Further trigonometry

Solutions Main Menu

22 What is the exact value of $\cos 165°$?

(A) $\dfrac{-\sqrt{2}-\sqrt{6}}{4}$

(B) $\dfrac{-\sqrt{2}+\sqrt{6}}{4}$

(C) $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$

(D) $\dfrac{-\sqrt{6}+\sqrt{2}}{4}$

23 What is the exact value of $\tan 75°$?

(A) $2-\sqrt{3}$

(B) $4-\sqrt{3}$

(C) $2+\sqrt{3}$

(D) $4+\sqrt{3}$

24 What is the minimum value of $\cos\theta+2\cos(\theta+240°)$?

(A) $-2\sqrt{3}$

(B) $-\sqrt{3}$

(C) $-\dfrac{\sqrt{3}}{2}$

(D) $-\dfrac{\sqrt{3}}{4}$

25 What is the exact value of $\tan(\theta-180)$, if $4\cos\theta=-3$ and $\tan\theta>0$?

(A) $-\dfrac{\sqrt{7}}{3}$

(B) $\dfrac{\sqrt{7}}{3}$

(C) $-\dfrac{3}{\sqrt{7}}$

(D) $\dfrac{3}{\sqrt{7}}$

26 If $3\cos\theta+2=0$ and $\tan\theta>0$, what is the exact value of $\sin(\theta+180)$?

(A) $-\dfrac{\sqrt{5}}{3}$

(B) $-\dfrac{\sqrt{5}}{2}$

(C) $\dfrac{\sqrt{5}}{2}$

(D) $\dfrac{\sqrt{5}}{3}$

27 Which of the following is equivalent to the expression $\sin 3\theta$?

(A) $2\sin\theta-\sin^3\theta$

(B) $2\sin\theta-4\sin^3\theta$

(C) $3\sin\theta-\sin^3\theta$

(D) $3\sin\theta-4\sin^3\theta$

28 Given $x=\sqrt{2\cos 2\theta}$ and $y=3\sin^2\theta$, which of the following equations is correct?

(A) $y=\dfrac{3}{2}(1-x^2)$

(B) $y=\dfrac{3}{2}(2-x^2)$

(C) $y=\dfrac{3}{4}(1-x^2)$

(D) $y=\dfrac{3}{4}(2-x^2)$

29 Which of the following is equivalent to the expression $\sqrt{\dfrac{4+4\cos 2x}{1-\cos 2x}}$?

(A) $2\cot^2 x$

(B) $2\cot x$

(C) $2\tan x$

(D) $2\tan^2 x$

30 Which of the following is equivalent to the expression $\dfrac{\sin 2\theta+\sin\theta}{\cos 2\theta+\cos\theta+1}$?

(A) $\cot\theta$

(B) $\sec\theta$

(C) $\sin\theta$

(D) $\tan\theta$

31 If $t = \tan\frac{1}{2}\theta$ which of the following expressions is equivalent to $\sec\theta$?

(A) $\frac{1+t^2}{1-t^2}$

(B) $\frac{2t}{1+t^2}$

(C) $\frac{1-t^2}{1+t^2}$

(D) $\frac{2t}{1-t^2}$

32 If $t = \tan\theta$ which of the following expressions is equivalent to $1-\sin 2\theta$?

(A) $\frac{1-t^2}{1+t^2}$

(B) $\frac{(t-1)^2}{1+t^2}$

(C) $\frac{1+t^2}{1-t^2}$

(D) $\frac{(t+1)^2}{1+t^2}$

33 If $t = \tan\frac{\theta}{2}$ which of the following expressions is equivalent to $4\sin\theta+3\cos\theta+5$?

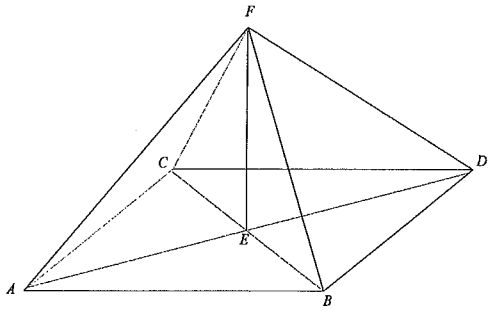
(A) $\frac{2(t+2)^2}{1-t^2}$

(B) $\frac{(t+4)^2}{1-t^2}$

(C) $\frac{2(t+2)^2}{1+t^2}$

(D) $\frac{(t+4)^2}{1+t^2}$

34 A rectangular pyramid shown below has $AB = 16$ cm, $BD = 12$ cm and $\angle FBL = 50°$.



What is the perpendicular height of the pyramid?

