GRAPHS – WORKSHEET #5

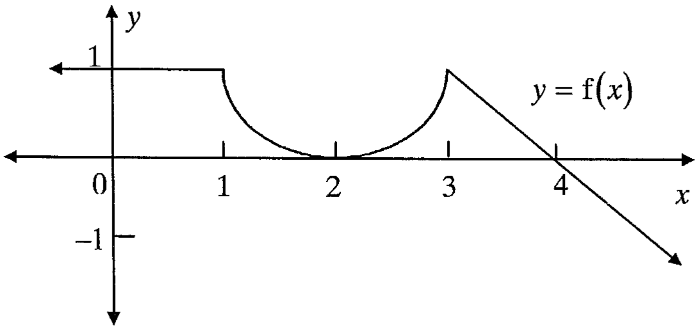
COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics Extension 2.

TOPIC

Graphs

1.



For the function $y = f(x)$ sketched above, draw separate sketches for the following.

(i) $y = f(x) - \frac{1}{2}$

(ii) $y = f\left(x - \frac{1}{2}\right)$

(iii) $y = f(-x)$

(iv) $y = \left(f(x)\right)^2$

(v) $y = \frac{1}{f(x)}$

(vi) $y = f(|x|)$

(vii) $y^2 = f(x)$

(viii) $y = 2^{-f(x)}$

2. Use the graphs of $y = |x+2|$ and $y = |x-1|$ to sketch the graph of $y = |x+2| + |x-1|$. Hence, or otherwise, solve the inequality $|x+2| + |x-1| > 4$.

3. Draw neat separate sketches of the following sketches, without using calculus. Include features such as any x– and y– intercepts and the equations of any asymptotes.

(i) $y = x^3$

(ii) $y = \frac{1}{x^3}$

(iii) $y = \frac{x^3 - 1}{x^3}$

(iv) $y = \frac{x^3}{x^3 - 1}$

(v) $y = x^3 - \frac{1}{x^3}$

4. Given that $f(x) = x^2(x^2 - 1)(x+1)$, sketch the following functions. (It is not necessary to locate turning points).

(i) $y = f(x)$

(ii) $y = f(|x|)$

(iii) $y = |f(x)|$

(iv) $y^2 = f(x)$

(v) $y.f(x) = 1$

5. Suppose the curve $y = \mathrm{f}(x)$ has a stationary point at $x = a$. Show that $\mathrm{g}(x) = \dfrac{1}{\mathrm{f}(x)}$ also has a stationary point at $x = a$ provided $\mathrm{f}(a) \neq 0$.

6. Let $f(x) = x - 2 + \dfrac{3}{x+2}$.

   (i) Find the points at which $f(x) = 0$.

   (ii) Find the turning points of $f(x)$, if any, and identify them.

   (iii) Find the asymptotes.

   (iv) Sketch the curve, marking all the features you have found in parts (i) - (iii) above.

7. Draw separate sketches of $y = \dfrac{x(x-1)}{x-2}$ and $y = \sqrt{\dfrac{x(x-1)}{x-2}}$. Include any x- or y-intercepts and asymptotes. (It is not necessary to find the co-ordinates of any stationary points.)

8. Let $\min(a,b)$ denote the minimum of the numbers a and b. Sketch the function $y = \min(2, x)$ over the interval $0 \leq x \leq 3$ and evaluate $\int_0^3 \min(2,x)\,dx$.

9. If $\mathrm{f}(x) = (x-2)(x+1)$, sketch separate graphs of the following:

   (i) $y = \mathrm{f}(x)$ (ii) $y = \dfrac{1}{|\mathrm{f}(x)|}$ (iii) $y = 2^{-\mathrm{f}(x)}$

   In each sketch, include the co-ordinates of any stationary or critical points.

10. If $f(x) = \frac{x+1}{x-2}$, sketch the following functions.

   (i) $y = f(x)$ (ii) $y = \ln(f(x))$ (iii) $y = e^{f(x)}$ (iv) $y = \sin^{-1}(f(x))$

11. Sketch the following:

   (i) $y = e^{-1/x}$ (ii) $y = e^{-\sin x}$ (iii) $y = \cos(x^2)$ (iv) $y = \cos(\sin^{-1} x)$

12. Without calculus, sketch the curve which represents $y = \dfrac{1}{\sqrt{(x+1)(x+3)}}$.