TRIGONOMETRY EXPRESSIONS AND IDENTITIES– WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 11 Preliminary Mathematics.
Syllabus reference: 5.1 – 5.2.

1. Simplify the following trigonometric expressions:

	I	II	III
(a)	$\dfrac{1}{\sec\theta}$	$\dfrac{2}{\cos\theta}$	$(\operatorname{cosec}\theta)^{-1}$
(b)	$\sin\theta\sec\theta$	$\sin x\cot x$	$\tan\alpha\cot\alpha$
(c)	$\dfrac{\sin y}{\tan y}$	$\dfrac{2}{\sin\theta\cot\theta}$	$\dfrac{\sin\beta}{\tan\beta\cos\beta}$
(d)	$1-\cos^2 x$	$\dfrac{\sin^2\phi}{\cos^2\phi-1}$	$\dfrac{-\sin^2 A}{\cos^2 A-1}$
(e)	$\sin^2 z+\tan^2 z+\cos^2 z$	$\cos^2\delta+\cos^2\delta\tan^2\delta$	$\dfrac{1}{\sin^2 x}-\dfrac{1}{\tan^2 x}-\dfrac{1}{\sec^2 x}$
(f)	$5-5\cos^2 A$	$2+\cos^2 A-2\sin^2 A$	$2+\dfrac{2\sin^2 x}{\cos^2 x}$
(g)	$\cos\theta(1+\tan^2\theta)$	$\tan^2\theta(1-\sin^2\theta)$	$\dfrac{\tan^2 x-\cot^2 x}{\sec^2 x-\operatorname{cosec}^2 x}$
(h)	$\dfrac{\sqrt{1-\cos^2\alpha}}{\sin\alpha}$	$\sqrt{1+\dfrac{\sin^2 A}{\cos^2 A}}$	$\sqrt{\dfrac{4+4\tan^2 B}{9\sec^2 B-9}}$
(i)	$\dfrac{\sec x}{\tan x+\cot x}$	$(1-\sin x)(\sec x+\tan x)$	$\dfrac{\cos x}{1+\sin x}+\tan x$
(j)	$\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}$	$\dfrac{\cos\theta}{1-\sin\theta}-\dfrac{\cos\theta}{1+\sin\theta}$	$\dfrac{1}{\operatorname{cosec}\theta-1}-\dfrac{1}{\operatorname{cosec}\theta+1}$

2. Simplify the following expressions:

	I	II
(a)	$4-u^2$ if $u=2\sin\theta$	a^2-u^2 if $u=a\sin\theta$
(b)	$\sqrt{9-u^2}$ if $u=3\cos\theta$	$\sqrt{a^2-u^2}$ if $u=a\cos\theta$
(c)	$\dfrac{2}{\sqrt{4+x^2}}$ if $x=2\tan\theta$	$\dfrac{a}{\sqrt{a^2+x^2}}$ if $x=a\tan\theta$
(d)	$\left(4-x^2\right)^{\frac{3}{2}}$ if $x=2\sin\theta$	$\left(a^2-x^2\right)^{\frac{3}{2}}$ if $x=a\sin\theta$

3. Verify the following identities:

	I	II
(a)	$\cot x = \operatorname{cosec} x \cos x$	$\operatorname{cosec} x = \cot x \sec x$
(b)	$\sin x = \dfrac{\tan x}{\sec x}$	$\sin x = \dfrac{\tan x}{\sec x}$
(c)	$\cos^2 x\left(1+\tan^2 x\right)=1$	$\sin^2 x\left(1+\cot^2 x\right)=1$
(d)	$\dfrac{\cos\alpha\sec\alpha}{1+\tan^2\alpha}=\cos^2\alpha$	$\dfrac{1+\tan^2\theta}{1+\cot^2\theta}=\tan^2\theta$
(e)	$(1+\cos\theta)(1-\cos\theta)\sec^2\theta=\tan^2\theta$	$\dfrac{\cos x\left(1+\tan^2 x\right)}{\sec x}=1$
(f)	$(\cos\alpha+\sin\alpha)^2+(\cos\alpha-\sin\alpha)^2=2$	$\left(\sin^2 x+\cos^2 x\right)^2=1$
(g)	$\dfrac{\cos A}{1-\sin A}-\tan A=\sec A$	$\dfrac{\sin x}{1-\cos x}-\cot x=\operatorname{cosec} x$
(h)	$1-\sin\theta=(1+\sin\theta)(\sec\theta-\tan\theta)^2$	$1+\cos\theta=(1-\cos\theta)(\operatorname{cosec}\theta+\cot\theta)^2$

