MATHEMATICS PRELIMINARY EXTENSION 1
ASSESSMENT TASK
TEST 2

COURSE/LEVEL
NSW Secondary High School Year 11 Preliminary Extension Mathematics.

TOPICS
- Basic Arithmetic (Syllabus Reference: 1.1, 1.2)
- Algebra and Surds (Syllabus Reference: 1.3)
- Equations (Syllabus Reference: 1.4, 1.4E)
- Geometry 1 (Syllabus Reference: 2.1, 2.2, 2.3, 2.4)
- Functions and Graphs (Syllabus Reference: 4.1, 4.2, 4.3, 4.4, 6.4, 8.1, 8.2)
- Straight Line Graphs (Syllabus Reference: 6.1, 6.2, 6.3, 6.5, 6.6E, 6.7, 6.7E, 6.8)

TOTAL TIME: 45 MINUTES

INSTRUCTIONS

Attempt all questions

Show all necessary working

Approved calculators may be used.

Marks may be deducted for careless or poorly arranged work

QUESTION ONE **Marks**

(a) Simplify $\sqrt{(x+2)^2+2x+5}$. **2**

(b) Solve $(2x-1)(x+4)<0$ by first sketching a parabola. **2**

(c) Express the following with a common denominator and simplify: **3**

$$\frac{2}{x^2-x}+\frac{2}{x^2-3x+2}.$$

(d) Find the following limits: **2**

(i) $\lim_{x\to 2}\frac{x^2-x-2}{x-2}$.

(ii) $\lim_{x\to\infty}\frac{2x+1}{3x^2+x-1}$.

(e) If $f(x)=x^2+3x-7$, **3**

(i) find $f(-3)$,

(ii) find $f(k+1)$ in terms of k, expressed in its simplest form,

(iii) solve for x if $f(x)=3$.

QUESTION TWO **Marks**

(a) Let $f(x) = \dfrac{(3-x)(3+x)}{x^2-1}$. **3**

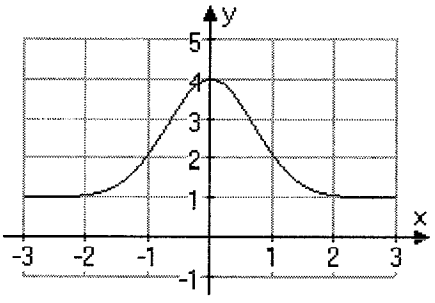
(i) Find the domain of this function.

(ii) Show that this is an even function. What does this tell you about the graph of this function?

(b) Sketch the graph of $y = \dfrac{x-2}{x-3}$ and clearly label all asymptotes and intercepts. **3**

(c) Show that the line $3x + 4y + 5 = 0$ is a tangent to the circle with centre (0, 0) and a radius of 1. **2**

(d) **1**



Shown above is a sketch of the function $y = f(x)$. The domain is all real x and the line $y = 1$ is an asymptote. The y–intercept is $y = 4$.
Write down the range of the function

(e) The angle between the straight lines $2x + y + 3 = 0$ and $y = mx + b$ is equal to θ where $\tan\theta = \sqrt{5}$. Show that **3**

$$m = \frac{\sqrt{5}-2}{2\sqrt{5}+1} \quad \text{or} \quad m = \frac{\sqrt{5}+2}{2\sqrt{5}-1}$$

