Projectile motion

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36 A rock is projected with a velocity of 29.4 ms^{-1} at an angle of 30° to the horizontal. What is the maximum height reached by the rock? Let g be 9.8 ms^{-2}.

(A) 0.375 metres
(B) 7.653 metres
(C) 11.025 metres
(D) 22.050 metres

37 A football is kicked at an angle of α to the horizontal. The position of the ball at time t seconds is given by $x = Vt\cos\alpha$ and $y = Vt\sin\alpha - \frac{1}{2}gt^2$ where g m/s² is the acceleration due to gravity and v m/s is the initial velocity of projection. What is the maximum height reached by the ball?

(A) $\dfrac{V\sin\alpha}{g}$

(B) $\dfrac{g\sin\alpha}{V}$

(C) $\dfrac{V^2\sin^2\alpha}{2g}$

(D) $\dfrac{g\sin^2\alpha}{2V^2}$

38 A particle is projected from a window 9 metres above the horizontal ground, at an angle of θ to the horizontal, where $\tan\theta = \frac{3}{4}$ and the initial velocity is 20 metres/second.
What is the maximum height above the ground reached the particle?
Assume $g = 10$ ms^{-2}.

(A) 6.5 metres
(B) 7.2 metres
(C) 15.5 metres
(D) 16.2 metres

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39 A particle is projected from a horizontal plane at an angle of elevation of 30° with a speed of 100 m/s. Assume the acceleration due to gravity is 10 m/s^{-2}. What is the equation of the trajectory?

(A) $y = -\dfrac{x}{\sqrt{3}} - \dfrac{x^2}{500}$

(B) $y = -\dfrac{x}{\sqrt{3}} - \dfrac{x^2}{1500}$

(C) $y = \dfrac{x}{\sqrt{3}} - \dfrac{x^2}{500}$

(D) $y = \dfrac{x}{\sqrt{3}} - \dfrac{x^2}{1500}$

40 A ball is thrown from the origin O with a velocity V and angle of elevation of θ, where $\theta \neq \frac{\pi}{2}$. What is the equation of flight of the ball?

(A) $y = -\frac{1}{2}gt^2 - Vt\sin\theta$

(B) $y = -\frac{1}{2}gt^2 + Vt\sin\theta$

(C) $y = x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$

(D) $y = x\tan\theta + \frac{1}{4h}x^2(1+\tan^2\theta)$

41 A body is projected horizontally from the top of a cliff 20 metres above the water with a velocity of 30 m/s. What is the horizontal distance travelled?

(A) 20 m
(B) 30 m
(C) 60 m
(D) 120 m

42 A particle is projected with a speed of 20 m/s and passes through a point P whose horizontal distance from the point of projection is 30 m and whose vertical height above the point of projection is $8\frac{3}{4}$ m. What is the angle of elevation θ? Take $g = 10\ \text{m/s}^2$.

(A) $\theta = \tan^{-1}(\frac{3}{4})$

(B) $\theta = \tan^{-1}(\frac{4}{3})$

(C) $\theta = \tan^{-1}(\frac{2}{3})$

(D) $\theta = \tan^{-1}(\frac{3}{2})$

43 A stone is thrown from the top of a 15m cliff with an initial velocity of 26ms^{-1} at an angle of projection equal to $\tan^{-1}\left(\frac{5}{12}\right)$ above the horizontal. The equations of motion of the stone are $\ddot{x} = 0$ and $\ddot{y} = -10$. What is the time when the stone will reach the ground?

(A) 2 seconds
(B) 3 seconds
(C) 4 seconds
(D) 5 seconds

44 A missile is fired from the edge of a cliff 100 m high with a velocity of 200 m/s. If the angle of projection is 20° above the horizontal, what is the time of flight? Assume the acceleration due to gravity is 10 m/s^2.