Trigonometric Expression & Identities

1a. I. $\frac{1}{\sec\theta} = \cos\theta$ ✓

II. $\frac{2}{\cos\theta} = 2\sec\theta$ ✓

III. $(\operatorname{cosec}\theta)^{-1}$

$= \frac{1}{\operatorname{cosec}\theta} = \sin\theta$ ✓

b. I. $\cancel{se}\ \sin\theta\sec\theta$

$= \sin\theta \times \frac{1}{\cos\theta}$

$= \tan\theta$ ✓

II. $\sin x \cot x$

$= \sin x \times \frac{1}{\tan x}$

$= \cancel{\sin x} \times \frac{\cos x}{\cancel{\sin x}}$

$= \cos x$ ✓

III. $\tan\alpha \cot\alpha$

$\frac{\cancel{\sin\alpha}}{\cancel{\cos\alpha}} \times \frac{\cancel{\cos\alpha}}{\cancel{\sin\alpha}}$

$= 1$ ✓

c. I. $\frac{\sin y}{\tan y}$

$= \sin y \div \frac{\sin y}{\cos y}$

$= \cancel{\sin y} \times \frac{\cos y}{\cancel{\sin y}}$

$= \cos y$ ✓

II. $\frac{2}{\sin\theta\cot\theta}$

$= 2 \div \left(\cancel{\sin\theta} \times \frac{\cos\theta}{\cancel{\sin\theta}}\right)$

$= \frac{2}{\cos\theta}$

$= 2\sec\theta$ ✓

III. $\frac{\sin\beta}{\tan\beta\cos\beta}$

$= \sin\beta \div \left(\frac{\sin\beta}{\cancel{\cos\beta}} \times \cancel{\cos\beta}\right)$

$= 1$ ✓

d. I. $1 - \cos^2 x$

$= \sin^2 x$ ✓

II. $\frac{\sin^2\phi}{\cos^2\phi - 1}$

$= \frac{\sin^2\phi}{-\sin^2\phi}$

$= -1$ ✓

III. $\frac{-\sin^2 A}{\cos^2 A - 1}$

$= \frac{-\sin^2 A}{-\sin^2 A}$

$= 1$ ✓

e. I. $\sin^2 z + \tan^2 z + \cos^2 z$

$= 1 + \tan^2 z$

$= \cancel{co}\ \sec^2 z$ ✓

II. $\cos^2\delta + \cos^2\delta\tan^2\delta$

$= \cos^2\delta\,(1 + \tan^2\delta)$

$= \cos^2\delta\,(\sec^2\delta)$ ✓

$= \cancel{\cos^2\delta} \times \frac{1}{\cancel{\cos^2\delta}}$

$= 1$ ✓

III. $\frac{1}{\sin^2 x} - \frac{1}{\tan^2 x} - \frac{1}{\sec^2 x}$

$= \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$

$= \frac{1 - \cos^2 x}{\sin^2 x} - \cos^2 x$ ✓

$= \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}} - \cos^2 x$

$= 1 - \cos^2 x = \sin^2 x$ ✓

f. I. $5(1-\cos^2 A)$

$= 5(\sin^2 A)$ ✓

II. $2+\cos^2 A - 2\sin^2 A$

$= 2-2\sin^2 A + \cos^2 A$

$= 2(1-\sin^2 A) + \cos^2 A$ ✓

$= 2(\cos^2 A) + \cos^2 A$

$= 3\cos^2 A$ ✓

III. $2 + \dfrac{2\sin^2 x}{\cos^2 x}$

$= 2\left(1 + \dfrac{\sin^2 x}{\cos^2 x}\right)$

$= 2(\sec^2 x)$

$= 2\sec^2 x$ ✓

g. I. $\cos\theta(\sec^2\theta)$

$\cancel{\cos\theta} \times \dfrac{1}{\cos^{\cancel{2}}\theta}$

$= \sec\theta$ ✓

II. $\tan^2\theta(1-\sin^2\theta)$

$= \dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}\left(\cancel{\cos^2\theta}\right)$

