# PREPARATORY PRELIMINARY MATHEMATICS WORKSHEET #5

## COURSE/LEVEL

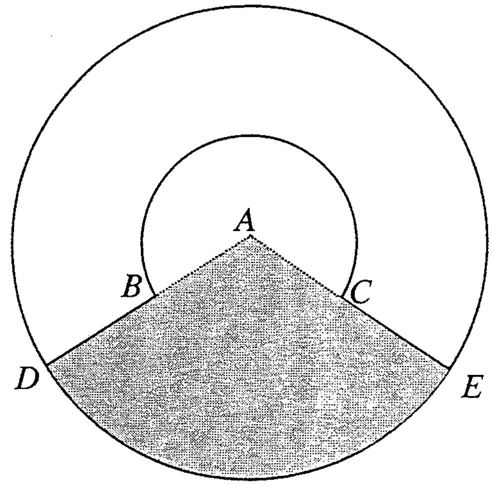
NSW Secondary High School Year 11 Preliminary Mathematics.

1. Factorise: (a) $x^2 - \dfrac{1}{x^2}$ (b) $a^4 - 9$ (c) $\left(\dfrac{x}{y}\right)^3 - \left(\dfrac{y}{x}\right)^3$

2. $2x^2 - 9x + 14 - \dfrac{9}{x} + \dfrac{2}{x^2} = 0$ is an interesting equation with at least three solutions. By letting $t = x + \dfrac{1}{x}$, find an expression for $t^2$, and then replace all $x$ terms in the equation to find a simpler equation in terms of $t$ only. Solve for $t$ first and then solve for $x$. Check that your solutions are correct.

3. A square with side $b$ units is inscribed inside a circle. Find an expression for the circumference of the circle.

4. The smaller circle has half the radius of the larger circle and it also has the same area as the shaded annular sector. Find the size of angle $BAC$.




5. Simplify $\dfrac{1}{k} - \dfrac{2+k}{k^2} + \dfrac{3}{k^3}$.

6. Simplify $\left(\dfrac{a^{-1}}{b^2}\right)^{-1}\left(\dfrac{a}{b^{-2}}\right)^{-3}\left(\dfrac{b}{a^4}\right)^0$.

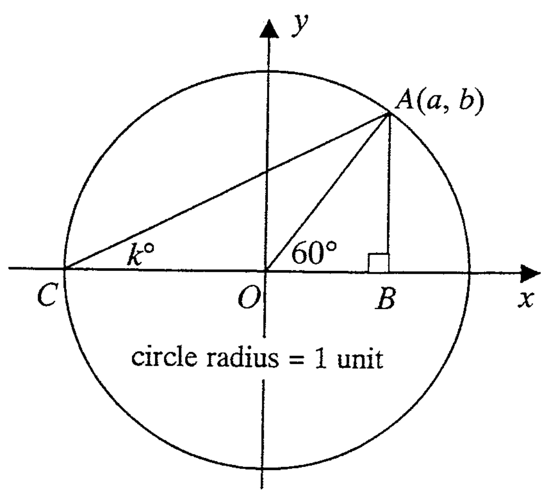
7. Rationalise the denominator: $\dfrac{1}{\sqrt{y} - \sqrt{y-1}}$.

8. In the diagram, $ABC$ is a right angled triangle and points $A$ and $C$ lie on the circle which has its centre at the origin $O$ and a radius of 1 unit. $\angle AOB = 60°$. Point $A$ has coordinates $(a, b)$.

   (i) Find $\angle ACB$, giving reasons.

   (ii) Show that $CA$ has gradient $\dfrac{1}{\sqrt{3}}$.

   (iii) Hence show that $\dfrac{b^2}{(1+a)^2} = \dfrac{1}{3}$, and that $a = \frac{1}{2}$.




9. Find four consecutive odd integers whose sum is 128.

Stephanie Pham

# Preparatory Preliminary Maths - Worksheet #5

V. Good work!

See corrections on next page.

1) a. $x^2 - \frac{1}{x^2} = \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$ ✓

b. $a^4 - 9 = (a^2-3)(a^2+3) = (a+\sqrt{3})(a-\sqrt{3})(a^2+3)$ ✓

c. $\left(\frac{x}{y}\right)^3 - \left(\frac{y}{x}\right)^3 = \left(\frac{x}{y} - \frac{y}{x}\right)\left(\frac{x^2}{y^2} + \frac{xy}{xy} + \frac{y^2}{x^2}\right)$

$= \left(\frac{x}{y} - \frac{y}{x}\right)\left(\frac{x^2}{y^2} + \frac{y^2}{x^2} + 1\right)$ ✓

