Parametric representation

Solutions   Main Menu

63 A curve has parametric equations $x = t + 1$ and $y = 2t^2$.
What is Cartesian equation of this curve?

(A) $y = 2\sqrt{(x-1)}$

(B) $y = 2\sqrt{(x+1)}$

(C) $y = 2(x-1)^2$

(D) $y = 2(x+1)^2$

64 A curve has parametric equations $x = \frac{2}{t}$ and $y = 2t^2$.
What is Cartesian equation of this curve?

(A) $y = \frac{4}{x}$

(B) $y = \frac{8}{x}$

(C) $y = \frac{4}{x^2}$

(D) $y = \frac{8}{x^2}$

65 What is the equation of the chord of the parabola joining the points with parameters 1 and -3 on $x = 4t$ and $y = 2t^2$?

(A) $x - y - 3 = 0$

(B) $x + y - 3 = 0$

(C) $x - y - 6 = 0$

(D) $x + y - 6 = 0$

66 A parabola has the parametric equations $x = 12t$ and $y = -6t^2$.
What are the coordinates of the focus?

(A) $(-6, 0)$

(B) $(0, -6)$

(C) $(6, 0)$

(D) $(0, 6)$

67 What is the equation of the chord of contact of $x^2 = 4y$ from the point $(-2, -1)$?

(A) $x + y - 1 = 0$

(B) $x + y + 4 = 0$

(C) $x + 2y - 1 = 0$

(D) $x + 2y + 4 = 0$

68 What is the equation of the tangent to $x^2 = 4y$ at the point $(2t, t^2)$?

(A) $y = tx - 2t$

(B) $y = tx + 2t$

(C) $y = tx - t^2$

(D) $y = tx + t^2$

69 What is the equation of the normal to $x = 2at$, $y = at^2$ at the point $t = p$?

(A) $x - py = 2ap + ap^2$

(B) $x - py = 2ap + ap^3$

(C) $x + py = 2ap + ap^2$

(D) $x + py = 2ap + ap^3$

70 Which of the following is the correct equation for $y = mx + b$ to be a tangent to the parabola $x^2 = 4ay$?

(A) $am - b = 0$

(B) $am^2 - b = 0$

(C) $am + b = 0$

(D) $am^2 + b = 0$

71 $P(2ap, ap^2)$ is a variable point on the parabola $x^2 = 4ay$. The tangent to the parabola at P meets the y-axis at T. What are the coordinates of T?

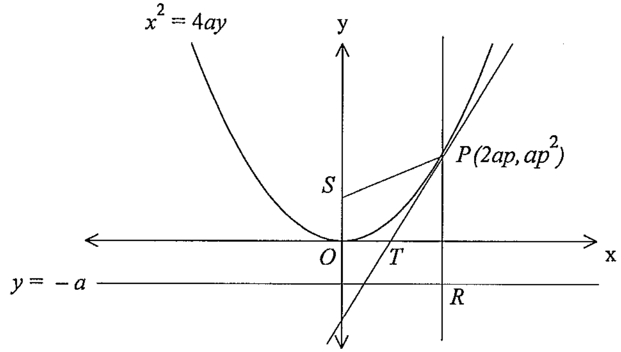
(A) $(0, -ap)$

(B) $(0, -ap^2)$

(C) $(0, -2ap)$

(D) $(0, -2ap^2)$

72 The diagram shows the parabola $x^2 = 4ay$ with focus $S(0, a)$ and directrix $y = -a$. The point $P(2ap, ap^2)$ is a point on the parabola. The line PR is parallel to the y-axis and R is on the directrix. The tangent at P meets the x-axis at T.



What is the equation of the tangent at P?

(A) $y = ax - ap^2$

(B) $y = px - ap^2$

(C) $y = ax - 2ap$

(D) $y = px - 2ap$

73 $P(2at, at^2)$ is any point on the parabola $x^2 = 4ay$. The line k is parallel to the tangent at P and passes through the focus S of the parabola. The line k intersects the x-axis at the point Q. What are the coordinates of the midpoint, M, of the interval QS?

(A) $(-\frac{a}{2}, -\frac{a}{2t})$

(B) $(-\frac{a}{2t}, \frac{a}{2})$

(C) $(\frac{a}{2}, -\frac{a}{2t})$

(D) $(-\frac{a}{2t}, \frac{a}{2})$

74 A straight line is drawn from a point $P(2at, at^2)$ on the parabola $x^2 = 4ay$ to the vertex. This line intersects on the directrix at D. What are the coordinates of D?

(A) $(\frac{-2a}{p}, -a)$

(B) $(\frac{2a}{p}, -a)$

(C) $(\frac{-p}{2a}, -a)$

(D) $(\frac{p}{2a}, -a)$

75 What is the equation of the locus of a point $P(x, y)$ which moves such that its distances from the point $A(1, 4)$ is four times its distance from the point $B(-2, 1)$?

(A) $x^2 + 6x + y^2 + 1 = 0$

(B) $3x^2 + 14x + 3y^2 - 8y + 3 = 0$

(C) $5x^2 + 22x + 5y^2 - 8y + 21 = 0$

(D) $15x^2 + 66x + 15y^2 - 8y + 63 = 0$

76 $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$. PQ is a focal chord of this parabola. PT and QT are parallel to the y-axis and x-axis respectively. What is the locus of T?