(A) 15 seconds
(B) 16 seconds
(C) 17 seconds
(D) 18 seconds

45 An object falling from rest in air is subjected to an acceleration $\ddot{x} = g - \frac{v}{k}$, where g and k are constants and v is the velooity at time t. Which of the following is the correct equation for velocity?

(A) $v = \frac{g}{k}(1 - e^{-\frac{t}{k}})$

(B) $v = gk(1 - e^{-\frac{t}{k}})$

(C) $v = \frac{g}{k}(1 - e^{\frac{t}{k}})$

(D) $v = gk(1 - e^{\frac{t}{k}})$

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	Solution	**Criteria**
36	$h=\dfrac{V^2\sin^2\alpha}{2g}$ $=\dfrac{(29.4)^2\times\sin^2 30}{2\times 9.8}$ $=11.025$ metres	1 Mark: C
37	$y=Vt\sin\alpha-\frac{1}{2}gt^2$ $\dot{y}=V\sin\alpha-gt$ Maximum height when $\dot{y}=0$ $0=V\sin\alpha-gt$ $t=\dfrac{V\sin\alpha}{g}$ Maximum height $h=V\sin\alpha\times\dfrac{V\sin\alpha}{g}-\dfrac{1}{2}g\times\left(\dfrac{V\sin\alpha}{g}\right)^2$ $=\dfrac{V^2\sin^2\alpha}{2g}$	1 Mark: C
38	$\ddot{y}=-10$ $\quad\tan\theta=\frac{3}{4}$ or $\sin\theta=\frac{3}{5}$ or $\cos\theta=\frac{4}{5}$ $\dot{y}=-10t+V\sin\theta$ When $t=0$, $V=20$ implies $V\sin\theta=20\times\frac{3}{5}=12$ $\dot{y}=-10t+12$ $y=-5t^2+12t+c$ When $t=0$, $y=9$ implies $c=9$ $y=-5t^2+12t+9$ Maximum height occurs when $\dot{y}=0$ $0=-10t+12$ or $t=\frac{6}{5}$ $y=-5\times\left(\frac{6}{5}\right)^2+12\times\frac{6}{5}+9$ $=16.2$	1 Mark: D

39	$x=V\cos\theta t$ $=100\cos 30t$ $=50\sqrt{3}t$ (1) $y=-\frac{1}{2}gt^2+V\sin\theta t$ $=-\frac{1}{2}\times 10\times t^2+100\sin 30t$ $=-5t^2+50t$ (2) From eqn (1) $t=\dfrac{x}{50\sqrt{3}}$ sub into eqn (2) $y=-5\times\left(\dfrac{x}{50\sqrt{3}}\right)^2+50\times\dfrac{x}{50\sqrt{3}}$ $=\dfrac{x}{\sqrt{3}}-\dfrac{x^2}{1500}$	1 Mark: D
40	$x=Vt\cos\theta$ (1) $y=-\frac{1}{2}gt^2+Vt\sin\theta$ (2) From eqn (1) $t=\dfrac{x}{V\cos\theta}$ sub into eqn (2) $y=-\frac{1}{2}g(\dfrac{x}{V\cos\theta})^2+V(\dfrac{x}{V\cos\theta})\sin\theta$ $=-\dfrac{gx^2}{2V^2\cos^2\theta}+\dfrac{\sin\theta x}{\cos\theta}$ $=-\dfrac{gx^2\sec^2\theta}{2V^2}+\tan\theta x$ $=-\dfrac{2gx^2\sec^2\theta}{4V^2}+\tan\theta x$ $=-\dfrac{x^2\sec^2\theta}{4h}+\tan\theta x$ $=x\tan\theta-\dfrac{1}{4h}x^2(1+\tan^2\theta)$	1 Mark: C

