# THE QUADRATIC POLYNOMIAL – WORKSHEET

## COURSE/LEVEL

NSW Secondary High School Year 11 Preliminary Mathematics.
Syllabus reference: 9.1 – 9.4.

**1.** For each of the following quadratic equations determine the number of real roots by considering the value of the discriminant $\Delta = b^2 - 4ac$. If real roots exist find them using the quadratic formula.

| I | II |
|---|---|
| $x^2 - 5x + 6 = 0$ | $-x^2 + 7x - 6 = 0$ |
| $-2x + 1 + x^2 = 0$ | $4 - 12x + 9x^2 = 0$ |
| $x^2 + x + 1 = 0$ | $x - 1 + x^2 = 0$ |
| $x^2 - 9 = 0$ | $x^2 + 6x = 9$ |

**2.** Find the exact solutions of the equation $2x^2 - x - 1 = 0$.

**3.** (i) Sketch the graph of $y = 6 - 5x - x^2$. Make sure to show all essential features including the $x$- and $y$- intercepts and the co-ordinates of the vertex.

(ii) Use your sketch to solve the inequality $6 - 5x - x^2 \leq 0$.

**4.** Solve these equations:

(a) $x^2 \geq (x+2)(x-3)$ (b) $(x-1)(x+1) \geq 0$

(c) $x^2 + x - 6 < 0$ (d) $12 + 4x - x^2 > 0$

**5.** Without solving the equation $2x^2 - 5x - 8$, determine whether the roots are:

(a) Real or unreal (b) equal or unequal (c) rational or irrational.

**6.** Find $k$ if $x^2 + 3x + k = 0$ has equal roots.

**7.** Find the values of $m$ if the quadratic equation $x^2 + (m-1)x - 2(m+1) = 0$ has equal roots.

**8.** Find the two possible values of $k$ for which the equation $9 + kx + x^2 = 0$ has one real root.

**9.** Find all possible values of $k$ for which the equation $2x^2 + 4x + k = 0$ has:

(i) two roots (ii) one root (iii) no roots

**10.** Find all possible values of $m$ for which the equation $mx^2 + 2x - 1 = 0$ has:

(i) two roots (ii) one root (iii) no roots

**11.** Find all possible values of $k$ for which the equation $x+\frac{x^2}{3}=k$ has:

(i) two roots (ii) one root (iii) no roots

**12.** Find all possible values of $k$ for which the equation $x^2+kx+(k+3)=0$ has no roots.

**13.** Find all possible values of $k$ for which the quadratic equation $kx^2+(k+1)x+(2-k)=0$ has two real roots.

**14.** The quadratic function $y=ax^2+ax+a$ has all of its coefficients equal. Prove that it has no real zeros.

**15.** Show that the equation $x-\frac{1}{x}=k$ has two solutions for all possible values of $k$.

**16.** Consider the quadratic equation $(k+2)x^2+(k+3)x+1=0$:

(i) Show that the equation always has at least one real root for all values of $k$.

(ii) Find the roots if $k=\sqrt{2}-2$.

**17.** Find the values of $m$ such that the expression $x^2+mx+16$ is:

(i) positive definite (ii) ~~positive~~ negative definite (ii) indefinite

**18.** Show that if the quadratic equation $ax^2+bx+c=0$ has two real roots then the roots are equally spaced about the axis of symmetry and the distance between the roots is $\Delta$.

**20.** If $\alpha$ and $\beta$ are the roots of the quadratic equation $3x^2-6x+2=0$, find the values of

(i) $\alpha+\beta$ (ii) $\alpha\beta$

(iii) $\alpha^{-1}+\beta^{-1}$ (iv) $\alpha^2+\beta^2$

**21.** Find the values of $k$ if the square of the sum of the roots of the equation $x^2-(4-k)x+k-3=0$ is equal to three times the product of the roots.

**22.** Find numbers $a$, $b$, $c$ such that $a(x+1)^2+b(x+1)+c=(2x+1)^2$ for all values of $x$.

**23.** For what values of $m$ will the straight line with equation $y=mx+4$

(i) touch (ii) intersect (iii) not intersect

the parabola with equation $y=2x^2-2x+5$.

**24.** Solve the equations: (i) $x^4-5x^2+6=0$ (ii) $9^x-4.3^x+3=0$.

Eva

# The Quadratic Polynomial

Q1 I.

