GEOMETRICAL SIGNIFICANCE OF THE SECOND DERIVATIVE – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics.
Syllabus reference: 10.4.

1. (a) Sketch the graph of a function which is:
   (i) increasing with an increasing gradient,
   (ii) increasing with a decreasing gradient,
   (iii) decreasing with an increasing gradient,
   (iv) decreasing with a decreasing gradient.

   (b) For each function in 1(a) state whether the function:
   (i) is concave up, concave down or neither,
   (ii) has a positive or negative first derivative,
   (iii) has a positive or negative second derivative.

2. Draw a sketch of the function $y = \mathrm{f}(x)$ where, for all x in its domain,

   (i) $\mathrm{f}''(x) > 0$,
   (ii) $\mathrm{f}''(x) < 0$,
   (iii) $\mathrm{f}'(x) > 0$ and $\mathrm{f}''(x) > 0$,
   (iv) $\mathrm{f}'(x) > 0$ and $\mathrm{f}''(x) < 0$,
   (v) $\mathrm{f}'(x) < 0$ and $\mathrm{f}''(\mathrm{x}) > 0$,
   (vi) $\mathrm{f}'(x) < 0$ and $\mathrm{f}''(x) < 0$,
   (vii) $\mathrm{f}(x) > 0$, $\mathrm{f}'(x) > 0$ and $\mathrm{f}''(x) > 0$,
   (viii) $\mathrm{f}(x) > 0$, $\mathrm{f}'(x) > 0$ and $\mathrm{f}''(x) < 0$,
   (ix) $\mathrm{f}(x) < 0$, $\mathrm{f}'(x) < 0$ and $\mathrm{f}''(x) > 0$,
   (x) $\mathrm{f}(x) < 0$, $\mathrm{f}'(x) < 0$ and $\mathrm{f}''(x) < 0$.

3. Draw sketches of the following functions $y = \mathrm{f}(x)$ where $0 \le x \le 4$.

   (i) $\mathrm{f}''(x) > 0$ for $0 \le x < 2$ and $\mathrm{f}''(x) < 0$ for $2 < x \le 4$. What do you think is the value of the second derivative at $x = 2$?

   (ii) $\mathrm{f}''(x) < 0$ for $0 \le x < 2$ and $\mathrm{f}''(x) > 0$ for $2 < x \le 4$. What do you think is the value of the second derivative at $x = 2$?

   (iii) $\mathrm{f}''(x) > 0$ for $0 \le x \le 4$ and $f'(2) = 0$.

   (iv) $\mathrm{f}''(x) < 0$ for $0 \le x \le 4$ and $f'(2) = 0$.

4. Classify the stationary points where $x = 2$ for the functions in 3(iii) and 3(iv). Could these functions have more than one stationary point?

5. Can you draw the graph of a function that is concave up at a maximum turning point? What about a function that is concave down at a maximum turning point?

6. (i) The graph of a function $y = \mathrm{f}(x)$ is concave up for all x and has one stationary point at $x = a$ (where a is some number). What sort of stationary point is this point? What can you say about the sign of $\mathrm{f}''(x)$?

   (ii) The graph of a function $y = \mathrm{f}(x)$ is concave down for all x and has one stationary point at $x = a$ (where a is some number). What sort of stationary point is this point? What can you say about the sign of $\mathrm{f}''(x)$?

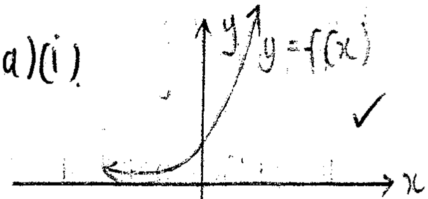
7. (i) Is the second derivative of a function positive or negative at a minimum turning point? Is this true for all minimum turning points?

   (ii) Is the second derivative of a function positive or negative at a maximum turning point? Is this true for all maximum turning points?

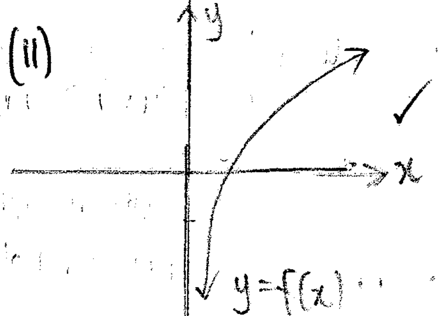
8. Use your answers to Question 7 to describe how you would determine the nature of a turning point (that is, whether it is a maximum or minimum turning point).

