Other inequalities   <u>Solutions</u>   <u>Main Menu</u>

1 What is the solution to the inequality $\dfrac{5}{x+1} \le 1$?

(A) $x > -1$ and $x \le 4$

(B) $x < -1$ and $x \ge 4$

(C) $x > 1$ and $x \le -4$

(D) $x < 1$ and $x \ge -4$

2 What is the solution to the inequality $\dfrac{x^2-4}{2x} < 0$?

(A) $-2 < x < 0$ or $x > 2$

(B) $-2 < x < 0$ or $x > 4$

(C) $-4 < x < 0$ or $x > 2$

(D) $-4 < x < 0$ or $x > 4$

3 What is the solution to the inequality $\dfrac{3}{x-2} \le 4$?

(A) $x < -2$ and $x \ge -\dfrac{11}{4}$

(B) $x > -2$ and $x \le -\dfrac{11}{4}$

(C) $x < 2$ and $x \ge \dfrac{11}{4}$

(D) $x > 2$ and $x \le \dfrac{11}{4}$

4 What is the solution to the inequality $\dfrac{2-x}{x^2+2} \ge 1$?

(A) $x \ge -2$ and $x \le 0$

(B) $x \le -2$ or $x \ge 0$

(C) $x \ge -1$ and $x \le 0$

(D) $x \le -1$ or $x \ge 0$

5 What is the solution to the inequality $\dfrac{3-x}{x^2+3} \ge 1$?

(A) $0 \ge x \ge 1$ (B) $0 \le x \le 1$

(C) $-1 \ge x \ge 0$ (D) $-1 \le x \le 0$

6 What is the solution to the inequality $\dfrac{x+1}{x-3} \le 1$?

(A) $x \le -1$

(B) $x < -1$

(C) $x \le 3$

(D) $x < 3$

7 What is the solution to the inequality $\dfrac{3}{x(x-2)} < 1$?

(A) $-3 < x < -2$, $-1 < x < 0$

(B) $x < -3$, $-2 < x < -1$, $x > 0$

(C) $-1 < x < 0$, $2 < x < 3$

(D) $x < -1$, $0 < x < 2$, $x > 3$

8 What is the solution to the inequality $3 - x \ge \dfrac{2}{x}$?

(A) $x < 0$ or $1 \le x \le 2$

(B) $x \ge 2$ or $0 < x \le 1$

(C) $x > 0$ or $-2 \le x \le -1$

(D) $x \le -2$ or $-1 \le x < 0$

9 What is the solution to the inequality $\dfrac{3}{x(2x-1)} \ge 1$?

(A) $x \le -1$, $0 < x < \dfrac{1}{2}$ and $x \ge \dfrac{3}{2}$

(B) $-1 \le x < 0$ and $\dfrac{1}{2} < x \le \dfrac{3}{2}$

(C) $-\dfrac{3}{2} \le x \le -\dfrac{1}{2}$ and $0 < x \le 1$

(D) $x \le -\dfrac{3}{2}$, $-\dfrac{1}{2} \le x < 0$ and $x \ge 1$

10 What is the solution to the equation $|2x-5| = -3x$?

(A) $x = -5$

(B) $x = -1$

(C) $x = 1$

(D) $x = 5$

11 What is the solution to the equation $3x + 4 = |2 - 2x|$?

    (A) $x = -6$

    (B) $x = -\dfrac{2}{5}$

    (C) $x = \dfrac{2}{5}$

    (D) $x = 6$

12 What is the solution to the equation $|x - 2| = 2x - 1$?

    (A) $x = -3$

    (B) $x = -1$

    (C) $x = 1$

    (D) $x = 3$

13 What is the solution to the equation $|2x - 5| = x + 2$?
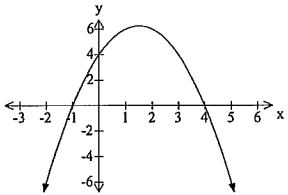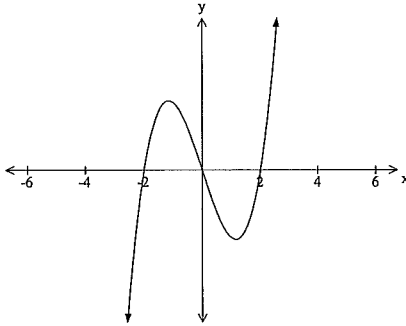
(A) $x = 1$
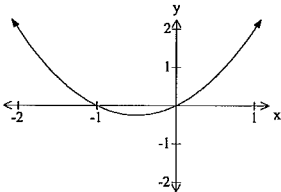
(B) $x = 7$