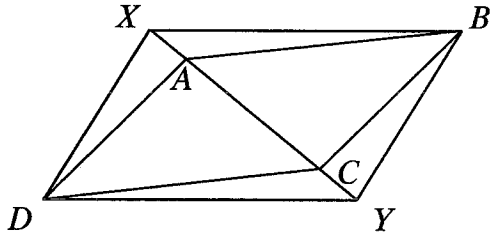
QUESTION THREE **Marks**

(a) Sketch the following on separate number planes: **3**

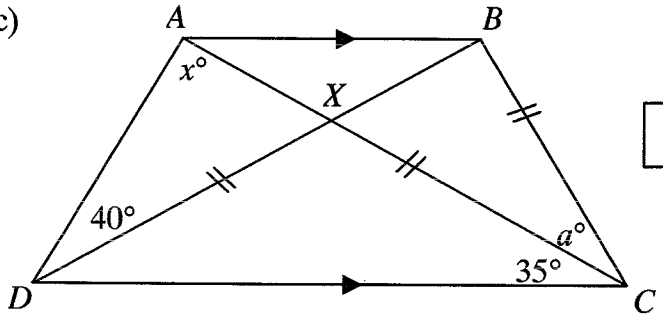
(i) $y = |x+1|$.

(ii) $|x+y| = 1$.

(b) $ABCD$ is a parallelogram, AC is produced to Y and CA to X such that such that $AX = CY$. Prove that $XBYD$ is a parallelogram. **3**



(c) **6**



NOT TO SCALE

In the diagram, $DX = CX = CB$ and $AB \parallel DC$. Several angles are labelled. Give reasons for your answer to the following.

(i) Explain why ΔAXB is isosceles.

(ii) Name two congruent triangles in the diagram.

(iii) Find a.

(iv) Find x.

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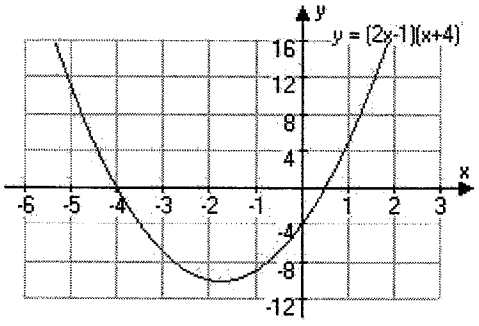
QUESTION ONE

(a)

$$\begin{aligned}\sqrt{(x+2)^2+2x+5} &= \sqrt{x^2+4x+4+2x+5} \\ &= \sqrt{x^2+6x+9} \\ &= \sqrt{(x+3)^2} \\ &= |x+3|\end{aligned}$$

(b)



The parabola $y = (2x - 1)(x + 4)$ has x intercepts $x = -4$ and $x = \frac{1}{2}$.

From the graph, $(2x - 1)(x + 4) < 0 \Rightarrow -4 < x < \frac{1}{2}$.

(c)

$$\frac{2}{x^2-x}+\frac{2}{x^2-3x+2}=\frac{2}{x(x-1)}+\frac{2}{(x-1)(x-2)}$$

$$=\frac{2(x-2)+2x}{x(x-1)(x-2)}$$

$$=\frac{2x-4+2x}{x(x-1)(x-2)}$$

$$=\frac{4x-4}{x(x-1)(x-2)}$$

$$=\frac{4(x-1)}{x(x-1)(x-2)}$$

$$=\frac{4}{x(x-2)}$$

(d) (i)

$$\lim_{x\to 2}\frac{x^2-x-2}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+1)}{x-2}$$

$$=\lim_{x\to 2}x+1$$

$$=3$$

(ii)

$$\lim_{x\to\infty}\frac{2x+1}{3x^2+x-1}=\lim_{x\to\infty}\frac{2x+1}{3x^2+x-1}\div\frac{x^2}{x^2}$$

$$=\lim_{x\to\infty}\frac{\frac{2}{x}+\frac{1}{x^2}}{3+\frac{1}{x}-\frac{1}{x^2}}$$

$$=\frac{0}{3}$$

$$=0$$

(e) $f(x)=x^2+3x-7$

(i) $f(x)=(-3)^2+3(-3)-7=-7$

(ii) $f(k+1) = (k+1)^2+3(k+1)-7$

$$=k^2+2k+1+3k+3-7$$

$$=k^2+5k-3$$

(ii) $f(x) = 3$

$x^2 + 3x - 7 = 3$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = -5, 2$

QUESTION TWO

(a) Let $f(x) = \dfrac{(3-x)(3+x)}{x^2-1}$.