Geometrical Significance of the 2nd Derivative

Qu 1 (a)(i)



(ii)



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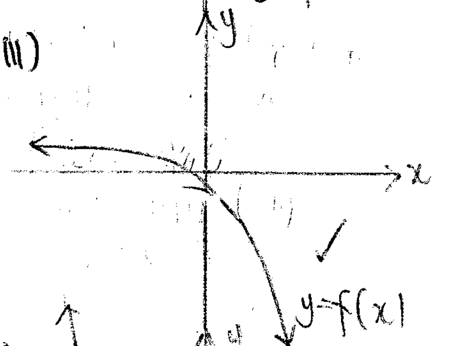

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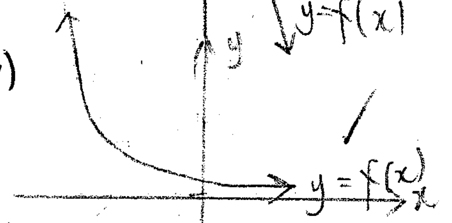


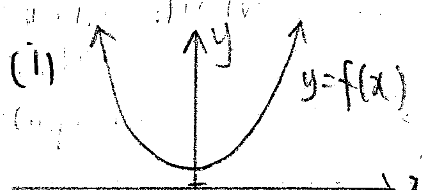
(b)(i) concave up, +ve $f'(x)$, $f''(x) > 0$

(ii) concave down, $f'(x) > 0$, $f''(x) < 0$

(iii) concave down, $f'(x) < 0$, $f''(x) < 0$

(iv) concave up, $f'(x) < 0$, $f''(x) > 0$

Qu 2 (i)



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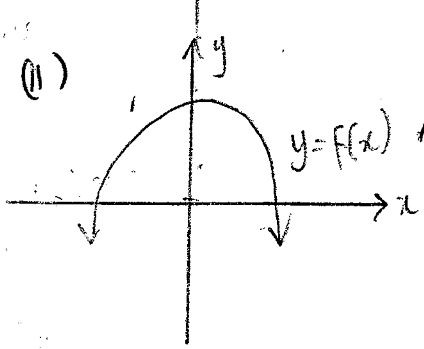

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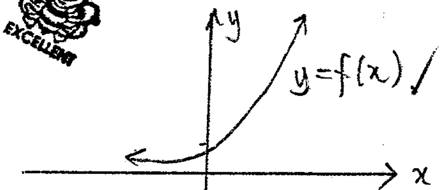

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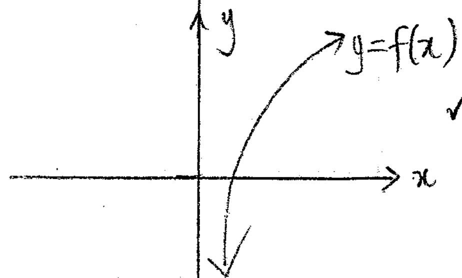

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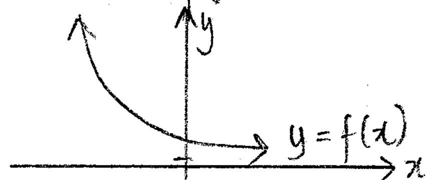

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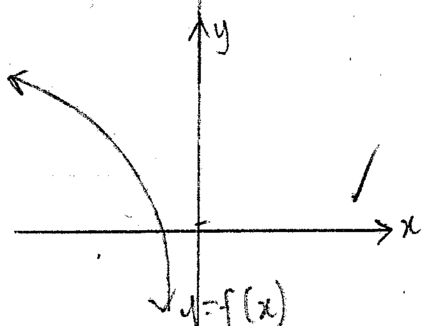

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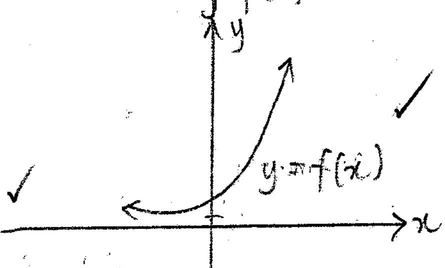