2) $t^2 = \left(x + \frac{1}{x}\right)^2$

$= x^2 + 2 + \frac{1}{x^2}$

$t^2 - 2 = x^2 + \frac{1}{x^2}$ ✓

$2x^2 + \frac{2}{x^2} = 2\left(x^2 + \frac{1}{x^2}\right)$

$\therefore 2(t^2-2) = 2x^2 + \frac{2}{x^2}$

$-9x - \frac{9}{x} = -9\left(x + \frac{1}{x}\right)$

$= -9t$

$2x^2 - 9x + 14 - \frac{9}{x} + \frac{2}{x^2} = 0$

$2(t^2-2) - 9t + 14 = 0$ ✓

$2t^2 - 4 - 9t + 14 = 0$

$2t^2 - 9t + 10 = 0$

$2t^2 - 5t - 4t + 10 = 0$

$2t(t-2) - 5(t-2) = 0$

$(2t-5)(t-2) = 0$

$\therefore t = \frac{5}{2}$ or $2$ ✓

$t = x + \frac{1}{x}$

$\frac{5}{2} = x + \frac{1}{x}$

$5x = 2x^2 + 2$

$2x^2 - 4x - x + 2 = 0$

$2x(x-2) - (x-2) = 0$ ✓

$(2x-1)(x-2) = 0$

$\therefore x = \frac{1}{2}$ or $2$ ✓

$t = x + \frac{1}{x}$

$2 = x + \frac{1}{x}$ ✓

$2x = x^2 + 1$

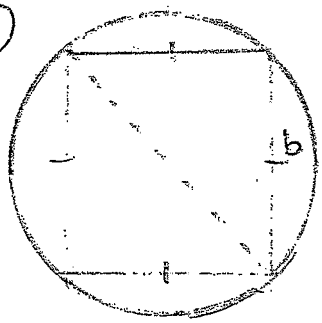
$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$\therefore x = 1$ ✓

V. Good work!

3)



diagonal of square $= b^2 + b^2$

$d^2 = 2b^2$ ✗

$d = \sqrt{2}b$

$\sqrt{2}b$ is also diameter of circle.

$\therefore$ circumference $=$ ~~$2\sqrt{2}\pi$~~ units

$= \pi d$

$= \sqrt{2}\pi b$

4) let radius of smaller circle be $x$,

$\therefore$ area of smaller circle $= \pi x^2$

$\therefore$ radius of larger circle $= 2x$

area of larger circle $= 4x^2\pi$

$\frac{\pi x^2}{4x^2\pi} = \frac{1}{4}$

$\therefore \angle BAC = \frac{1}{4} \times 360$

$= 90°$

5) $\frac{1}{k} - \frac{2+k}{k^2} + \frac{3}{k^3} = \frac{k^2 - 2k - k^2 + 3}{k^3}$ ✓

$= \frac{-2k+3}{k^3}$ ✓

6) $\left(\frac{a^{-1}}{b^2}\right)^{-1}\left(\frac{a}{b^{-2}}\right)^{-3}\left(\frac{b}{a^4}\right)^0 = \left(\frac{1}{ab^2}\right)^{-1} \times \frac{1}{a^3b^6}$

$= ab^2 \times \frac{1}{a^3b^6}$ ✓

$= \frac{ab^2}{a^3b^6}$

$= \ldots$ ✓

7) $\dfrac{1}{\sqrt{y}-\sqrt{y-1}} \times \dfrac{\sqrt{y}+\sqrt{y-1}}{\sqrt{y}+\sqrt{y-1}}$

$= \dfrac{\sqrt{y}+\sqrt{y-1}}{y-(y-1)}$

$= \dfrac{\sqrt{y}+\sqrt{y-1}}{+1}$

) i. $\cos 60° = \dfrac{OB}{1}$

$OB = \cos 60°$

$= \dfrac{1}{2}$

$\therefore COB = 1\frac{1}{2}$ ✓

$\tan 60° = \dfrac{AB}{\frac{1}{2}}$

$AB = \frac{1}{2} \tan 60$

$= \dfrac{\sqrt{3}}{2}$

$\tan k° = \dfrac{\frac{\sqrt{3}}{2}}{1\frac{1}{2}}$

$= \dfrac{\sqrt{3}}{2} \div \dfrac{3}{2}$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{2}{3}$

$= \dfrac{2\sqrt{3}}{6}$

$= \dfrac{\sqrt{3}}{3}$ ✓

$\therefore \underline{k = 30°}$ ($\angle ACB = k°$)

or alternatively

$k° + k° + 30° = 90°$

($\angle$ sum of $\Delta$)

$\therefore 2k° = 60°$

$\underline{k° = 30°}$

ii. $m_{CA} = \dfrac{\sqrt{3}}{2} \div \dfrac{3}{2}$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{2}{3}$ ✓

$= \dfrac{2\sqrt{3}}{6}$

$= \dfrac{\sqrt{3}}{3}$ (or $\dfrac{1}{\sqrt{3}}$) ✓

iii. $\dfrac{b^2}{(1+a)^2} = \dfrac{1}{3}$

$\dfrac{\sqrt{3}/2}{(1+a)} = \dfrac{1}{\sqrt{3}}$ ✓

$\boxed{\begin{aligned} \sin 60 &= \frac{b}{1} \\ b &= \frac{\sqrt{3}}{2} \end{aligned}}$

To show that

$\dfrac{3}{2} = 1+a$

$\therefore a = \dfrac{1}{2}$ ✓

* $\dfrac{b^2}{(1+a)^2} = \dfrac{1}{3}$

(9) let the smallest odd integer be $x$, ✓

$x + x+2 + x+4 + x+6 = 128$

$4x + 12 = 128$

$4x = 116$ ✓

$\therefore x = 29$

$\therefore$ 29, 31, 33 and 35 are 4 consecutive odd integers whose sum is 128. ✓

* In $\Delta ABC$, ~~using Pyth Theorem.~~

Using gradient of AC

$\dfrac{1}{\sqrt{3}} = \dfrac{b}{1+a}$

Squaring both sides

$\dfrac{1}{3} = \dfrac{b^2}{(1+a)^2}$