$.

3. Find the non-negative values of θ not greater than 360^0 that satisfy the following equations.

   (a) $\cos^2\theta = \dfrac{1}{2}$ (b) $\sin^2\dfrac{x}{2} = \dfrac{1}{4}$

4. Solve these equations for θ if $0° \le \theta < 360°$.

   (a) $\sin\theta = \dfrac{-\sqrt{3}}{2}$ (b) $\tan\dfrac{\theta}{2} = 2\sin\dfrac{\theta}{2}$

Elementary Trig Eqtns
$$0° \le \theta < 360°$$

1a. I. $\sin\theta = 0$
$\theta = 0, 180°$ ✓

II. $\tan\theta = 0$
$\theta = 0, 180°$ ✓

b. I. $\sin\theta = 1$
$\theta = 90°$ ✓

II. $\tan\theta = 1$
$\theta = 45°, 225°$ ✓

c. I. $\cos\theta = \frac{1}{\sqrt{2}}$
$\theta = 45°, 315°$ ✓

II. $\sec\theta = \sqrt{2}$
$\cos\theta = \frac{1}{\sqrt{2}}$
$\theta = 45°, 315°$ ✓

d. I. $\sin\theta = -1$
$\theta = 270°$ ✓

II. $\tan\theta = -1$
$\theta = 135°, 315°$ ✓

e. I. $\tan\theta = -\sqrt{3}$
$\theta = 120°, 300°$ ✓

II. $\tan(\theta - 30°) = -\sqrt{3}$
$\theta - 30° = 120°, 300°$
$\theta = 150°, 330°$ ✓

f. I. $2\sin\theta = 1$
$\sin\theta = \frac{1}{2}$
$\theta = 30°, 150°$ ✓

II. $\sqrt{3}\tan\theta - 1 = 0$
$\tan\theta = \frac{1}{\sqrt{3}}$
$\theta = 30°, 210°$ ✓

g. I. $\sin\theta = \cos\theta$
$\frac{\sin\theta}{\cos\theta} = 1$
$\tan\theta = 1$ ✓
$\theta = 45°, 225°$ ✓

II. $\sin\theta = \sqrt{3}\cos\theta$
$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$
$\tan\theta = \sqrt{3}$
$\theta = 60°, 240°$ ✓

h. I. $\sin^2\theta = \frac{1}{2}$
$\sin\theta = \pm\frac{1}{\sqrt{2}}$ ✓
$\theta = 45°, 135°, 225°, 315°$ ✓

II. $4\sin^2\theta = 1$
$\sin^2\theta = \frac{1}{4}$
$\sin\theta = \pm\frac{1}{2}$ ✓
$\theta = 30°, 150°, 210°, 330°$ ✓

-1-

I. $\tan^2\theta - 3 = 0$

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$
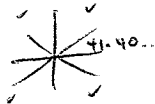
$\theta = 60°, 120°, 240°, 300°$ ✓

II. $9\sec^2\theta = 16$

$\sec^2\theta = \dfrac{16}{9}$

$\sec\theta = \pm\dfrac{4}{3}$

$\cos\theta = \pm\dfrac{3}{4}$

$\theta = 41.4°, 138.6°, 221.4°, 318.6°$ ✓

j. I. $\sin\theta = \csc\theta$

$\sin\theta = \dfrac{1}{\sin\theta}$

$\sin^2\theta = 1$

$\sin\theta = \pm 1$ ✓

$= 90°, 270°$ ✓

II. $3\tan\theta = \cot\theta$

$3\tan\theta = \dfrac{1}{\tan\theta}$

$3\tan^2\theta = 1$

$\tan^2\theta = \dfrac{1}{3}$

$\tan\theta = \pm\dfrac{1}{\sqrt{3}}$ ✓

$\theta = 30°, 150°, 210°, 330°$ ✓
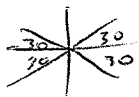
k. I. $\tan 2\theta = \sqrt{3}$ $(0° \le 2\theta < 720°)$

