Series and applications

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88 In the first week of the snow season 5 cm of snow falls. In each following weeks the snowfalls increase by 2 cm, so in the second week there is 7 cm, in the third week there is 9 cm. How much snow falls in the 11th week?

(A) 23 cm
(B) 25 cm
(C) 27 cm
(D) 29 cm

89 A marathon runner started a new training program. On the first day he completed 30 laps of the running track. Every succeeding day he increased his training by 5 laps, until their daily schedule reached 105 laps. On which day did he complete 105 laps in a single day?

(A) 15
(B) 16
(C) 20
(D) 21

90 Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether. Which of the following is the correct expression for the number of boxes in the bottom layer?

(A) $n+5$
(B) $n+6$
(C) $6n-1$
(D) $6n-5$

91 The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?

(A) -5
(B) 5
(C) 13
(D) 42

92 Owen starts on a salary of \$30 000 with an annual increment of \$1550. What is the total amount Owen would earn in eleven years of employment?

(A) \$45 000
(B) \$276 830
(C) \$415 250
(D) \$495 000

93 What is the value of $\sum_{r=1}^{3} 2^{1-r}$?

(A) $\frac{3}{4}$
(B) $1\frac{3}{4}$
(C) 6
(D) 7

94 What is the value of $\sum_{r=1}^{40} (3r-7)$?

(A) 109
(B) 2180
(C) 2260
(D) 2380

95 The first term of a geometric series is 81 and the fourth term is $\frac{1}{9}$. What is the common ratio?

(A) $\frac{1}{3}$
(B) $\frac{1}{9}$
(C) $\frac{1}{27}$
(D) $\frac{1}{81}$

96 The third and seventh terms of a geometric series are 2.5 and 40 respectively. What is the common ratio?

(A) ± 2
(B) ± 4
(C) ± 8
(D) ± 16

97 The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term?

(A) ± 2

(B) ± 4

(C) $\frac{5}{16}$

(D) $\frac{5}{1024}$

98 A brand of rechargeable batteries provides power for 20 hours when first purchased fully charged. After its first recharge it only provides power for a further 18 hours. After its second recharge it only supplies power for 16.2 hours. Each subsequent recharging results in the battery having 90% of its previous power available. How many hours could you expect to get out of the battery?

(A) 100 hours

(B) 150 hours

(C) 200 hours

(D) 250 hours

99 An infinite geometric series has a first term of 8 and a limiting sum of 12. What is the common ratio?

(A) $\frac{1}{6}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

100 A sum of \$26 000 is borrowed now at 6% per annum reducible interest. Payment is made by n equal annual instalments of \$4 200 beginning at the end of the first year. Which of the following is the correct expression for the amount after 2 years?

(A) $A_2 = 26000 \times 1.06^2 - 4200(1.06+1)$

(B) $A_2 = 26000 \times 1.06^3 - 4200(1.06^2+1.06+1)$

(C) $A_2 = 4200 \times 1.06^2 - 26000(1.06+1)$

(D) $A_2 = 4200 \times 1.06^3 - 26000(1.06^2+1.06+1)$

101 Abby has set up her superannuation fund and after 10 years she has accumulated \$134 000. However due to an accident she is no longer able to work and make further contributions to the fund. Abby is leaving the money in the superannuation fund to accumulate interest at 8% p.a. compounded annually. However she needs to withdraw \$24 000 at the end of each year for normal living expenses. Which of the following is the correct expression for the amount in the fund after 3 years?

(A) $A_3 = 24000(1.08^3+1.08^2+1.08+1) - 134000 \times 1.08^2$

(B) $A_3 = 134000 \times 1.08^3 - 24000(1.08^2+1.08+1)$

(C) $A_3 = 134000(1.08^3+1.08^2+1.08+1) - 24000 \times 1.08^2$

(D) $A_3 = 24000 \times 1.08^3 - 134000(1.08^2+1.08+1)$

102 John invests \$$P$ at 7% per annum compounded annually. He plans to withdraw \$5000 at the end of each year for six years to cover university fees. Which of the following is the correct expression for the amount \$$A_6$ remaining in the account after the sixth withdrawal?

