PROPERTIES OF QUADRILATERALS – WORKSHEET

COURSE/LEVEL

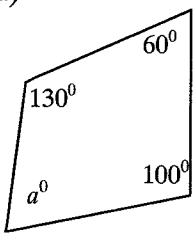
NSW Secondary High School Year 11 Preliminary Mathematics.

TOPIC

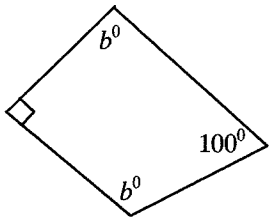
Plane Geometry: Properties of Quadrilaterals. (Syllabus Ref: 2.2)

1. Find the value of each pronumeral.

(a)



(b)



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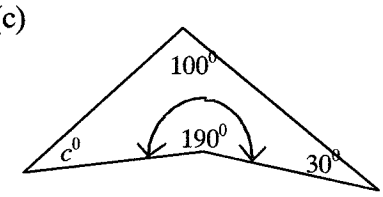
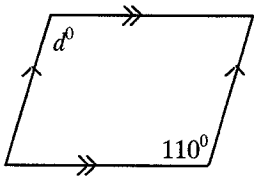
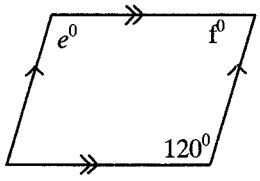


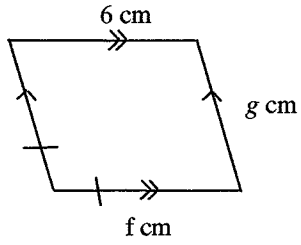
(d)



(e)



(f)



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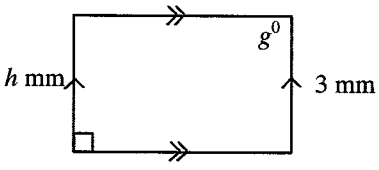

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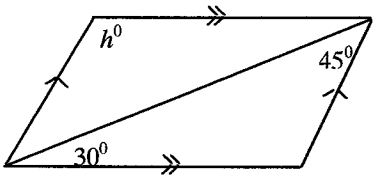
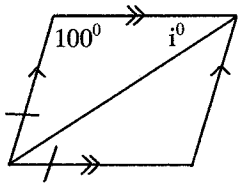
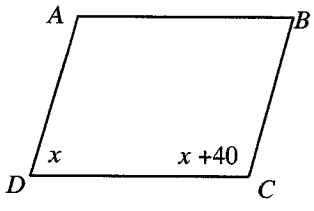


(i)

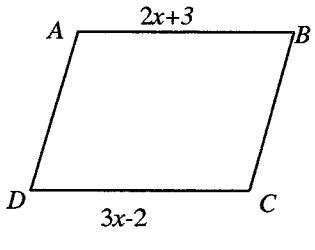


2. In each of the following, *ABCD* is a parallelogram. Find x, giving reasons.

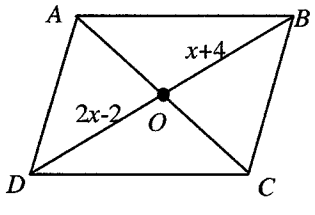
(a)



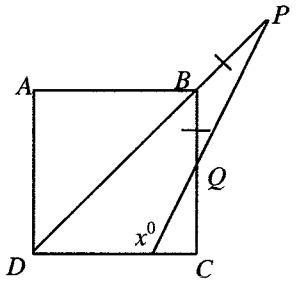
(b)



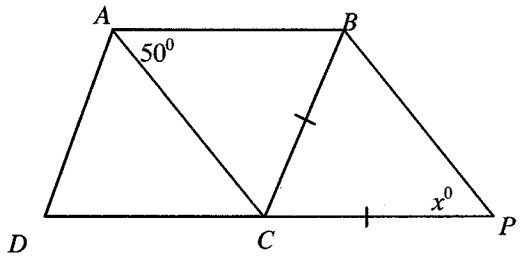
(c)



3. (a) $ABCD$ is a square and $BP = BQ$.
Find x, giving reasons.



3. (b) $ABCD$ is a rhombus and $CP = BC$.
Find x, giving reasons.