1. $x^2 - 5x + 6 = 0$

$\Delta = 25 - 4 \times 6$

$= 1$ ✓

$x = \frac{5 \pm \sqrt{1}}{2}$

$x = 3$ ✓ or $x = 2$ ✓

2. $-2x + 1 + x^2 = 0$

$\Delta = 4 - 4$

$= 0$ ✓ equal roots

$x = \frac{2 \pm \sqrt{0}}{2}$

$= 1$ ✓

3. $x^2 + x + 1 = 0$

$\Delta = 1 - 4 \times 1$

$= -3$ ✓

no real roots. ✓

4. $x^2 - 9 = 0$

$\Delta = 0 - 4 \times -9$

$= 36$ ✓

$x = \frac{0 \pm \sqrt{36}}{2}$

$x = 3$ or $-3$ ✓

II

1. $-x^2 + 7x - 6 = 0$

$\Delta = 49 - 4 \times -6 \times -1$

$= 25$ ✓

$x = \frac{-7 \pm \sqrt{25}}{-2}$

$= \frac{-7 \pm 5}{-2}$ ✗ ✗

$= +6$ or $+1$

2. $4 - 12x + 9x^2 = 0$

$\Delta = 144 - 4 \times 4 \times 9$

$= 0$ ✓

$x = \frac{12}{2} \frac{-b}{2a} = \frac{12}{2 \times 9} \times^2$

$= \frac{6}{\cdot} = \frac{2}{3}$

3. $x - 1 + x^2 = 0 \Rightarrow x^2 + x - 1 = 0$

$\Delta = 1 - 4 \times -1$

$= 5$ ✓

$x = \frac{-1 \pm \sqrt{5}}{2}$ ✓

$\therefore x = \frac{-1 - \sqrt{5}}{2}$ or $\frac{-1 + \sqrt{5}}{2}$ ✓

4. $x^2 + 6x - 9 = 0$

$\Delta = 36 - 4 \times -9$

$= 72$

$x = \frac{-6 \pm \sqrt{72}}{2} = \frac{-6 \pm 6\sqrt{2}}{2}$

$= -3 \qquad = -3 \pm 3\sqrt{2}$

Q2 $2x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1 - 4 \times 2 \times -1}}{4}$

$= \frac{1 \pm \sqrt{9}}{4}$

$= 1$ or $-\frac{1}{2}$ ✓

Q3 i) $6 - 5x - x^2 = (1 - x)(x + 6)$

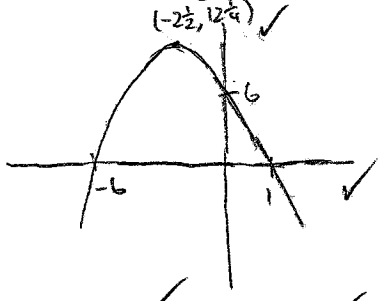
$\therefore x = 1$ or $-6$ ✓

at $x = 0 \quad y = 6$ ✓

$x = \frac{-b}{2a}$

$= \frac{5}{-2}$ ✓

at $x = \frac{-5}{2} \quad y = 12\frac{1}{4}$




✓ ✓ ✓

ii) $x \leq -6$ or $x \geq 1$ ✓

Q4 a) $x^2 \geq x^2 - x - 6$

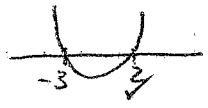
$x \geq -6$ ✓

b) 