(i) Domain: $x^2 - 1 \neq 0$

ie $x \neq \pm 1$

(ii) $f(-x) = \dfrac{(3-(-x))(3+(-x))}{(-x)^2-1}$

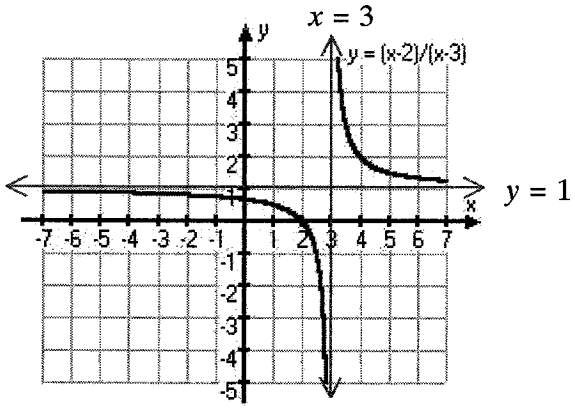
$= \dfrac{(3+x)(3-x)}{x^2-1}$

$= f(x)$

$\therefore$ $f(x)$ is even.

The graph of $y = f(x)$ is symmetric about the y–axis.

(b) $y = \dfrac{x-2}{x-3}$.



Working:

x intercept: When $y = 0$, $x = 2$.

y intercept: When $x = 0$, $y = \frac{2}{3}$.

$y \approx \frac{x}{x}$ for large x. Hence $y = 1$ is a horizontal asymptote.

Sign of y in "critical" regions:

For $x < 2$, $y > 0$.

For $2 < x < 3$, $y < 0$.

For $x > 3$, $y > 0$.

(c) Distance from point (0, 0) to line $3x + 4y + 5 = 0$:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|3 \times 0 + 4 \times 0 + 5|}{\sqrt{3^2 + 4^2}}$$

$$= 1,$$

which is equal to the radius of the circle. Hence the line is a tangent to the circle.

(d) $1 < y \leq 4$.

(e) $2x + y + 3 = 0$, $y = mx + b$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

or

$$\sqrt{5} = \left|\frac{m+2}{1-2m}\right|$$

$$\therefore \quad \frac{m+2}{1-2m} = \sqrt{5} \quad \text{or} \quad \frac{m+2}{1-2m} = -\sqrt{5}$$

$$m + 2 = \sqrt{5}(1 - 2m) \qquad m + 2 = -\sqrt{5}(1 - 2m)$$

$$m + 2 = \sqrt{5} - 2\sqrt{5}m \qquad m + 2 = 2\sqrt{5}m - \sqrt{5}$$

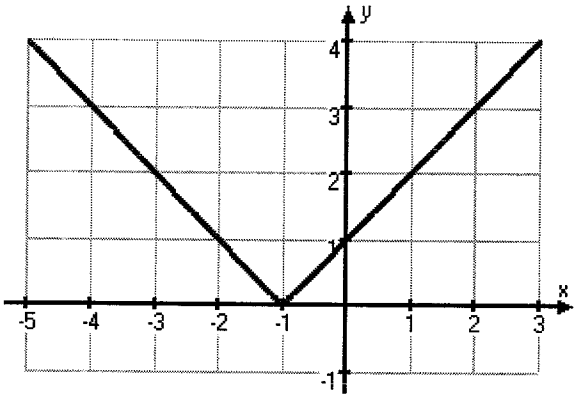
$$2\sqrt{5}m + m = \sqrt{5} - 2 \qquad 2\sqrt{5}m - m = \sqrt{5} + 2$$

$$m = \frac{\sqrt{5} - 2}{2\sqrt{5} + 1} \qquad m = \frac{\sqrt{5} + 2}{2\sqrt{5} - 1}$$

$$\text{ie } m = \frac{\sqrt{5} - 2}{2\sqrt{5} + 1} \quad \text{or} \quad m = \frac{\sqrt{5} + 2}{2\sqrt{5} - 1}$$

