MAXIMA AND MINIMA

- The absolute extrema of a continuous function f defined on a closed interval $[a, b]$ occur either at the endpoints of the interval or at a critical point of f.
- If $f'(c) = 0$ and $f''(c) < 0$ then $(c, f(c))$ is a local maximum.
- If $f'(c) = 0$ and $f''(c) > 0$ then $(c, f(c))$ is a local minimum.

225. Find the absolute maximum and minimum values of each of the following functions over the given domain:

a) $f(x) = x^2;\ -1 \leq x \leq 4$ b) $f(x) = x^2;\ 2 \leq x \leq 3$

c) $f(x) = x^3 - 3x;\quad 0 \leq x \leq 2$ d) $f(x) = x^3 - 3x;\quad -4 \leq x \leq 4$

226. Genetically altered bacteria have been designed to produce insulin which scientists hope to harvest. The bacteria in sensitive to temperature and the output I (in milligrams) of insulin is a function of the temperature T (in deg C) as follows:

$$I = -T^4 + 3200T^2.$$

The machinery controlling the system is only capable of producing temperatures between $10°C$ and $50°C$. Determine the maximal and minimal possible outputs of insulin.

227. (*) A new drug is known to reduce the body temperature of feverish patients, however it is not clear which dose is most effective. In a clinical trial a number of patients with a temperature of $40°C$ were given varying doses of the drug and their temperature was subsequently measured. It was found that the temperature T (in deg C) one hour after a dose of D milligrams was

$$T = 40 - De^{-D/6}.$$

(a) Show that $\dfrac{dT}{dD} = e^{-D/6}\{D/6 - 1\}$.

(b) Determine the dose D which will minimise the temperature and the value of this minimum temperature.

(c) What would happen (to the temperature at least) to a patient that is given a massive overdose?

228. The level of pollution in the air near Sydney varies dramatically over any workday. From midnight to dawn it drops, then begins to rise again as commuters get to work in cars, buses and industrial plants begin to operate. It peaks at sunset and then declines again to midnight. We will model this situation using the equation

$$P = -\frac{t^3}{6} + 6t^2 - 48t + 120$$

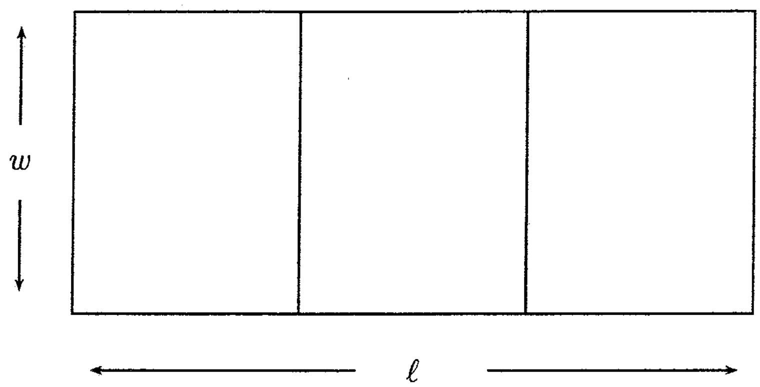
where $0 \leq t \leq 24$ is time measured in hours from midnight and P is measured in pollutant standard index (P.S.I.).

(a) Show that $P(0) = P(24)$ (as you'd expect).

(b) Determine the lowest and highest levels of pollution during the day and the approximate time they occur (you may find the quadratic formula useful).

(c) Draw a sketch of P vs t.

229. A scientist is conducting an experiment on chickens. She needs to construct a rectangular pen of length ℓ and width w which has three equal compartments as shown



Only 400m of fencing is available for the job.

(a) Explain why $4w + 2\ell = 400$.

(b) Show that the area A of the pen is given by

$$A = 200w - 2w^2.$$

(c) What dimensions should the pen have in order to maximise its area?

(d) What is this maximum area?

230. Two garbage dumps A and B are 1km apart on a straight road. When you stand at a distance of x from dump A the intensity of the smell (measured in Nostrums) is $I_A = \frac{1}{x}$. Dump B smells four times as bad, so that when you stand at a distance of x from dump B the intensity of the smell is $I_B = \frac{4}{x}$.

Pepe wants to build his house at a point P between A and B. Suppose that P is a distance of t km from A.

(a) Show that the total intensity of the smell at P is

$$I = \frac{1}{t} + \frac{4}{1-t}.$$

(b) Where should Pepe build his house in order to minimise the total smell?

231. John is a bookmaker. The Lahore test match starts soon and in order to set his odds he wishes to determine the critical stage during the match where the moisture in the pitch will be minimal. He rings his good mate Shane who informs him that the moisture M_1 (in grams of water/ m^3 of pitch) at time t (in days) will be given by $M_1 = 2e^t$. However when chatting to Mark at a local casino he is informed (elegantly) that the moisture content will be $M_2 = 3000e^{-t}$. Unsure how to proceed, John decides to take the average of his two pieces of inside information to determine a formula for the moisture content M .