(A) 12 cm

(B) 14 cm

(C) 19 cm

(D) 24 cm

35 AB is a vertical pole 12 m high. AC is the shadow of the pole when the direction of the sun is due north and the sun's angle of elevation is 60°. AD is the shadow cast by the pole when the direction of the sun is on a bearing of 345° and the sun's angle of elevation is 50°. What is the size of $\angle CBD$?

(A) 15°

(B) 110°

(C) 285°

(D) 295°

36 Jade is on a ship and observes two lighthouses on the shore. The lighthouse at Addison Head has a bearing of 224° from the ship. The lighthouse at Blake Beach has a bearing of 195° from the ship and 165° from Addison Head. The lighthouses are 3.4 km apart. What is the distance of Jade's ship from the Addison Head lighthouse?

(A) 3.3 km

(B) 3.4 km

(C) 3.5 km

(D) 3.6 km

37 Point A is due south of a hill and the angle of elevation from A to the top of the hill is 35°. Another point B is a bearing 200° from the hill and the angle of elevation from B to the top of the hill is 46°. The distance AB is 220 m. Which of the following are the correct expressions for OA and OB?

(A) $OA = \dfrac{\tan 35^\circ}{h}$ and $OB = \dfrac{\tan 46^\circ}{h}$

(B) $OA = \dfrac{h}{\tan 35^\circ}$ and $OB = \dfrac{h}{\tan 46^\circ}$

(C) $OA = h\tan 35^\circ$ and $OB = h\tan 46^\circ$

(D) $OA = 220\tan 35^\circ$ and $OB = 220\tan 46^\circ$

38 Which of the following is the correct expression for $2\sin x - \cos x$?

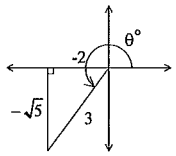
(A) $\sqrt{3}\sin(x - 26^\circ 34')$

(B) $\sqrt{3}\sin(x - 30^\circ)$

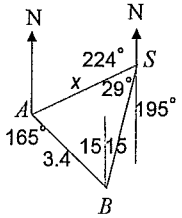
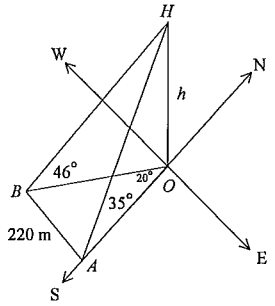
(C) $\sqrt{5}\sin(x - 26^\circ 34')$

(D) $\sqrt{5}\sin(x - 30^\circ)$

Further trigonometry		Main Menu
	Solution	**Criteria**
22	$\cos 165^\circ = \cos(180^\circ - 15^\circ)$ $= -\cos 15^\circ$ $= -\cos(45^\circ - 30^\circ)$ $= -(\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ)$ $= -(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2})$ $= \frac{-\sqrt{3}-1}{2\sqrt{2}}$ $= \frac{-\sqrt{6}-\sqrt{2}}{4}$	1 Mark: C
23	$\tan 75^\circ = \tan(45^\circ + 30^\circ)$ $= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$ $= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $= 2 + \sqrt{3}$	1 Mark: C
24	$\cos\theta + 2\cos(\theta + 240) = \cos\theta + 2[\cos\theta \cos 240 - \sin\theta \sin 240)]$ $= \cos\theta + 2[\cos\theta \times -\frac{1}{2} - \sin\theta \times -\frac{\sqrt{3}}{2})]$ $= \sqrt{3}\sin\theta$ Smallest value of $\sin\theta$ is -1. Minimum value of $\cos\theta + 2\cos(\theta + 240^\circ)$ is $-\sqrt{3}$	1 Mark: B
25	$4\cos\theta = -3$ and $\tan\theta > 0$ Therefore $\tan\theta = \frac{\sqrt{7}}{3}$ $\cos\theta = -\frac{3}{4}$ $\tan(\theta - 180) = \frac{\tan\theta - \tan 180^\circ}{1 + \tan\theta \tan 180^\circ}$ $(\tan 180 = 0)$ $= \tan\theta$ $= \frac{\sqrt{7}}{3}$	1 Mark: B

