PREPARATORY PRELIMINARY MATHEMATICS
WORKSHEET #4

COURSE/LEVEL

NSW Secondary High School Year 11 Preliminary Mathematics.

1. The following statements seem quite familiar in many ways but there is actually an enormous difference between them. Explain the difference.

Statement 1: $(x-3)^2 = x^2 - 6x + 9$ **Statement 2**: $(x-3)^2 = 2x^2 - 7x + 3$

2. Find a and b to satisfy $a - b\sqrt{3} = \dfrac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$.

3. If θ is any angle, show that $\sin^2\theta + \cos^2\theta = 1$.

4. Find a quadratic equation which has $3+\sqrt{5}$ and $3-\sqrt{5}$ as its solutions.

5. (a) If $y = 3 - \sqrt{x+7}$, find an expression for y^2 in terms of x.

   (b) Solve for x if $\sqrt{x} + \sqrt{x+7} = 3$. (There is a bit of a trick to this .... see what you can find).

6. There's an easy way to find the sum of all the integers from 1 up to 10 000. Find this sum.

7. Find the remainder when the polynomial $P(x) = 4x^3 - x^2 + 2x - 5$ is divided by $(x+1)$.

8. Simplify: (a) $\log_3 36 - 2\log_3 2$ (b) $\dfrac{\log_5 8 - \log_5\left(\frac{1}{64}\right) - \log_5 4}{\log_5 2}$ (c) $5\log_{32} 4$

9. A hollow tank in the shape of an inverted square pyramid has a maximum depth of 9 metres and a width of 12 metres.

   (a) What is the total volume of the tank in metres?

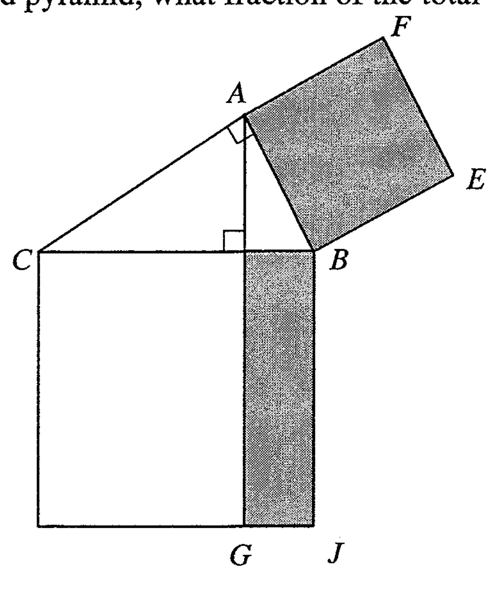
   (b) Given that the tank contains water to a depth of 3 metres, and that the space occupied by the water is aslo in the shape of an inverted pyramid, what fraction of the total volume of the tank is filled by water?

10. ABC is a right angled triangle. Sides AB and CB also form the sides of two adjacent squares. AH is perpendicular to CB and meets the opposite side of the larger square at G.

    (a) Show that the triangles ABC and HBA are similar.

    (b) Show that rectangle $HGJB$ and square $ABEF$ have the same area.

    (c) Hence prove Pythagoras' Theorem.



Prepatory Preliminary Mathematics Worksheet #4

V. Good effort!

$(x-3)^2 = x^2-6x+9$

1. $(x-3)^2 = 2x^2-7x+3$

$= x^2 - x - 6$

$\neq (x-3)(x+2)$ $\therefore$ the difference is shown.

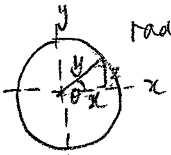
2. $\sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}} = a - b\sqrt{3}$

$\dfrac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} \times \dfrac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}} = \dfrac{4-4\sqrt{3}+3}{1}$

$= 7-4\sqrt{3}$ ✓

$\therefore a = 7$ ✓

$b = 4$ ✓

3. Using  radius = 1

LHS : $\sin^2\theta + \cos^2\theta$

$= \left(\dfrac{z}{y}\right)^2 + \left(\dfrac{x}{y}\right)^2$

$= \dfrac{z^2}{y^2} + \dfrac{x^2}{y^2}$ ✓

$= \dfrac{z^2+x^2}{y^2}$

But, $x^2+z^2 = y^2$ ✓

$\therefore \dfrac{y^2}{y^2} = 1$ ✓

$\therefore$ LHS = RHS

4. $ax^2+bx+c=0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\alpha = 3+\sqrt{5}$, $\beta = 3-\sqrt{5}$