(A) $xy = 4a^2$

(B) $xy = 4a^3$

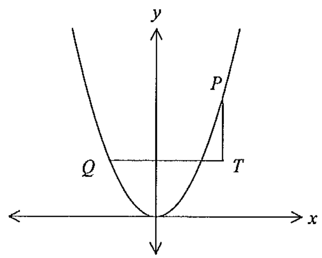
(C) $x^2y = 4a^2$

(D) $x^2y = 4a^3$

Parametric representation		Main Menu
	Solution	**Criteria**
63	$x=t+1$ $t=x-1$ Substitute $t-1$ for x into $y=2t^2$ $y=2(x-1)^2$	1 Mark: C
64	$x=\frac{2}{t}$ or $t=\frac{2}{x}$ Substitute $\frac{2}{x}$ for x into $y=2t^2$ $y=2(\frac{2}{x})^2$ $=\frac{8}{x^2}$	1 Mark: D
65	Parameter of 1 the point is $(4,2)$ Parameter of -3 the point is $(-12,18)$ $\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$ $\frac{y-2}{x-4}=\frac{18-2}{-12-4}$ $\frac{y-2}{x-4}=-1$ $y-2=-x+4$ $x+y-6=0$	1 Mark: D
66	$x=12t$ and $y=-6t^2$ $a=6$ and the parabola is concave downwards Focus is $(0,-6)$	1 Mark: B
67	$a=1$ given $x^2=4y$ Equation of the chord of contact $xx_0=2a(y+y_0)$ $x\times-2=2\times1\times(y+-1)$ $-2x=2y-2$ $x+y-1=0$	1 Mark: A

68	To find the gradient of the tangent $y=\frac{1}{4}x^2$ $\frac{dy}{dx}=\frac{1}{2}x$ At $(2t,t^2)$ $\frac{dy}{dx}=\frac{1}{2}\times2t=t$ Equation of the tangent at $(2t,t^2)$ $y-y_1=m(x-x_1)$ $y-t^2=t(x-2t)$ $y-t^2=tx-2t^2$ $y=tx-t^2$	1 Mark: C
69	Parabola is $x^2=4ay$ To find the gradient of the tangent $y=\frac{1}{4a}x^2$ $\frac{dy}{dx}=\frac{1}{2a}x$ At $(2ap,ap^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times2ap=p$ Gradient of the normal is $-\frac{1}{p}$ Equation of the normal at $(2ap,ap^2)$ $y-y_1=m(x-x_1)$ $y-ap^2=-\frac{1}{p}(x-2ap)$ $py-ap^3=-x+2ap$ $x+py=2ap+ap^3$	1 Mark: D
70	Tangent intersects the parabola at one point. Substitute $y=mx+b$ into $x^2=4ay$ $x^2=4a(mx+b)$ $x^2-4amx-4ab=0$ Now $\Delta=b^2-4ac$ $=(-4am)^2-4\times1\times-4ab$ $=16a^2m^2+16ab$ $=16a(am^2+b)$ The discriminant must equal zero (one solution). Hence $am^2+b=0$	1 Mark: D

71	To find the gradient of the tangent $y=\frac{1}{4a}x^2,\ \frac{dy}{dx}=\frac{1}{2a}x$ At $P(2ap,ap^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2ap=p$ Equation of the tangent at $P(2ap,ap^2)$ $y-y_1=m(x-x_1)$ $y-ap^2=p(x-2ap)$ $y=px-ap^2$ PT meets the y-axis when $x=0$ Coordinates of T are $(0,-ap^2)$	1 Mark: B
72	To find the gradient of the tangent $y=\frac{1}{4a}x^2,\ \frac{dy}{dx}=\frac{1}{2a}x$ At $P(2ap,ap^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2ap=p$ Equation of the tangent at $P(2ap,ap^2)$ $y-y_1=m(x-x_1)$ $y-ap^2=p(x-2ap)$ $y=px-ap^2$	1 Mark: B
73	To find the gradient of the tangent $y=\frac{1}{4a}x^2,\ \frac{dy}{dx}=\frac{1}{2a}x$ At $P(2at,at^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2at=t$ Line k has a gradient of t and passes through $S(0,a)$ $y-y_1=m(x-x_1)$ $y-a=t(x-0)$ $y=tx+a$ To find the coordinates of Q Substitute $y=0$ into $y=tx+a$ then $x=-\frac{a}{t}$ or $Q(-\frac{a}{t},0)$ To find the coordinates of M $x=\frac{x_1+x_2}{2}$ $y=\frac{y_1+y_2}{2}$ $=\frac{-\frac{a}{t}+0}{2}=-\frac{a}{2t}$ $=\frac{0+a}{2}=\frac{a}{2}$	1 Mark: D

74	The vertex of $x^2=4ay$ is $O(0,0)$. Gradient of OP is $m=\frac{ap^2-0}{2ap-0}=\frac{p}{2}$ Equation of OP is $y-y_1=m(x-x_1)$ $y-0=\frac{p}{2}(x-0)$ $y=\frac{p}{2}x$ OP meets the directrix when $y=-a$ $-a=\frac{p}{2}x$ or $x=\frac{-2a}{p}$ Coordinates of D is $(\frac{-2a}{p},-a)$	1 Mark: A
75	$PA=4PB$ $\sqrt{(x-1)^2+(y-4)^2}=4\sqrt{(x+2)^2+(y-1)^2}$ $x^2-2x+1+y^2-8y+16=16(x^2+4x+4+y^2-2y+1)$ $15x^2+66x+15y^2-24y+63=0$ $5x^2+22x+5y^2-8y+21=0$	1 Mark: C
76	 Coordinates of T $(2ap,aq^2)$ Focal chord $pq=-1$ Now $x=2ap$ and $y=aq^2$ $p=\frac{x}{2a}$ $=a\times\left(-\frac{1}{p}\right)^2=\frac{a}{p^2}$ Eliminating p $y=\frac{a}{\left(\frac{x}{2a}\right)^2}=\frac{4a^3}{x^2}$ $x^2y=4a^3$	1 Mark: D