(C) $x = 1$ or $x = 7$

(D) $x = 1$ and $x = 7$

Objective Response Bank

Year 11 Mathematics Extension 1

Worked solutions

Other inequalities		Main Menu
	Solution	**Criteria**
1	$(x+1)^2 \times \dfrac{5}{(x+1)} \leq 1 \times (x+1)^2$ $(x+1) \times 5 \leq 1(x+1)^2 \quad x \neq -1$ $(x+1)(5-x-1) \leq 0$ $(x+1)(4-x) \leq 0$ From the graph $x < -1$ and $x \geq 4$	1 Mark: B
2	Multiply both sides by x^2 $x^2 \times \dfrac{x^2-4}{2x} < 0 \times x^2$ $x \times \dfrac{x^2-4}{2} < 0$ $x(x^2-4) > 0$ $x(x-2)(x+2) > 0$ $x = 0, 2, -2$ $-2 < x < 0$ or $x > 2$	1 Mark: A

3	$(x-2)^2 \times \dfrac{3}{(x-2)} \leq 4 \times (x-2)^2$ $\quad (x-2)3 \leq 4(x-2)^2 \quad x \neq 2$ $(x-2)(3-4x+8) \leq 0$ $\quad (x-2)(11-4x) \leq 0$ $x < 2$ and $x \geq \dfrac{11}{4}$	1 Mark: C
4	$(x^2+2) \times \dfrac{2-x}{(x^2+2)} \geq 1 \times (x^2+2) \qquad (x^2+3)$ is always positive $\quad 2-x \geq x^2+2$ $\quad x^2+x \leq 0$ $\quad x(x+1) \leq 0$ Critical points are 0 and -1 Test values in each region $x \geq -1$ and $x \leq 0$	1 Mark: C
5	$(x^2+3) \times \dfrac{3-x}{(x^2+3)} \geq 1 \times (x^2+3) \qquad (x^2+3)$ is always positive $\quad 3-x \geq x^2+3$ $\quad x^2+x \leq 0$ $\quad x(x+1) \leq 0$ From the graph $-1 \leq x \leq 0$	1 Mark: D
6	$(x-3)^2 \times \dfrac{x+1}{x-3} \leq 1 \times (x-3)^2 \qquad x \neq 3$ $\quad (x-3)(x+1) \leq (x-3)^2$ $(x-3)(x+1) - (x-3)^2 \leq 0$ $(x-3)[(x+1)-(x-3)] \leq 0$ $\quad\quad 4(x-3) \leq 0$ Critical point is 3 Test values in each region $x < 3$	1 Mark: D

7	$x^2(x-2)^2 \times \dfrac{3}{x(x-2)} < 1 \times x^2(x-2)^2$ $\qquad 3x(x-2) < x^2(x-2)^2 \qquad x \neq 0 \text{ and } x \neq 2$ $3x(x-2) - x^2(x-2)^2 < 0$ $\quad x(x-2)[3 - x(x-2)] < 0$ $\quad x(x-2)(x^2 - 2x - 3)] > 0$ $\quad x(x-2)(x-3)(x+1) > 0$ Critical points are $-1, 0, 2$ and 3 Test values in each region $x < -1,\ 0 < x < 2,\ x > 3$	1 Mark: D
8	$3 - x \geq \dfrac{3}{x} \qquad\qquad x \neq 0$ $\qquad x^2 \times (3-x) \geq \dfrac{2}{x} \times x^2$ $\qquad x^2(3-x) \geq 2x$ $\quad x^2(3-x) - 2x \geq 0$ $\quad x[x(3-x) - 2] \geq 0$ $\quad x(x^2 - 3x + 2) \leq 0$ $\quad x(x-2)(x-1) \leq 0$ Critical points are $0, 1$ and 2 Test values in each region $x < 0$ and $1 \leq x \leq 2$	1 Mark: A
9	$\dfrac{3}{x(2x-1)} \geq 1 \qquad\qquad x \neq 0 \text{ or } x \neq \dfrac{1}{2}$ $\quad x^2(2x-1)^2 \times \dfrac{3}{x(2x-1)} \geq 1 \times x^2(2x-1)^2$ $\qquad 3x(2x-1) \geq x^2(2x-1)^2 \qquad x \neq 0 \text{ and } x \neq 2$ $3x(2x-1) - x^2(2x-1)^2 \geq 0$ $\quad x(2x-1)[3 - x(2x-1)] \geq 0$ $\quad x(2x-1)(2x^2 - x - 3)] \leq 0$ $\quad x(2x-1)(2x-3)(x+1) \leq 0$ Critical points are $-1, 0, \dfrac{1}{2}$ and $\dfrac{3}{2}$ Test values in each region $-1 \leq x < 0$ and $\dfrac{1}{2} < x \leq \dfrac{3}{2}$	1 Mark: B