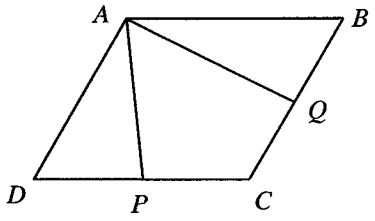


4. $ABCD$ is a rhombus. P and Q are midpoints of sides CD and BC respectively. Show that

(i) $\Delta ADP \equiv \Delta ABQ$

(ii) $AP = AQ$

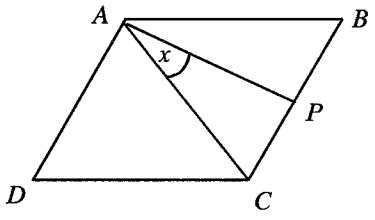
(iii) $\angle APC = \angle AQC$



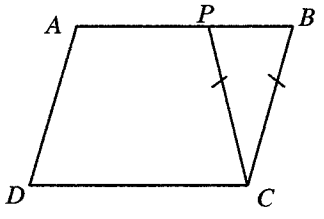
5. $ABCD$ is a rhombus. AP bisects $\angle CAB$. Let $x = \angle CAP$ and show that

(i) $\angle BAD = 4x$

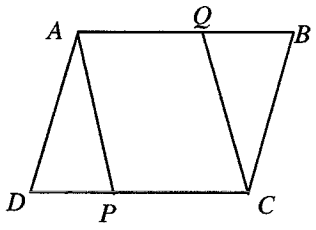
(ii) $\angle APB = 3x$



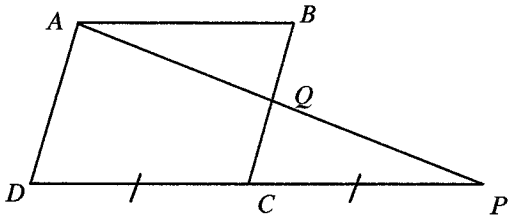
6. $ABCD$ is a parallelogram. $CP = BC$. Show that $\angle ADC = \angle PCD$.



7. $ABCD$ and $AQCP$ are parallelograms. Show that $\Delta ADP \equiv \Delta QBC$.



8. $ABCD$ is a parallelogram. DC is produced to P such that $CP = CD$. AP intersects BC at Q.
Show that $CQ = BQ$.

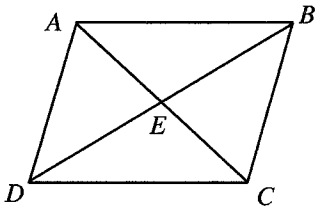


9. $ABCD$ is a parallelogram. Diagonals AC and BD intersect at E.

(i) Show that $\Delta AEB \equiv \Delta DEC$

(ii) Hence show that $DE = BE$ and $AE = CE$.

(iii) Which property of parallelograms does this prove?

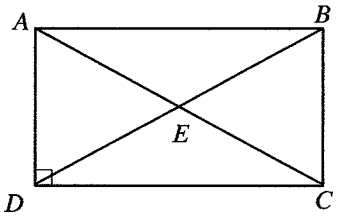


10. $ABCD$ is a rectangle. Diagonals AC and BD intersect at E.

(i) Show that $\Delta ABC \equiv \Delta ADB$.

(ii) Hence show that $AC = BD$.

(iii) Which property of rectangles does this prove?



Excellent work!

Properties of Quadrilaterals – Worksheet

1 a) $a^\circ = 360 - (100 + 130 + 60)$ ✓
$= 70^\circ$ (∠ sum of quadrilateral) ✓

b) $2b = 360 - (90 + 100)$ ✓
$b = 85^\circ$ (∠ sum of quadrilateral) ✓

c) $c^\circ = 360 - (190 + 100 + 30)$ ✓
$= 40^\circ$ ✓

d) $d^\circ = 110^\circ$ ✓ (opposite ∠s, prop. of ||gram)

e) $e = 120^\circ$ ✓ (" ")
$f = 180 - 120$
$= 60^\circ$ ✓ (co-interior ∠s, || lines)

f) $f = 6$ cm (opp. sides of parallelogram)
$g = 6$cm ✓

g) $g^\circ = 90^\circ$ ✓ (opp. ∠ of rectangle)
$h = 3$mm ✓ (opp. sides of rectangle)

h) $h = 105^\circ$ ✓ (∠ sum of Δ, opp. ∠'s of parallelogram)

i) $i^\circ = (180 - 100) \div 2$ (∠ sum of isoceles Δ)
$= 40^\circ$ ✓

2. a) $x + x + 40 = 180$ (co-interior ∠s, || lines) ✓
$2x = 140$
$x = 70^\circ$ ✓