$2\theta = 60°, 240°, 420°, 600°$

$\theta = 30°, 120°, 210°, 300°$ ✓
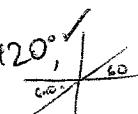
II. $2\sin 2\theta - \sqrt{2} = 0$

$2\sin 2\theta = \sqrt{2}$

$\sin 2\theta = \dfrac{\sqrt{2}}{2}$

$= \dfrac{1}{\sqrt{2}}$ ✓

$2\theta = 45°, 135°, 405°, 495°$ ✓

$\theta = 22.5°, 67.5°, 202.5°, 247.5°$ ✓
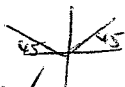
l. I. $\sin 3\theta = \dfrac{1}{2}$

$3\theta = 30°, 150°, 390°, 510°, 750°, 870°$ ✓

$\theta = 10°, 50°, 130°, 170°, 250°, 290°$ ✓
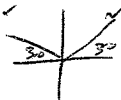
II. $\sin^2 2\theta = \dfrac{3}{4}$

$\sin 2\theta = \pm\dfrac{\sqrt{3}}{2}$ ✓

$2\theta = 60°, 120°, 240°, 300°, 420°, 480°, 600°, 660°$ ✓

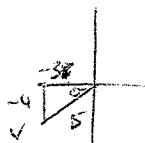$\theta = 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°$ ✓
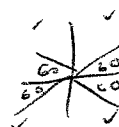
2a.

$$\cos\theta = \frac{4}{5} \checkmark$$
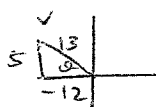
$$\tan\theta = \frac{3}{4} \checkmark$$

b.

$$\cos\theta = -\frac{3}{5} \checkmark$$
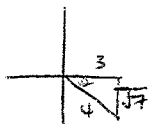
$$\tan\theta = \frac{4}{3} \checkmark$$

c.

$$\sin\theta = \frac{5}{13} \checkmark$$
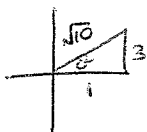
$$\cos\theta = -\frac{12}{13} \checkmark$$

d.

$$\sin\theta = \frac{-\sqrt{7}}{4}$$

$$\tan\theta = \frac{-\sqrt{7}}{3}$$

$$\csc\theta = \frac{-4}{\sqrt{7}} \checkmark$$

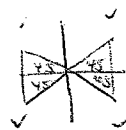**e.**

$$\sin\theta = \frac{3}{\sqrt{10}}$$

3a. $\cos^2\theta = \frac{1}{2}$
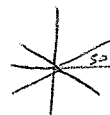
$$\cos\theta = \pm\frac{1}{\sqrt{2}} \checkmark$$

$$\theta = 45°, \ 135°, \ 225°, \ 315° \checkmark$$

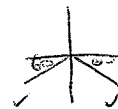**b.** $\sin^2\frac{x}{2} = \frac{1}{4}$ $\qquad (0 \le \frac{x}{2} < 180°) \checkmark$

$$\sin\frac{x}{2} = \pm\frac{1}{2} \checkmark$$

$$\frac{x}{2} = 30°, \ 150° \checkmark$$

$$x = 60°, \ 300° \checkmark$$

4a. $\sin\theta = -\frac{\sqrt{3}}{2}$

$$\theta = 240°, \ 300° \checkmark$$

b. $\tan\frac{\theta}{2} = 2\sin\frac{\theta}{2}$

$$\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = 2\sin\frac{\theta}{2}$$

$$\sin\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$0 = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\frac{\theta}{2}$$

$$= \sin\frac{\theta}{2}\left(2\cos\frac{\theta}{2} - 1\right) \checkmark$$

$$0 = 2\cos\frac{\theta}{2} - 1$$

$$\frac{1}{2} = \cos\frac{\theta}{2} \qquad (\text{nb. } 0 \le \frac{\theta}{2} \le 180)$$

$$\frac{\theta}{2} = 60°$$

$$\therefore \theta = 120° \checkmark$$

Also $\sin\frac{\theta}{2} = 0,$

$$\frac{\theta}{2} = 0°, \ 180°$$

$$\Rightarrow \underline{\theta = 0°}.$$