10	$|2x - 5| = -3x$ $2x - 5 = 3x \qquad\qquad$ or $\qquad 2x - 5 = -3x$ $\quad x = -5 \qquad\qquad\qquad\qquad\qquad x = 1$ Test solutions When $x = -5 \qquad\qquad |2x - 5| = -3x$ $\qquad\qquad\qquad |2 \times -5 - 5| = -3 \times -5$ $\qquad\qquad\qquad\qquad 15 = 15 \text{ (correct)}$ When $x = 1 \qquad\qquad |2x - 5| = -3x$ $\qquad\qquad\qquad |2 \times 1 - 5| = -3 \times 1$ $\qquad\qquad\qquad\qquad 3 = -3 \text{ (incorrect)}$ Solution is $x = -5$	1 Mark: A
11	$3x + 4 = |2 - 2x|$ $|2 - 2x| = 3x + 4$ $2 - 2x = 3x + 4 \qquad$ or $\qquad 2 - 2x = -(3x + 4)$ $\quad 5x = -2 \qquad\qquad\qquad\qquad 2 - 2x = -3x - 4$ $\quad x = -\dfrac{2}{5} \qquad\qquad\qquad\qquad\qquad x = -6$ Test solutions When $x = -\dfrac{2}{5} \qquad\qquad 3x + 4 = |2 - 2x|$ $\qquad\qquad\qquad 3 \times -\frac{2}{5} + 4 = |2 - 2 \times -\frac{2}{5}|$ $\qquad\qquad\qquad\qquad \dfrac{14}{5} = \dfrac{14}{5} \text{ (correct)}$ When $x = -6 \qquad\qquad 3x + 4 = |2 - 2x|$ $\qquad\qquad\qquad 3 \times -6 + 4 = |2 - 2 \times -6|$ $\qquad\qquad\qquad\qquad -14 = 14 \text{ (incorrect)}$ Solution is $x = -\dfrac{2}{5}$	1 Mark: B
12	$|x - 2| = 2x - 1$ $x - 2 = 2x - 1 \qquad$ or $\qquad x - 2 = -(2x - 1)$ $\quad x = -3 \qquad\qquad\qquad\qquad x - 2 = -2x + 1$ $\qquad\qquad\qquad\qquad\qquad\qquad x = 1$ Test solutions $x = -3 \qquad |x - 2| = 2x - 1 \qquad\qquad x = 1 \qquad |x - 2| = 2x - 1$ $\qquad\qquad |-1 - 2| = 2 \times -1 - 1 \qquad\qquad\qquad |1 - 2| = 2 \times 1 - 1$ $\qquad\qquad\qquad 3 = -3 \text{ (incorrect)} \qquad\qquad\qquad\qquad 1 = 1 \text{ (correct)}$ Solution is $x = 1$	1 Mark: C

13	$\|2x-5\| = x+2$	1 Mark: C

$\|2x-5\| = x+2$

$2x-5 = x+2 \qquad$ or $\qquad 2x-5 = -(x+2)$

$\qquad x = 7 \qquad\qquad\qquad\qquad x = 1$

Test solutions

When $x = 7 \qquad\qquad \|2x-5\| = x+2$

$\qquad\qquad\qquad\qquad \|2\times 7-5\| = 7+2$

$\qquad\qquad\qquad\qquad\qquad 9 = 9$ (correct)

When $x = 1 \qquad\qquad \|2x-5\| = x+2$

$\qquad\qquad\qquad\qquad \|2\times 1-5\| = 1+2$

$\qquad\qquad\qquad\qquad\qquad 3 = 3$ (correct)

Solution is $x = 1$ or $x = 7$