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65 Ten kilograms of chlorine is placed in water and begins to dissolve. After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$. What is the value of k given that $A = 3.6$ and $t = 5$?

(A) −0.717
(B) −0.204
(C) 0.204
(D) 0.717

66 The population of a colony of bugs is increasing continuously at a rate proportional to the existing population. The present population is 20 000 and the population 3 months ago was 8000. What is the value of k?

(A) −0.916
(B) −0.305
(C) 0.305
(D) 0.916

67 The population of a town is falling at a constant rate, so that after 25 years the population will have halved, and $\frac{dP}{dt} = -kP$, where P is the population of the town and t is the time in years. What is the value of k?

(A) $\frac{\ln 0.5}{-25}$
(B) $\frac{\ln 0.5}{25}$
(C) $\frac{\ln 2}{-25}$
(D) $\frac{\ln 2}{25}$

68 It is assumed that the number $N(t)$ of ants in a certain nest at time $t \geq 0$ is given by $N(t) = \frac{A}{1+e^{-t}}$ where A is a constant and t is measured in months. At time $t = 0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A?

(A) 2×10^5
(B) 2×10^{-5}
(C) 4×10^5
(D) 4×10^{-5}

69 A flat circular disc is being heated so that the rate of increase of the area (A in m^2), after t hours, is given by $\frac{dA}{dt} = \frac{1}{8}\pi t$. Initially the disc has a radius of 2 metres. Which of the following is the correct expression for the area after t hours?

(A) $A = \frac{1}{8}\pi t^2$
(B) $A = \frac{1}{16}\pi t^2$
(C) $A = \frac{1}{8}\pi t^2 + 4\pi$
(D) $A = \frac{1}{16}\pi t^2 + 4\pi$

70 A circular metal plate of area A cm^2 is being heated. It is given that $\frac{dA}{dt} = \frac{\pi t}{32}$ cm^2/h. What is the exact area of the plate after 8 hours, if initially the plate had a radius of 6 cm?

(A) π (B) 0.25π
(C) 36π (D) 37π

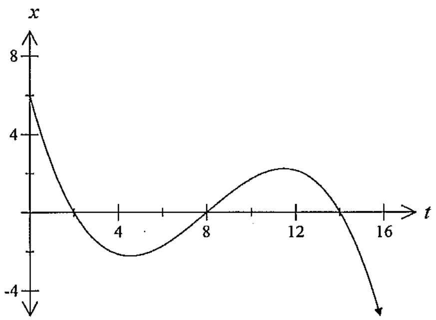
71 A particle moves along a straight line so that its distance x, in metres from a fixed point O is given by $x = \cos t + t$, where t is the time measured in seconds. When does the particle first come to rest?

(A) $\frac{\pi}{4}$ seconds
(B) $\frac{\pi}{2}$ seconds
(C) $\frac{3\pi}{4}$ seconds
(D) $\frac{3\pi}{2}$ seconds

72 A particle moves along the x-axis with acceleration $3t - 2$. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. What is the position of the particle after 5 seconds?

(A) 37.5 units to the right
(B) 37.5 units to the left
(C) 51.5 units to the right
(D) 51.5 units to the left

73 The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph.



When was the particle at rest?

(A) $t = 4.5$ and $t = 11.5$

(B) $t = 0$

(C) $t = 2$, $t = 8$ and $t = 14$

(D) $t = 1.5$ and $t = 8$

74 A particle is moving in a straight line, starting from the origin. At time t seconds the particle has a displacement of x metres from the origin, velocity of v ms^{-1} and acceleration of a ms^{-2}. The displacement is given by $x = t - 2\log_e(t^2 + 1)$. Which of the following is the correct expression for the velocity v?

(A) $v = 1 - \dfrac{2}{t^2 + 1}$

(B) $v = 1 - \dfrac{4t}{t^2 + 1}$

(C) $v = t - \dfrac{2}{t^2 + 1}$

(D) $v = t - \dfrac{4t}{t^2 + 1}$

75 The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$. Initially the particle is at $x = 5$ metres and the initial velocity of the particle is -36 m/s. When is the particle at rest?

(A) $t = 0$

(B) $t = 1$

(C) $t = 2$

(D) $t = 3$

76 A particle moves along a straight line about a fixed point O so that its acceleration, a ms^{-2}, at time t seconds is given by $a = 4\cos\left(2t + \dfrac{\pi}{6}\right)$. Initially the particle is moving to the right with a velocity of 1 ms^{-1} from a position $\dfrac{\sqrt{3}}{2}$ metres to the left of O. Which of the following is the correct expression for the velocity of the particle after t seconds?