26	$3\cos\theta + 2 = 0$ $3\cos\theta = -2$ $\cos\theta = -\frac{2}{3}$ Angle is in the third quadrant ($\cos\theta < 0$ and $\tan\theta > 0$.  $\sin(\theta + 180) = \sin\alpha$ (α is the acute angle) $= \frac{\sqrt{5}}{3}$	1 Mark: D
27	$\sin(3\theta) = \sin(\theta + 2\theta)$ $= \sin\theta\cos 2\theta + \cos\theta \sin 2\theta$ $= \sin\theta(1 - 2\sin^2\theta) + \cos\theta \times 2\sin\theta\cos\theta$ $= \sin\theta - 2\sin^3\theta + \cos^2\theta \times 2\sin\theta$ $= \sin\theta - 2\sin^3\theta + (1 - \sin^2\theta) \times 2\sin\theta$ $= \sin\theta - 2\sin^3\theta + 2\sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$	1 Mark: D
28	$x = \sqrt{2\cos 2\theta}$ $x^2 = 2\cos 2\theta$ $= 2(1 - 2\sin^2\theta)$ $= 2 - 4\sin^2\theta$ $4\sin^2\theta = 2 - x^2$ $\sin^2\theta = \frac{2 - x^2}{4}$ Now $y = 3 \times \frac{2 - x^2}{4} = \frac{3}{4}(2 - x^2)$	1 Mark: D
29	$\sqrt{\frac{4 + 4\cos 2x}{1 - \cos 2x}} = 2\sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}}$ $\left(\begin{matrix}\cos 2x = 2\cos^2 x - 1 \\ \cos 2x = 1 - 2\sin^2 x\end{matrix}\right)$ $= 2\sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$ $= 2\sqrt{\cot^2 x}$ $= 2\cot x$	1 Mark: B

30	$\dfrac{\sin 2\theta+\sin\theta}{\cos 2\theta+\cos\theta+1}=\dfrac{2\sin\theta\cos\theta+\sin\theta}{2\cos^2\theta-1+\cos\theta+1}$ $=\dfrac{\sin\theta(2\cos\theta+1)}{\cos\theta(2\cos\theta+1)}$ $=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	1 Mark: D
31	$\cos\theta=\dfrac{1-t^2}{1+t^2}$ $\sec\theta=\dfrac{1+t^2}{1-t^2}$	1 Mark: A
32	$1-\sin 2\theta=1-\dfrac{2t}{1+t^2}$ $=\dfrac{1+t^2-2t}{1+t^2}$ $=\dfrac{(t-1)^2}{1+t^2}$	1 Mark: B
33	$4\sin\theta+3\cos\theta+5=4\times\dfrac{2t}{1+t^2}+3\times\dfrac{1-t^2}{1+t^2}+5$ $=\dfrac{8t+3(1-t^2)+5(1+t^2)}{1+t^2}$ $=\dfrac{8t+3-3t^2+5+5t^2}{1+t^2}$ $=\dfrac{2t^2+8t+8}{1+t^2}$ $=\dfrac{2(t^2+4t+4)}{1+t^2}$ $=\dfrac{2(t+2)^2}{1+t^2}$	1 Mark: C
34	$BC^2=16^2+12^2$ $=400$ $BC=20$ $ED=10$ (half of BC) $\tan 50^\circ=\dfrac{FE}{10}$ $FE=10\tan 50^\circ$ $=11.91753593...\approx 12$ cm	1 Mark: A
35	$\angle CBD=360^\circ-345^\circ$ $=15^\circ$	1 Mark: A

36	 $\dfrac{x}{\sin 30}=\dfrac{3.4}{\sin 29}$ $x=\dfrac{3.4\sin 30}{\sin 29}$ $=3.506531077$ ≈ 3.5 km	1 Mark: C
37	 In ΔHOA In ΔHOB $\tan 35^\circ=\dfrac{h}{OA}$ $\tan 46^\circ=\dfrac{h}{OB}$ $OA=\dfrac{h}{\tan 35^\circ}$ $OB=\dfrac{h}{\tan 46^\circ}$	1 Mark: B
38	$A\sin(x-\alpha)=A\sin x\cos\alpha-A\cos x\sin\alpha$ Hence $A\cos\alpha=2$ and $A\sin\alpha=1$ Dividing these equations $\tan\alpha=\dfrac{1}{2}$ $\alpha=26^\circ 34'$ Squaring and adding the equations $A^2=2^2+1^2$ $A=\sqrt{5}$ $2\sin x-\cos x=\sqrt{5}\sin(x-26^\circ 34')$	1 Mark: C