$x \leq -1$ or $x \geq 1$ ✓ ✓

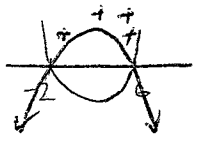
c) $(x + 3)(x - 2) < 0$

$-3 < x < 2$ ✓




✓

d) $(6-x)(x+2)>0$



~~$x<-2$ or $x>6$~~

$-2<x<6$

Q5a) $\Delta = 25 - 4\times -8\times 2$

$= 89$ ✓

$\therefore$ its real roots as $\Delta > 0$

b) $\frac{5 \pm \sqrt{89}}{4}$

unequal ✓

c) irrational. ✓

Q6 $\Delta = 0$

$0 = 9 - 4\times k$ ✓

$0 = 9 - 4k$ ✓

$4k = 9$

$k = \frac{9}{4}$ ✓

Q7. $\Delta = 0$

$0 = (m-1)^2 - 4\times -2(m+1)$

$0 = m^2 - 2m + 1 + 8m + 8$ ✓

$= m^2 + 6m + 9$

$= (m+3)(m+3)$ ✓

$\therefore m = -3$ ✓

Q8. $\Delta \geq 0$

$0 \leq k^2 - 4\times 9$

$0 \leq k^2 - 36 \implies (k+6)(k-6) \geq 0$

$36 \leq k^2$ X

$k \geq \pm 6$

$k \leq -6$ or $k \geq 6$

Q9. i) $\Delta > 0$

$16 - 4\times 2k > 0$

$16 > 8k$

$2 > k$ ✓

ii) $16 - 8k = 0$

$8k = 16$

$k = 2$ ✓

iii) $\Delta < 0$

$16 - 8k < 0$

$16 < 8k$

$k > 2$ ✓

10 i) $\Delta > 0$

$4 - 4\times -1\times m > 0$

$4 + 4m > 0$

$4m > -4$ ✓

$m > -1$ ✓

ii) $\Delta = 0$

$0 = 4 + 4m$

$m = -1$ ✓

iii) $\Delta < 0$

$4 + 4m < 0$

$4m < -4$

$m < -1$ ✓

11 i) $3x + x^2 = 3k$

$x^2 + 3x - 3k = 0$ ✓

$\Delta > 0$

$9 - 4\times -3k > 0$

$9 + 12k > 0$

$12k > -9$

$k > -\frac{3}{4}$ ✓

ii) $\Delta = 0$

$0 = 9 + 12k$

$k = -\frac{3}{4}$ ✓

iii) $\Delta < 0$

$9 + 12k < 0$

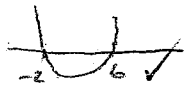
$k < -\frac{3}{4}$ ✓

12. $\Delta < 0$

$0 > k^2 - 4(k+3)$

$0 > k^2 - 4k - 12$

$0 > (k-6)(k+2)$

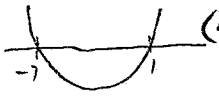


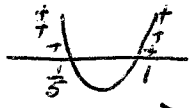
13. $\Delta > 0$

$0 < (k+1)^2 - 4k(2-k) \Rightarrow 0 < k^2+2k+1-8k+4k^2$

$0 < k^2+2k+1-8+4k \quad \therefore 5k^2-6k+1 > 0$

$0 < k^2+6k-7 \quad (k-1)(5k-1) > 0$




$0 < (k+7)(k-1)$

$k < -7$ and $k > 1$

$k \leq \frac{1}{5}$ or $k \geq 1$

14. $y = ax^2 + ax + a$

$\Delta = 0$

$0 = a^2 - 4a^2$

$0 = a^2(1-4)$

$a^2 = 0$ ✓

$(1-4) \neq 0$ ✓

$\therefore$ it has no real zeros

15. $x^2 - 1 = kx$

$x^2 - kx - 1 = 0$

$\Delta > 0$

$0 < k^2 - 4x - 1$ ✓

$0 < k^2 + 4$

$0 < (k-2)(k+2)$ ✓

$\therefore k = 2$ or $-2$ ✓

16 i) $\Delta \geq 0$

$\Delta$ ~~$\leq$~~ $(k+3)^2 - 4 \times (k+2)$

$= k^2+6k+9-4k-8$

$= k^2+2k+1 \Rightarrow (k+1)^2 \geq 0$ for all $k$

$0 \leq (k+1)(k+1) \quad \therefore k \geq -1$ for all $k$.

at least 1 real root.

~~$\Delta < 0$ $a > 0$ is a positive definite.~~

ii) $x = \frac{-k-3 \pm \sqrt{(k+1)^2}}{2(k+2)} = \frac{-k-3 \pm (k+1)}{2(k+2)}$

$= \frac{-(\sqrt{2}-2) - 3 \pm \sqrt{((\sqrt{2}-2)+1)^2}}{2(\sqrt{2}-2+2)} = \frac{-k-3+k+1}{2(k+2)}$ or

$= \frac{-\sqrt{2}+2-3 \pm \sqrt{2}-1}{2\sqrt{2}} \quad \frac{-k-3-k-1}{2(k+2)}$

$= \frac{-\sqrt{2}-1 \pm 1}{2\sqrt{2}}$

$\therefore x = \frac{-\sqrt{2}-2}{2\sqrt{2}} = \frac{-1}{2(k+2)}$ or $\frac{-2k-4}{2(k+2)}$