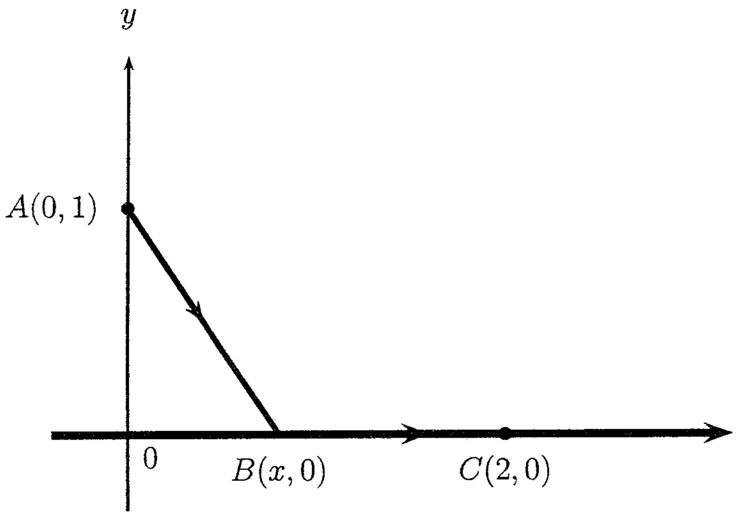
(a) Show that $M = e^t + 1500e^{-t}$.

(b) What will the moisture content be at the beginning and end of the match? (A test match runs for five days).

(c) What is the minimal moisture content and when does it occur?

(d) Sketch a graph of M versus t .

232. (**) In the diagram below the x -axis represents a major blood vessel whilst the line AB is a minor blood vessel joining the major vessel x units from 0 at $B(x,0)$. Consider the flow of blood from $A(0,1)$ to $B(x,0)$ and then to $C(2,0)$. The resistance to flow along the minor vessel is always five times the distance travelled while the resistance to flow on the major vessel is only four times the distance travelled.



(a) Show that the total resistance to flow for blood travelling from A to B to C is given by

$$R = 5\sqrt{x^2+1} + 8 - 4x.$$

(b) Verify that $\dfrac{dR}{dx} = \dfrac{5x}{\sqrt{x^2+1}} - 4.$

(c) At which point $B(x,0)$ should the minor vessel join the major vessel in order to minimise this resistance?

(d) If the resistance in the minor vessel increased would you expect your answer in c) to move left or right?

(e) A build-up of plaque in the **minor** vessel leads to an increase in the resistance along this vessel to 10 times the distance travelled. Find a new formula for the total resistance and determine the new point of contact $B(x,0)$ which minimises this resistance.

SOLUTIONS

Maxima and minima

225. (a) 16 and 0   (b) 9 and 4   (c) 2 and -2 (d) 52 and -52 .

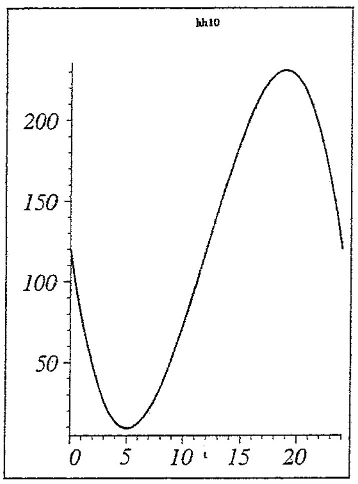
226. Maximum of 2560g; Minimum of 310g.

227. (a) Proof   (b) 6mg reduces the temperature to 37.79 ° .   (c) Nothing.

228. (a) Proof

 (b) Maximum of 230.85, 18.92 hours after midnight. Minimum of 9.15, 5.07 hours after midnight.

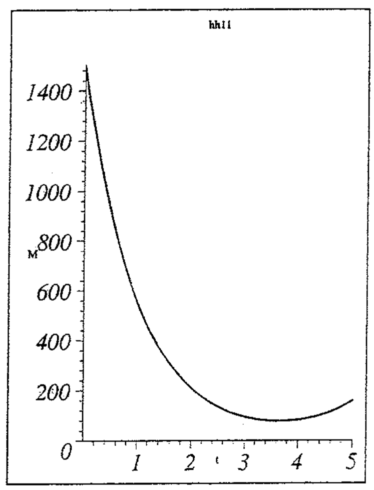
 (c)



229. (a) Proof   (b) Proof   (c) 50×100 m   (d) 5000m.

230. (a) Proof   (b) 1/3 km from A.

231. (a) Proof   (b) 1501 and 158.52   (c) 77.46 g/m ^{3} after 3.66 days   (d)



232. (a) Proof   (b) Proof   (c) (4/3,0)   (d) Left   e) (0.44, 0)