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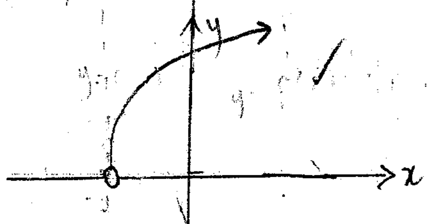

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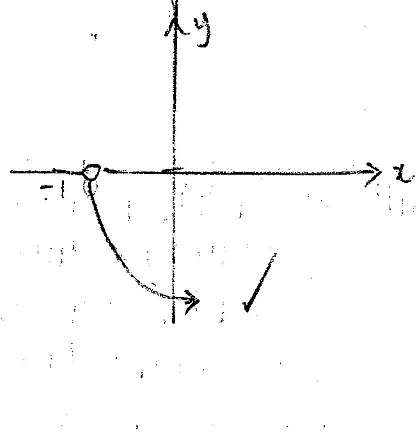
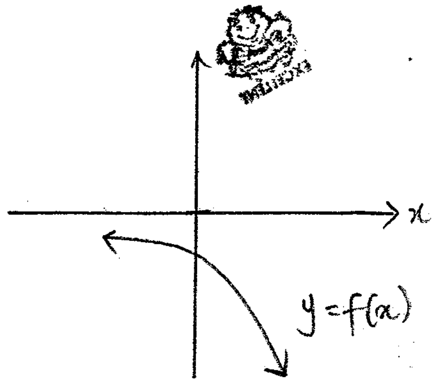
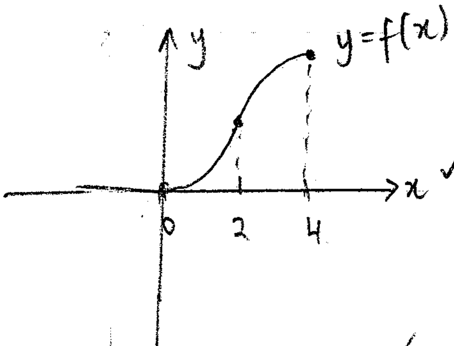


(x)



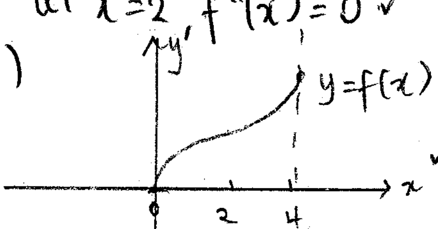
Qu3 (i)



$f''(x) > 0$ for $0 \le x < 2$

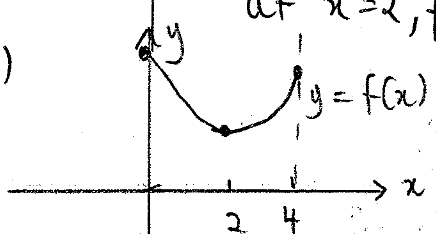
at $x = 2$, $f''(x) = 0$

(ii)



at $x = 2$, $f''(x) = 0$

(iii)



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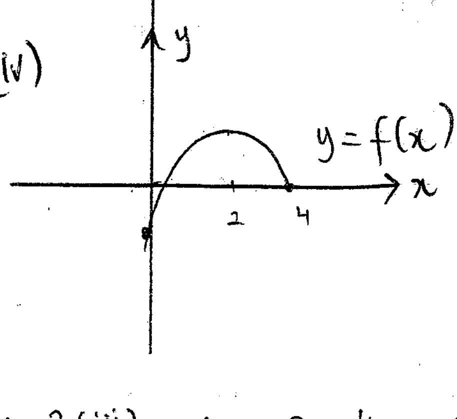

Qu4. in 3(iii), at $x = 2$, there is a minimum turning point.
In 3(iv), at $x = 2$, there is a maximum turning point.
No, not under these conditions.

Qu5. No, at max. turning pt, $f(x)$ must be concave down and at min. turning pt, $f(x)$ must be concave up.

Qu6(i) minimum turning point. $f''(x) > 0$
(ii) maximum turning point, $f''(x) < 0$

Qu7(i) $f''(x) > 0$ at a minimum turning pt ie positive. True for all min. turning pts
(ii) $f''(x) < 0$ (negative) at a max. turning pt. This is true for all max. turning pts.

Qu8. if $f''(x) > 0$, and $f'(x) = 0$, then min. turning pt.
if $f''(x) < 0$, and $f'(x) = 0$, then max. turning pt.