$= -2+2\sqrt{2} \quad -1$

$= \frac{-1+\sqrt{2}}{2}$ or

$x = \frac{-\sqrt{2}}{2\sqrt{2}}$

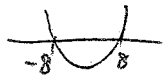
$= -\frac{2}{4}$

$= -\frac{1}{2}$

17 i) $a > 0 \quad \Delta < 0$

$0 > m^2 - 4 \times 16$

$0 > m^2 - 64$ ✓




$0 > (m-8)(m+8)$

$-8 < m < 8$ ✗

~~$m < -8 \quad m > 8$~~

(ii)

iii) $a > 0 \quad \Delta \geq 0$ ✓ Negative definite


$0 \leq m^2 - 64$

$0 \leq (m-8)(m+8)$

$-8 \leq m \leq 8$ ✗ $x < -8$ or $x > 8$

20 i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{6}{3}$

$= 2$ ✓

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{2}{3}$ ✓

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$= \frac{2}{\frac{2}{3}}$

$= \frac{6}{2}$

$= 3$ ✓

iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(\frac{2}{3})$

$= 4 - \frac{4}{3}$

$= 2\frac{2}{3}$ ✓

21. $(\alpha+\beta)^2 = 3\alpha\beta$

$\alpha + \beta = \frac{-b}{a}$

$= \frac{4-k}{1}$ ✓

$\alpha\beta = \frac{c}{a}$

$= \frac{k-3}{1}$

$= k-3$ ✓

$(4-k)^2 = 3(k-3) \Rightarrow 16 - 8k + k^2 = 3k - 9$

$4 - k = 3k - 9$ ✗

$13 = 4k$ ✗

$k = \frac{13}{4}$

$k^2 - 11k + 25 = 0$

$k = \frac{11 \pm \sqrt{121 - 100}}{2}$

$= \frac{11 \pm \sqrt{21}}{2}$

22. $a(x^2 + 2x + 1) + b(x+1) + c = 4x^2 + 4x + 1$

$ax^2 + 2ax + a + bx + b + c = 4x^2 + 4x + 1$

$ax^2 + 2ax + bx + a + b + c = 4x^2 + 4x + 1$

$ax^2 + (2a+b)x + (a+b+c) = 4x^2 + 4x + 1$

$a = 4$ ✓

$(2a + b) = 4$

$8 + b = 4$

$b = -4$ ✓

$a + b + c = 1$

$4 + (-4) + c = 1$

$c = 1$ ✓

$\therefore a = 4, b = -4, c = 1$

23 i) $mx + 4 = 2x^2 - 2x + 5$

$0 = 2x^2 - 2x - mx + 1$

$= 2x^2 - (2+m)x + 1$ ✓

$\Delta = 0$

$0 = (2+m)^2 - 4 \times 2 \times 1$

$= 4 + 4m + m^2 - 8$

$= m^2 + 4m - 4$ ✓

$m = \frac{-4 \pm \sqrt{16 - 4(-4)}}{2} = \frac{-4 \pm 4\sqrt{2}}{2}$

$= \frac{-4}{2}$ ✗

$= -2$

$= -2 \pm 2\sqrt{2}$ ✓

$= 2(-1 \pm \sqrt{2})$ ✓

ii) at $m = 2$

$2x + 4 = 2x^2 - 2x + 5$

$0 = 2x^2 - 4x + 1$

$\Delta > 0$

$0 < 0 - 4 \times 1 \times 2$

$0 < 8$

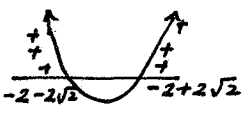
iii) $\Delta < 0$

$0 > 0 - 4 \times 1 \times 2$

$\Delta > 0$ $\therefore m^2 + 4m - 4 > 0$

~~$(m+2)^2 > 0$~~

~~True for all $m$.~~




$m > 2(-1+\sqrt{2})$ or $m < -2(1+\sqrt{2})$

(iii) $m^2 + 4m - 4 < 0 \Rightarrow -2-2\sqrt{2} < m < -2+2\sqrt{2}$

~~$(m+2)^2 < 0$~~

~~No values of $m$.~~

24 i) let $u = x^2$

$u^2 - 5u + 6 = 0$

$(u-3)(u-2) = 0$ ✓

$u = 3$ or $2$

$x^2 = 3$ or $x^2 = 2$

$= \pm\sqrt{3}$ ✓ $= \pm\sqrt{2}$ ✓

ii) let $u = 3^x$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$ ✓

$\therefore u = 3$ or $1$

$3^x = 3$ $3^x = 1$

$3^x = 3^1$ $3^x = 3^0$

$x = 1$ ✓ $x = 0$ ✓