BRIGIDINE COLLEGE RANDWICK

ear 12 Mathematics — Student ____________________

December 2003 — Teacher ____________________

ime: 45 Minutes

how all necessary working.
leatness may be taken into consideration in the awarding of marks.

There are 8 Questions.

Differentiate the following
(leaving answers completely simplified with positive indices)

a. $y = -4x^3 - 4x^{-2} + 7$ *2 Marks*

b. $g(x) = x^2\sqrt{x}$ *2 Marks*

Find the equation of the tangent to the curve $y = 2x^3 - 3x + 1$ at the point where is crosses the y-axis. *3 Marks*

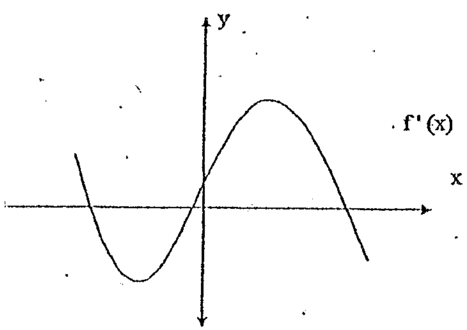
Find the primitives of the following

a. $2x + 11$ *2 Marks*

b. $6 - \sqrt[3]{x^2}$ *2 Marks*

The curve $y = 4x^2 + bx + 9$ has gradient of 16, when $x = 1$. *2 Marks*
Find the value of b.

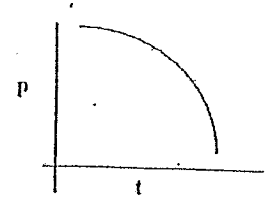
Below is the graph of $f'(x)$ the primitive function of $f(x)$. *3 Marks*



a. Copy this diagram onto your page.

b. Hence sketch the graph of $f(x)$, directly below your diagram.

6. To the right is a graph representing the effects of a drug to reduce the population p of rats over a period of time t. *2 Marks*



Comment on the success of this drug taking into account $\frac{dp}{dt}$ and $\frac{d^2p}{dt^2}$.

7. Consider the curve $f(x) = \frac{1}{2}x^4 - 4x^2$

a. Show that this curve represents an even function. *1 Mark*

b. Show that this curve has x intercepts at $x = \pm 2\sqrt{2}$ and $x = 0$. *1 Mark*

c. Determine $f'(x)$ and $f''(x)$. *2 Marks*

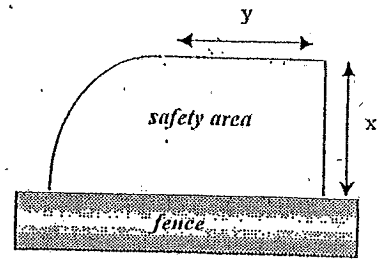
d. Show that there exists Stationary Values at $x = 0$ and $x = \pm 2$ and determine their nature. *2 Marks*

e. Determine the points of inflection for this curve. *2 Marks*

f. Sketch this curve showing all the above features. *2 Marks*

8. A council decides to provide a safety area for a park by using an existing fence and roping off the rest of the area.

This safety area is made up of a quadrant of a circle and a rectangle as shown to the right. *(not to scale)*



a. If there is 33 metres of rope available, show that $y = 33 - (\frac{\pi}{2} + 1)x$. *1 Marks*

b. Show that the area to be roped off is given by $A = 33x - (\frac{\pi}{4} + 1)x^2$. *2 Marks*

c. Find in simplest form the exact value for x for which the maximum area can be roped off. *3 Marks*

1) $y = -4x^3 - 4x^{-2} + 7$

$y' = -12x^2 + 8x^{-3}$ ✓

$y' = -12x^2 + \frac{8}{x^3}$ ✓

) $g(x) = x^2\sqrt{x}$

$g(x) = x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$

$g'(x) = \frac{5}{2}x^{\frac{3}{2}}$ ✓✓

$= \frac{5\sqrt{x^3}}{2}$

$= \frac{5x\sqrt{x}}{2}$

$y = 2x^3 - 3x + 1$

$y' = 6x^2 - 3$

$(0,1)$ $x = 0$, $m = -3$ ✓ ✓

$y - 1 = -3(x - 0)$

$y = -3x + 1$ ✓

a) $y = x^2 + 11x + c$ ✓✓

b) $y = 6x - \frac{3}{5}x^{\frac{5}{3}} + c$ ✓✓

(No "+c" lose 1 mark)