b) $2x + 3 = 3x - 2$ (equal opp. sides of ✓ parallelogram)
$-x = -5$
$x = 5$u. ✓

c) $2x - 2 = x + 4$ (diagonals bisect each other)
$x = 6$ ✓

3. a) $\angle PDC = 45^\circ$ (diagonals bisect ∠ADC)
$\angle QBP = 180 - 45^\circ$ (supplementary ∠s)
$= 135^\circ$
$\angle BQP = (180 - 135) \div 2$
$= 22.5^\circ$ ✓
$\therefore x^\circ = 180 - 22.5 - 45$ (∠ sum of Δ)
$= 112.5^\circ$ ✓

3 b) $\angle DAB = 2 \times \angle CAB$ (AC bisects $\angle DAB$)
$= 2 \times 50^\circ$
$= 100^\circ$ ✓

$\therefore \angle DCB = 100^\circ$
$\angle BCP = 180 - 100^\circ$ (supplementary $\angle$)
$= 80^\circ$ ✓

$x^\circ = \frac{180 - 80}{2}$ ($\angle$ sum of isosceles $\triangle$)

$= 50^\circ$ ✓

4. i. $DC = CB$ (equal sides of rhombus)
$\therefore DP = QB$ (P and Q bisect DC and CB) ✓
$\angle ADP = \angle ABQ$ (opp. $\angle$s of rhombus) ✓ ✓
$AD = AB$ (equal sides of rhombus)
$\therefore \triangle ADP \equiv \triangle ABQ$ (SAS) ✓
ii. $AP = AQ$ (corresp. sides, congruent $\triangle$'s) ✓
iii. $\angle APC = \angle AQC$ (corresp. $\angle$s, congruent $\triangle$s) ✓

5. i. $\angle DAC = \angle CAB$ (AC bisects $\angle DAB$)
$\angle CAP = \angle DAB$ (AP bisects $\angle CAB$)
$\angle DAC = 2x$. ✓
$\therefore \angle BAD = 2 \times 2x$
$= 4x$ ✓

ii. $\angle DAC = 2x$ (proven)
$\angle CAP = x$ (given)
$\angle APB = \angle DAC + \angle CAP$
$= 2x + x$
$= 3x$ ✓

6. $\angle PBC = \angle BPC$ (base $\angle$s of isosceles $\triangle$)
$\angle ADC = \angle ABC$ (opp. $\angle$s of parallelogram)
$= \angle PBC$ (proven) ✓
$\angle PBC = \angle BPC$ (proven)
$\angle BPC = \angle PCD$ (alt. $\angle$s, parallel lines)
$\therefore \angle ADC = \angle PCD$. ✓

7. $AB = DC$ (prop. of ||gram)
$AQ = \_$ (")
$AQ + QB = PC + DP$ ✓
$\therefore QB = DP$
$\angle ADP = \angle QPC$ (prop. of ||gram)
$AD = BC$ (")
$\therefore \Delta ADP = \Delta QBC$ (SAS) ✓

8. In Δs ABQ and PCQ:
$\angle AQB = \angle CQP$ (vert. opp. $\angle$s)
$DC = AB = CP$ (∥ prop. of ||gram, given)
$CP = AB$
$\angle ABQ = \angle PCQ$ (alt $\angle$s, $AB \parallel DP$)
$\therefore \Delta ABQ = \Delta PCQ$ (SAS) ✓

9. i. In Δs AEB and DEC:
$AB = DC$ (prop. of ||gram) ✓
$\angle BAC = \angle DCA$ (alt. $\angle$s, $AB \parallel DC$) ✓
$\angle DEC = \angle BEA$ (vert. opp. $\angle$s) ✓
$\therefore \Delta AEB \cong \Delta DEC$ (AAS) ✓
ii. $AE = EC$ (corresp. sides, congruent Δs) ✓
$DE = EB$ (") ✓
iii. That the diagonals of a parallelogram bisect each other.

10. In Δs ABC and ADB
$AD = BC$ (opp. sides of rectangle) ✓
$\angle DAC = \angle ABC$ (prop. of rectangle) ✓
AB is common. ✓
$\therefore \Delta ABC \cong \Delta ADB$ (SAS)
ii. $AC = BD$ (corresp. sides, congruent Δs) ✓
iii. That the diagonals of a rectangle are equal. ✓