(A) $A_6 = P \times 1.07^5 - 5000(1.07^6+1.07^5+\ldots+1)$

(B) $A_6 = P \times 1.07^6 + 5000(1.07^5 - 1.07^4 - \ldots - 1)$

(C) $A_6 = P \times 1.07^5 + 5000(1.07^6 - 1.07^5 - \ldots - 1)$

(D) $A_6 = P \times 1.07^6 - 5000(1.07^5+1.07^4+\ldots+1)$

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	Solution	**Criteria**
88	$a=5$, $n=11$ and $d=2$ $T_n=a+(n-1)d$ $T_{11}=5+(11-1)\times 2$ $=25$ 25 cm of snowfalls in week 11.	1 Mark: C
89	$a=30$, $T_n=105$ and $d=5$ $T_n=a+(n-1)d$ $105=30+(n-1)\times 5$ $105=30+5n-5$ $5n=80$ $n=16$ The runner completes 105 laps on the 16th day.	1 Mark: B
90	Number of boxes in each layer from the top are an AP: 6, 7, 8... $T_n=a+(n-1)d$ $=6+(n-1)\times 1$ $=6+n-1$ $=n+5$	1 Mark: A
91	$T_n=a+(n-1)d$ $T_4=a+3d=27$.....(1) $T_7=a+6d=12$.....(2) Eqn (2) – (1) $3d=-15$ $d=-5$	1 Mark: A
92	$a=30000$, $n=11$ and $d=1550$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{11}=\frac{11}{2}[60000+10\times 1550]$ $=\$415\ 250$	1 Mark: C
93	$\sum_{r=1}^{3}2^{1-r}=2^0+2^{-1}+2^{-2}$ $=1+\frac{1}{2}+\frac{1}{4}=1\frac{3}{4}$	1 Mark: B

94	$\sum_{r=1}^{40}(3r-7)=-4-1+2+5+...+113$ Arithmetic series with $a=-4$, $l=113$ and $n=40$ $S_n=\frac{n}{2}(a+l)$ $=\frac{40}{2}(-4+113)$ $=2180$	1 Mark: B
95	Geometric series with $a=81$ and $T_4=\frac{1}{9}$ $T_n=ar^{n-1}$ $T_4=81\times r^{4-1}=\frac{1}{9}$ $r^3=\frac{1}{729}$ $r=\frac{1}{9}$	1 Mark: B
96	$T_3=ar^2=2.5$ and $T_7=ar^6=40$ Divide the two equations $\frac{ar^6}{ar^2}=\frac{40}{2.5}$ $r^4=16$ $r=\pm 2$	1 Mark: A
97	$T_3=ar^2=1.5$ and $T_7=ar^6=20$ Divide the two equations $\frac{ar^6}{ar^2}=\frac{20}{1.25}$ $r^4=16$ $r=\pm 2$ $T_7=a\times(\pm 2)^6=20$ $a=\frac{20}{64}=\frac{5}{16}$	1 Mark: C
98	$a=20$ and $r=90\%=0.9$ $S=\frac{a}{1-r}$ $=\frac{20}{1-0.9}$ $=200$	1 Mark: C

99	$a=8$ and $S=12$ $S=\frac{a}{1-r}$ $12=\frac{8}{1-r}$ $12-12r=8$ $12r=4$ $r=\frac{1}{3}$	1 Mark: C
100	$A=P(1+r)^n$ $=26000\times(1+0.06)^1$ $=26000(1.06)$ After 1 year $A_1=26000\times1.06-42000$ After 2 years $A_2=(26000(1.06)-42000)\times1.06-42000$ $=26000\times1.06^2-42000(1.06+1)$	1 Mark: A
101	$A=P(1+r)^n$ $=134000\times(1+0.08)^1$ $=134000(1.08)$ After 1 year $A_1=134000\times1.08-24000$ After 2 years $A_2=(134000(1.08)-24000)\times1.08-24000$ $=134000\times1.08^2-24000(1.08+1)$ After 3 years $A_3=\left[(134000\times1.08^2-24000(1.08+1)\right]\times1.08-24000$ $=134000\times1.08^3-24000(1.08^2+1.08+1)$	1 Mark: B
102	$A=P(1+r)^n$ $=P\times(1+0.07)^1$ $=P(1.07)$ After 1 year $A_1=P(1.07)-5000$ After 2 years $A_2=(P(1.07)-5000)\times1.07-5000$ $=P\times1.07^2-5000(1.07+1)$ After 6 years $A_6=P\times1.07^6-5000(1.07^5+1.07^4+...+1)$	1 Mark: D