$= \sin^2\theta$ ✓

III. $\dfrac{\tan^2 x - \cot^2 x}{\sec^2 x - \operatorname{cosec}^2 x}$

$1+\tan^2 x = \sec^2 x$ (A)

$\cot^2 x + 1 = \operatorname{cosec}^2 x$ (B) ✓

(A) - (B)

$= \tan^2 x - \cot^2 x = \sec^2 x - \operatorname{cosec}^2 x$

$\therefore \dfrac{\tan^2 x - \cot^2 x}{\sec^2 x - \operatorname{cosec}^2 x}$

$= 1.$ ✓

h. I. $\dfrac{\sqrt{1-\cos^2\alpha}}{\sin\alpha}$

$= \dfrac{\sin\alpha}{\sin\alpha}$

$= 1$ ✓

II. $\sqrt{1+\tan^2 A}$

$= \sqrt{\sec^2 A}$

$= \sec A$ ✓

III. $\sqrt{\dfrac{4(1+\tan^2 B)}{9(\sec^2 B - 1)}}$

$= \sqrt{\dfrac{4(\sec^2 B)}{9(\tan^2 B)}}$

$= \dfrac{2(\sec B)}{3(\tan B)}$ ✓

$= \dfrac{2}{\cancel{\cos B}} \times \dfrac{\cancel{\cos B}}{3\sin B}$

$= \dfrac{2}{3\sin B}$

$= \dfrac{2}{3}\operatorname{cosec} B$ ✓

i. I. $\dfrac{\sec x}{\tan x + \cot x}$

$= \dfrac{1}{\cos x} \div \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)$

$= \dfrac{1}{\cos x} \div \left(\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)$ ✓

$= \dfrac{1}{\cos x} \div \dfrac{1}{\sin x \cos x}$

$= \dfrac{1}{\cancel{\cos x}} \times \dfrac{\sin x\,\cancel{\cos x}}{1}$

$= \sin x$ ✓

II. $(1-\sin x)(\sec x + \tan x)$

$(1-\sin x)\left(\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}\right)$ ✓

$= (1-\sin x)\left(\dfrac{1+\sin x}{\cos x}\right)$

$= \dfrac{1-\sin^2 x}{\cos x}$

$= \cos x$ ✓

III. $\dfrac{\cos x}{1+\sin x} + \tan x$

$= \dfrac{\cos x}{1+\sin x} + \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos^2 x + \sin x + \sin^2 x}{(1+\sin x)(\cos x)}$ ✓

$= \dfrac{\cancel{1+\sin x}}{(\cancel{1+\sin x})(\cos x)}$

$= \dfrac{1}{\cos x}$ ✓

$= \sec x$

j. I. $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A}$

$= \dfrac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1+\cos A)(\sin A)}$ ✓

$= \dfrac{1+1+2\cos A}{(1+\cos A)(\sin A)}$

$= \dfrac{2(\cancel{1+\cos A})}{(\cancel{1+\cos A})(\sin A)}$ ✓

$= \dfrac{2}{\sin A}$

$= 2\operatorname{cosec} A$ ✓

II. $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta}$

$\dfrac{\cos\theta + \sin\theta\cos\theta - \cos\theta + \sin\theta\cos\theta}{1-\sin^2\theta}$ ✓

$= \dfrac{2\sin\theta\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta}$ ✓

$= \dfrac{2\sin\theta}{\cos\theta}$

$= 2\tan\theta$ ✓

III. $\dfrac{1}{\operatorname{cosec}\theta - 1} - \dfrac{1}{\operatorname{cosec}\theta + 1}$

$= \dfrac{\operatorname{cosec}\theta + 1 - (\operatorname{cosec}\theta - 1)}{\operatorname{cosec}\theta - 1}$

$= \dfrac{\operatorname{cosec}\theta + 1 - \operatorname{cosec}\theta + 1}{\operatorname{cosec}^2\theta - 1}$ ✓