(A) $v = 2\sin\left(2t + \dfrac{\pi}{6}\right)$

(B) $v = 2\sin\left(2t + \dfrac{\pi}{6}\right) + 1 - \sqrt{3}$

(C) $v = 4\sin\left(2t + \dfrac{\pi}{6}\right)$

(D) $v = 4\sin\left(2t + \dfrac{\pi}{6}\right) - 1$

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	Solution	**Criteria**
65	$A = 10e^{-kt}$ $3.6 = 10e^{-k\times 5}$ $e^{-5k} = 0.36$ $-5k\log_e e = \log_e 0.36$ $k = \frac{\log_e 0.36}{-5}$ $= 0.2043302495 \approx 0.204$	1 Mark: C
66	$P = 8000e^{kt}$ $20000 = 8000e^{k\times 3}$ $e^{3k} = 2.5$ $3k\log_e e = \log_e 2.5$ $k = \frac{\log_e 2.5}{3}$ $= 0.305430244 \approx 0.305$	1 Mark: C
67	Exponential decay $P = P_0e^{-kt}$ When $t = 25$ then $P = \frac{P_0}{2}$ $P_0 = \frac{P_0}{2}e^{-k\times 25}$ $e^{-25k} = \frac{1}{2}$ $\ln e^{-25k} = \ln 0.5$ $-25k = \ln 0.5$ $k = \frac{\ln 0.5}{-25}$	1 Mark: A
68	$N(t) = \frac{A}{1+e^{-t}}$ $2\times 10^5 = \frac{A}{1+e^0}$ $A = 4\times 10^5$	1 Mark: C
69	$A = \pi r^2 = \pi \times 2^2 = 4\pi \text{ m}^2$ $A = \int \frac{1}{8}\pi t\,dt$ When $t = 0, A = 4\pi$ $4\pi = \frac{1}{16}\pi 0^2 + c$ $= \frac{1}{16}\pi t^2 + c$ $c = 4\pi$ Hence $A = \frac{1}{16}\pi t^2 + 4\pi$	1 Mark: D

70	$A = \pi r^2 = \pi \times 6^2 = 36\pi \text{ cm}^2$ $A = \int \frac{\pi t}{32}dt$ When $t = 0, A = 36\pi$ $36\pi = \frac{1}{64}\pi 0^2 + c$ $= \frac{1}{64}\pi t^2 + c$ $c = 36\pi$ Hence $A = \frac{1}{64}\pi t^2 + 36\pi$ $= \frac{1}{64}\pi \times 8^2 + 36\pi = 37\pi$	1 Mark: D
71	Particle comes to rest if $v = 0$ $v = \frac{dx}{dt}$ $0 = -\sin t + 1$ $\sin t = 1$ $t = \frac{\pi}{2}$ seconds	1 Mark: B
72	$a = 3t - 2$ $v = \frac{3t^2}{2} - 2t + c$ When $t = 0$ then $v = 2$ $2 = \frac{3\times 0^2}{2} - 2\times 0 + c$ or $c = 2$ $v = \frac{3t^2}{2} - 2t + 2$ $x = \frac{t^3}{2} - t^2 + 2t + k$ When $t = 0$ then $x = 4$ $4 = \frac{0^3}{2} - 0^2 + 2\times 0 + k$ or $k = 4$ $x = \frac{t^3}{2} - t^2 + 2t + 4$ When $t = 5$ $x = \frac{5^3}{2} - 5^2 + 2\times 5 + 4$ $= 51.5$ units	1 Mark: C
73	Particle at rest if $v = 0$ (stationary points of the curve) $t = 4.5$ and $t = 11.5$	1 Mark: A

74	$x = t - 2\log_e(t^2+1)$ $v = 1 - \frac{2}{t^2+1} \times 2t$ $= 1 - \frac{4t}{t^2+1}$	1 Mark: B
75	$a = 12t + 6$ $v = 6t^2 + 6t + c$ When $t = 0$ then $v = -36$ $-36 = 6 \times 0^2 + 6 \times 0 + c$ or $c = -36$ $v = 6t^2 + 6t - 36$ $= 6(t+3)(t-2)$ Particle at rest ($v = 0$) when $t = 2$	1 Mark: C
76	$v = \int 4\cos\left(2t + \frac{\pi}{6}\right)dt$ $= 2\sin\left(2t + \frac{\pi}{6}\right) + c$ When $t = 0, v = 1$ $1 = 2\sin\left(2\times 0 + \frac{\pi}{6}\right) + c$ $c = 1 - 2\sin\left(2\times 0 + \frac{\pi}{6}\right) = 0$ $\therefore\ v = 2\sin\left(2t + \frac{\pi}{6}\right)$	1 Mark: A