QUESTION THREE

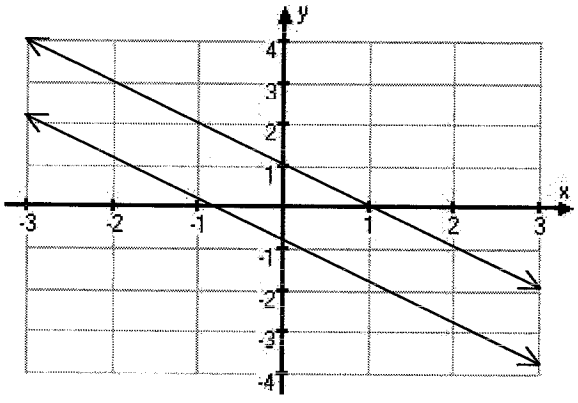
(a) (i) $y = |x+1|$.



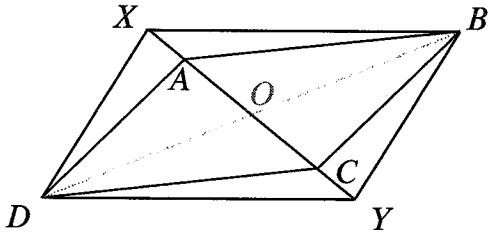
(ii) $|x+y|=1$.

$x + y = 1$ or $x + y = -1$

ie $y = 1 - x$ or $y = -x - 1$.



(b)



Construct diagonal BD bisected meeting AC at O.

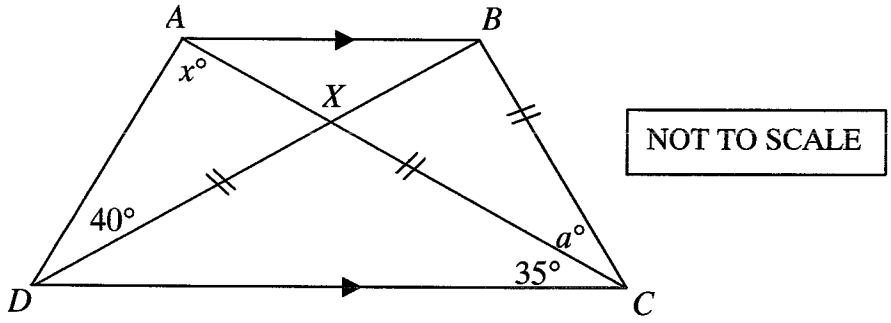
$OB = OD$ and $OA = OC$ (diagonals of ||gram $ABCD$ bisect each other)

But $OA + AX = OC + CY$ ($AX = CY$ as given)

$\therefore OX = OY$

$\therefore XBYD$ is a parallelogram (diagonals bisect each other)

(c)



(i) $\angle BDC = 35°$ (equal base angles of isos ΔDXC)

$\angle ABD = 35°$ (alt $\angle$s, $AB \parallel DC$)

$\angle CAB = 35°$ (alt $\angle$s, $AB \parallel DC$)

$\therefore \Delta AXB$ is isosceles (base angles equal)

(ii) $\Delta AXD \equiv \Delta BXC$

Reasons:

In ΔAXD and ΔBXC,

$AX = BX$	(equal sides of isos ΔDXC)	(S)
$\angle AXD = \angle BXC$	(vert. opp $\angle$s)	(A)
$DX = CX$	(given)	(S)

$\therefore \Delta AXD \equiv \Delta BXC$ (SAS)

(iii) $a = 40$ (corr $\angle$s in cong Δs)

(iv) $\angle ADC = 40° + 35° = 75°$ (adjacent angles)

$\therefore x° + 75° + 35° = 180°$ ($\angle$sum of ΔADC)

$\therefore x° = 70°$