$= \dfrac{2}{\cot^2\theta}$

$= 2\tan^2\theta$ ✓

2a. I. $4 - 4\sin^2\theta$

$= 4(1 - \sin^2\theta)$

$= 4\cos^2\theta$ ✓

II. $a^2 - a^2\sin^2\theta$

$= a^2(1-\sin^2\theta)$

$= a^2\cos^2\theta$

$= (a\cos\theta)^2$ ✓

b. I. $\sqrt{9 - 9\cos^2\theta}$

$= \sqrt{9(1-\cos^2\theta)}$

$= \sqrt{9\sin^2\theta}$

$= 3\sin\theta$ ✓

II. $\sqrt{a^2 - a^2\cos^2\theta}$

$= \sqrt{a^2(\sin^2\theta)}$

$= a\sin\theta$ ✓

c. I. $\dfrac{2}{\sqrt{4+4\tan^2\theta}}$

$= \dfrac{2}{\sqrt{4(\sec^2\theta)}}$ ✓ $= \dfrac{\cancel{2}}{\cancel{2}\sec\theta}$

$= \dfrac{1}{\sqrt{\sec^2\theta}}$: $\underline{\cos\theta}$.

$= \cos^{\cancel{2}}\theta$

II. $\dfrac{a}{\sqrt{a^2 + a^2\tan^2\theta}}$ ✓

$= \dfrac{\cancel{a}}{\cancel{a}\sqrt{\sec^2\theta}} = \dfrac{1}{\sec\theta}$

$= \cos^{\cancel{2}}\theta$

2a) I- $4 - 4\sin^2\theta$
$= 4(1-\sin^2\theta)$
$= 4\cos^2\theta$ ✓

II- $a^2 - a^2\sin^2\theta$
$= a^2(1-\sin^2\theta)$
$= a^2\cos^2\theta = (a\cos\theta)^2$ ✓

b) I- $\sqrt{9-9\cos^2\theta}$
$= \sqrt{9(1-\cos^2\theta)}$
$= \sqrt{9\sin^2\theta}$
$= 3\sin\theta$ ✓

II- $\sqrt{a^2-a^2\cos^2\theta}$
$= \sqrt{a^2(1-\cos^2\theta)}$
$= \sqrt{a^2\sin^2\theta}$
$= a\sin\theta$ ✓

c) I- $\dfrac{2}{\sqrt{4+4\tan^2\theta}}$
$= \dfrac{2}{\sqrt{4(1+\tan^2\theta)}}$
$= \dfrac{2}{\sqrt{4\sec^2\theta}}$
$= \dfrac{1}{2\sec\theta}$
$= \dfrac{\cos\theta}{2}$ ✓

II- $\dfrac{a}{\sqrt{a^2+a^2\tan^2\theta}}$
$= \dfrac{a}{\sqrt{a^2(1+\tan^2\theta)}}$
$= \dfrac{a}{\sqrt{a^2\sec^2\theta}}$
$= \dfrac{a}{a\sec\theta} = \cos\theta$ ✓

d) I- $(4-4\sin^2\theta)^{\frac{3}{2}}$
$= (4(1-\sin^2\theta))^{\frac{3}{2}}$
$= (4\cos^2\theta)^{\frac{3}{2}}$
$= 8\cos^3\theta$ ✓

II- $(a^2-a^2\sin^2\theta)^{\frac{3}{2}}$
$= (a^2(1-\sin^2\theta))^{3/2}$
$= (a^2\cos^2\theta)^{3/2}$
$= a^3\cos^3\theta$ ✓

3.a) I - $\cot x = \operatorname{cosec} x \cos x$
RHS $= \operatorname{cosec} x \cdot \cos x$
$= \dfrac{1}{\sin x} \cdot \cos x$ ✓
$= \dfrac{\cos x}{\sin x}$
$= \cot x =$ LHS

II $\operatorname{cosec} x = \cot x \sec x$
RHS $= \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x}$
$= \dfrac{1}{\sin x}$ ✓
$= \operatorname{cosec} x =$ LHS

b) I $\sin x = \dfrac{\tan x}{\sec x}$
RHS $= \dfrac{\sin x}{\cos x} \times \cos x$ ✓
$= \sin x =$ LHS

3 c) I - $\cos^2 x\,(1+\tan^2 x) = 1$

$LHS = \cos^2 x\left(1 + \frac{\sin^2 x}{\cos^2 x}\right)$

$= \cos^2 x + \sin^2 x$ ✓

$= 1 = RHS$

II $\sin^2 x\,(1+\cot^2 x) = 1$

$LHS = \sin^2 x\left(1 + \frac{\cos^2 x}{\sin^2 x}\right)$

$= \sin^2 x + \cos^2 x$ ✓

$= 1 = RHS$

d) I - $\dfrac{\cos\alpha \sec\alpha}{1+\tan^2\alpha} = \cos^2\alpha$

$LHS = \dfrac{\cos\alpha \frac{1}{\cos\alpha}}{1 + \frac{\sin^2\alpha}{\cos^2\alpha}}$