$\alpha+\beta = 6$ ✓ $\alpha\beta = 4$ ✓

$\therefore x^2-6x+4=0$ ✓

5. (a) $y^2 = 9 + (x+7) - 6\sqrt{x+7}$

$y^2 = 2 - x$ ✗

(b) $(\sqrt{x} + \sqrt{x+7})^2 = 3^2$

$x + 2\sqrt{x^2+7} + x+7 = 9$

$2x + 2\sqrt{x^2+7} = 2$

Try this ↙ $x + \sqrt{x^2+7x} = 1$

~~xx~~ $1-x = \sqrt{x^2+7x}$

squaring again

$x^2+x^2+7x=1$

$2x^2+7x-1=0$

$x = \dfrac{-7\pm\sqrt{57}}{4} = \dfrac{-7+\sqrt{57}}{4}$ ✗

Qu 6 $10001 \times 5000 = 50005000$

Qu 7 $(x+1)\overline{)\,4x^3-x^2+2x-5}$ quotient $4x^2-5x+1$

$4x^3+4x^2$ ✓

$-5x^2+2x$

$-5x^2-5x$

$7x-5$

$7x+7$

-12 ✓

How about using

$P(-1) = -4-1-2-5$

$= -12$?

$\therefore P(x) = (4x^2-5x+1)(x+1) - 12$

$\therefore R = -12$

Qu 8 (a) $\log_3 36 - 2\log_3 2$

$= \log_3 3^2 + \log_3 4 - \log_3 2^2$ ✓

$= 2 + \log_3 1$

$= 2$ ✓

(b) $\dfrac{3\log_5 2 - \log_5 2^{-6} - 2\log_5 2}{\log_5 2}$

$= \dfrac{3\log_5 2 + 6\log_5 2 - 2\log_5 2}{\log_5 2}$ ✓

$= \dfrac{\log_5 512 - 2\log_5 2}{\log_5 2}$

$= \dfrac{\log_5 128}{\log_5 2} = \dfrac{7\log_5 2}{\log_5 2}$ ✓

$= 7$

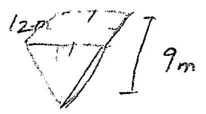
(c) $5\log_{32} 4$

$= \log_{32} 4^5$ ✓

$= \log_{32} 32^2$ ✓

$= 2$

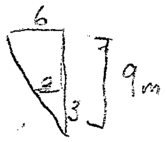
Qu9 (a)



$V = 12^2 \times 9 \times \frac{1}{3}$ ✓

$= 432 \text{ m}^3$

(b)



$\frac{16}{432} = \frac{1}{27}$ ✓

Qu10 In ΔABC, HBA,

$\angle B$ is common

$\angle BAC = \angle BHA$

$\therefore \Delta ABC \ ||| \ HBA$ ✓

(b) A of $ABEF = AB^2$

A of $HGJB$:

$HB \times CB = HB \cdot BC$ ✓

Since $\Delta ABC \ ||| \ \Delta HBA$,

$\frac{AB}{HB} = \frac{BC}{AB}$ ✓

$\therefore HB \cdot BC = AB^2$

$\therefore$ A of $ABEF$ = A of $HGJB$

(c) In similar Δs AHC, ΔBAC,

$\frac{AC}{BC} = \frac{HC}{AC}$ ✓

$AC^2 = HC \cdot BC$ — ①

In similar Δs ΔABH, ΔCBA

$\frac{AB}{BC} = \frac{HB}{AB}$ ✓

$\therefore AB^2 = HB \cdot BC$ — ②

① + ② : $AB^2 + AC^2 = HC \cdot BC + HB \cdot BC$ ✓ V. Good!

$= BC\,(HC + HB)$

$= BC^2$

$\therefore BC^2 = AB^2 + AC^2$ ✓