41	$\ddot{x}=0$ $\dot{x}=V\cos\theta$ When $t=0$, $V=30$, $\theta=0$ implies $V\cos\theta=30$ $\dot{x}=30$ $x=30t+c$ When $t=0$, $x=0$ implies $c=0$ $x=30t$ $\ddot{y}=-10$ $\dot{y}=-10t+V\sin\theta$ When $t=0$, $V=30$, $\theta=0$ implies $V\sin\theta=0$ $\dot{y}=-10t$ $y=-5t^2+c$ When $t=0$, $y=20$ implies $c=20$ $y=-5t^2+20$ Particle reaches the water when $y=0$ $0=-5t^2+20$ or $t=2$ Horizontal distance travelled $x=30t=30\times 2=60$ m	1 Mark: C
42	$x=Vt\cos\theta$ $=20t\cos\theta$ $y=-\frac{1}{2}gt^2+Vt\sin\theta$ $=-\frac{1}{2}\times 10\times t^2+20t\sin\theta$ $=5t^2+20t\sin\theta$ At $P(30,\frac{35}{4})$ $30=20t\cos\theta$ or $t=\frac{3}{2\cos\theta}$ $\frac{35}{4}=-5t^2+20t\sin\theta$ $7=-4t^2+16t\sin\theta$ $7=-4\left(\frac{3}{2\cos\theta}\right)^2+16\times\left(\frac{3}{2\cos\theta}\right)\times\sin\theta$ $7=-9\sec^2\theta+24\tan\theta$ $7=-9(\tan^2\theta+1)+24\tan\theta$ $9\tan^2\theta-24\tan\theta+16=0$ $(3\tan\theta-4)^2=0$ $3\tan\theta=4$ $\tan\theta=\frac{4}{3}$ or $\theta=\tan^{-1}(\frac{4}{3})$	1 Mark: B

43	Impact occurs when $y=0$ $\ddot{y}=-10$ $\dot{y}=-10t+c$ When $t=0$, $V=26$ and $\tan\theta=\frac{5}{12}$ $(\sin\theta=\frac{5}{13})$ Hence $\dot{y}=V\sin\theta=26\times\frac{5}{13}=10$ and $c=10$ $\dot{y}=-10t+10$ $y=-5t^2+10t+c$ When $t=0$, $y=15$ and hence $c=15$ $y=-5t^2+10t+15$ $=-5(t^2-2t-3)$ $=-5(t-3)(t+1)$ Therefore if $y=0$ then $t=3$	1 Mark: B
44	Impact occurs when $y=0$ $\ddot{y}=-10$ $\dot{y}=-10t+c$ When $t=0$, $V=200$ and $\theta=20^\circ$ Hence $\dot{y}=V\sin\theta=200\sin 20^\circ$ and $c=200\sin 20^\circ$ $\dot{y}=-10t+200\sin 20^\circ$ $y=-5t^2+200\sin 20^\circ t+c$ When $t=0$, $y=100$ and hence $c=100$ $y=-5t^2+200\sin 20^\circ t+100$ $=-5(t^2-40\sin 20^\circ t-20)$ Therefore if $y=0$ then $t=\frac{40\sin 20\pm\sqrt{(-40\sin 20)^2-4\times 1\times -20}}{2}$ $=15.012985...$ or $-1.3321800...$ ≈ 15	1 Mark: A

45	$\ddot{x} = g - \frac{v}{k}$ $\frac{dv}{dt} = g - \frac{v}{k} = \frac{kg - v}{k}$ $\frac{dt}{dv} = \frac{k}{kg - v}$ $t = \int \frac{k\,dv}{kg - v}$ $= -k\ln(kg - v) + c$ Initial conditions $t = 0$ and $v = 0$ $0 = k\log_e(kg) + c$ or $c = k\log_e(kg)$ $t = -k\ln(kg - v) + k\log_e(kg)$ $\frac{t}{k} = \ln\frac{kg}{kg - v}$ $e^{\frac{t}{k}} = \frac{kg}{kg - v}$ $kg - v = kge^{-\frac{t}{k}}$ $v = kg - kge^{-\frac{t}{k}}$ $= gk(1 - e^{-\frac{t}{k}})$	1 Mark: B