$= \dfrac{1}{\frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha}}$

$= \dfrac{1}{\frac{1}{\cos^2\alpha}}$ ✓

$= \cos^2\alpha = RHS$

II - $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \tan^2\theta$

$LHS = \dfrac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$

$= \dfrac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$ ✓

$= \tan^2\theta = RHS$

Quicker $\dfrac{\sec^2\theta}{\operatorname{cosec}^2\theta}$

$= \dfrac{1}{\cos^2\theta} \times \dfrac{\sin^2\theta}{1}$

$= \tan^2\theta$

e) I - $(1+\cos\theta)(1-\cos\theta)\sec^2\theta = \tan^2\theta$

$LHS = (1-\cos^2\theta)\frac{1}{\cos^2\theta}$

$= \dfrac{1-\cos^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$ ✓ $= \tan^2\theta$

$= RHS$

II - $\dfrac{\cos x\,(1+\tan^2 x)}{\sec x} = 1$

$LHS = \dfrac{\cos x \cdot \sec^2 x}{\sec x}$

$= \dfrac{\cos x \frac{1}{\cos^2 x}}{\frac{1}{\cos x}}$ ✓

$= \dfrac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$ ✓

$= 1 = RHS$

f) I - $(\cos\alpha + \sin\alpha)^2 + (\cos\alpha - \sin\alpha)^2 = 2$

$LHS = \cos^2\alpha + 2\cos\alpha\sin\alpha + \sin^2\alpha + \cos^2\alpha - 2\cos\alpha\sin\alpha + \sin^2\alpha$

$= 2\cos^2\alpha + 2\sin^2\alpha$ ✓

$= 2(\cos^2 + \sin^2)$ ✓

$= 2 = RHS$

II - $(\sin^2 x + \cos^2 x)^2 = 1$

$LHS = 1^2$ ✓

$= 1 = RHS$

g) I- $\frac{\cos A}{1-\sin A} - \tan A = \sec A$

$$LHS = \frac{\cos A}{1-\sin A} - \frac{\sin A}{\cos A}$$

$$= \frac{\cos^2 A - \sin A + \sin^2 A}{\cos A\,(1-\sin A)}$$

$$= \frac{\cancel{1-\sin A}}{\cos A\,(\cancel{1-\sin A})}$$

$$= \frac{1}{\cos A}$$

$$= \sec A = RHS$$

II- $\frac{\sin x}{1-\cos x} - \cot x = \operatorname{cosec} x$

$$LHS = \frac{\sin x}{1-\cos x} - \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x - \cos x + \cos^2 x}{\sin x\,(1-\cos x)}$$

$$= \frac{\cancel{1-\cos x}}{\sin x\,(\cancel{1-\cos x})}$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x = RHS$$

h) I- $1-\sin\theta = (1+\sin\theta)(\sec\theta - \tan\theta)^2$

$$RHS = (1+\sin\theta)\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (1+\sin\theta)\left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= (1+\sin\theta)(1-\sin\theta)\left(\frac{1-\sin\theta}{\cos^2\theta}\right)$$

$$= (1-\sin^2\theta)\left(\frac{1-\sin\theta}{\cos^2\theta}\right)$$

$$= \cancel{\cos^2\theta}\left(\frac{1-\sin\theta}{\cancel{\cos^2\theta}}\right)$$

$$= 1-\sin\theta = LHS$$

II- $1+\cos\theta = (1-\cos\theta)(\operatorname{cosec}\theta + \cot\theta)^2$

$$RHS = (1-\cos\theta)\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= (1-\cos\theta)\left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= (1-\cos\theta)(1+\cos\theta)\left(\frac{1+\cos\theta}{\sin^2\theta}\right)$$

$$= (1-\cos^2\theta)\left(\frac{1+\cos\theta}{\sin^2\theta}\right)$$

$$= \cancel{\sin^2\theta}\left(\frac{1+\cos\theta}{\cancel{\sin^2\theta}}\right)$$

$$= 1+\cos\theta = LHS$$