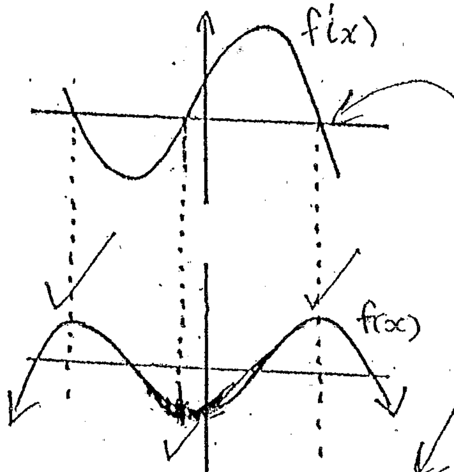
$y = 4x^2 + bx + 4$

$y' = 8x + b$ $\therefore = 1$

$16 = 8 + b$ ✓

$b = 8$ ✓

5



looking for where it crosses x-axis to match turning pts.

6 $P' < 0$, $P'' < 0$ ✓

Rate of success increases over time or population decreasing at an increasing rate. ✓

* NEED TO MENTION SOMETHING ABOUT RATE.

7 a) $f(x) = \frac{1}{2}x^4 - 4x^2$

$f(-x) = \frac{1}{2}(-x)^4 - 4(-x)^2$

$= \frac{1}{2}x^4 - 4x^2$ ✓

$\therefore f(x) = f(-x)$

NOTE substituting numbers $= \underline{0}$.

b) $0 = x^4 - 8x^2$

$0 = x^2(x^2 - 8)$ ✓

$x = \pm 2\sqrt{2}, 0$

c) $f'(x) = 2x^3 - 8x$ ✓

$f''(x) = 6x^2 - 8$ ✓

d) $2x(x^2 - 4) = 0$ ✓

$x = 0$, $x = 2$, $x = -2$

Test: $y'' < 0$ $y'' > 0$ $y'' > 0$ ✓

max min

$(0, 0)$ $(2, -8)$ $(-2, -8)$

e) $y'' = 0 = 6x^2 - 8$

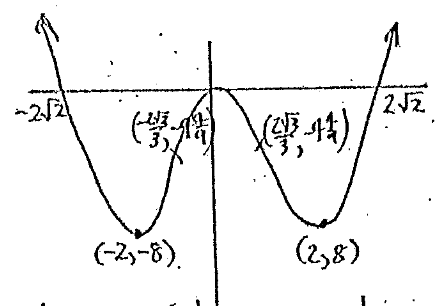
$x^2 = \frac{8}{6}$

$x = \pm\sqrt{\frac{8}{6}} = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$

1 mark.

Test ✓ or put y-values in ✓

ie $\left(\frac{2\sqrt{3}}{3}, -4\frac{4}{9}\right)$, $\left(-\frac{2\sqrt{3}}{3}, -4\frac{4}{9}\right)$



1 mark for marking x-values.

1 mark for the rest.

8 a) $33 = y + x + \frac{1}{4} \times 2\pi r$, $r = x$

$33 = y + x + \frac{2\pi x}{4}$

$y = 33 - x - \frac{\pi x}{2}$

$y = 33 - (\frac{\pi}{2} + 1)x$ ✓

b) $A = \frac{1}{4}\pi x^2 + xy$ ✓

$= \frac{\pi x^2}{4} + x(33 - (\frac{\pi}{2} + 1)x)$

$= \frac{\pi x^2}{4} + 33x - \frac{\pi x^2}{2} - x^2$

$= 33x - \frac{\pi x^2}{4} - x^2$

$A = 33x - (\frac{\pi}{4} + 1)x^2$ ✓

c) $A = 33x - \frac{\pi x^2}{4} - x^2$

$A' = 33 - \frac{\pi x}{2} - 2x$ ✓

$0 = 33 - \frac{\pi x}{2} - 2x$

$0 = 66 - \pi x - 4x$

$(\pi + 4)x = 66$

$x = \frac{66}{\pi + 4}$ ✓

$A'' = -\frac{\pi}{2} - 2 < 0$ $\therefore$ MAX ✓