Binomial theorem

Solutions Main Menu

73 What is the sixth term in the expansion of $(2x-3y)^9$?

(A) ${}^9C_3\times 2^6\times(-3)^3x^6y^3$

(B) ${}^9C_4\times 2^5\times(-3)^4x^5y^4$

(C) ${}^9C_5\times 2^4\times(-3)^5x^4y^5$

(D) ${}^9C_6\times 2^3\times(-3)^6x^3y^6$

74 What is the term independent of x in the expansion of $(x^2-\frac{2}{x})^9$

(A) ${}^9C_3(-2)^3$

(B) ${}^9C_6(-2)^6$

(C) ${}^9C_3(2)^3$

(D) ${}^9C_6(2)^6$

75 What is the coefficient of x^8 in the expansion of $\left(2x^3-\frac{1}{x}\right)^{12}$?

(A) $-{}^{12}C_8\times 2^4$

(B) ${}^{12}C_8\times 2^4$

(C) ${}^{12}C_8\times 2^5$

(D) $-{}^{12}C_7\times 2^5$

76 What is the coefficient of x^{-5} in the expansion of $\left(2x^2-\frac{1}{x}\right)^{20}$?

(A) $-{}^{20}C_{16}\times 2^4$

(B) $-{}^{20}C_{16}\times 2^5$

(C) $-{}^{20}C_{15}\times 2^4$

(D) $-{}^{20}C_{15}\times 2^5$

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77 In the expansion of $(2x+k)^6$ the coefficients of x and x^2 are equal.
What is the value of k?

(A) 5 (B) 6

(C) 11 (D) 12

78 In the expansion $(1+x+kx^2)^9$ the coefficient of x^2 is zero. What is the value of k?

(A) −4 (B) −1

(C) 1 (D) 4

79 What is the coefficient of x^5 in the expansion of $(1-3x+2x^3)(1-2x)^6$?

(A) −792 (B) −720

(C) 120 (D) 312

80 What is the greatest coefficient in the expansion of $(5+2x)^{12}$?

(A) ${}^{12}C_3\times 5^9\times 2^3$ (B) ${}^{12}C_4\times 5^8\times 2^4$

(C) ${}^{12}C_5\times 5^7\times 2^5$ (D) ${}^{12}C_6\times 5^9\times 2^3$

81 Consider the binomial expansion $(1+x)^n=1+{}^nC_1x+{}^nC_2x^2+\ldots+{}^nC_nx^n$.
Which of the following expressions is correct?

(A) ${}^nC_1+2\,{}^nC_2+\ldots+n\,{}^nC_n=n2^{n-1}$

(B) ${}^nC_1+2\,{}^nC_2+\ldots+n\,{}^nC_n=n2^{n+1}$

(C) ${}^nC_1+{}^nC_2+\ldots+{}^nC_n=2^{n-1}$

(D) ${}^nC_1+{}^nC_2+\ldots+{}^nC_n=2^{n+1}$

82 Consider the binomial expansion
$(1+x)^{2n}={}^{2n}C_0x+{}^{2n}C_1x+{}^{2n}C_2x^2+\ldots+{}^{2n-1}C_{2n-1}x^n+{}^{2n}C_{2n}x^n$.
Which of the following expressions is correct?

(A) $\sum_{r=0}^{n}{}^{2n}C_r=2^{2n-2}+\frac{(2n)!}{2(n!)^2}$

(B) $\sum_{r=0}^{n}{}^{2n}C_r=2^{2n-2}+\frac{(2n-1)!}{2(n!)^2}$

(C) $\sum_{r=0}^{n}{}^{2n}C_r=2^{2n-1}+\frac{(2n)!}{2(n!)^2}$

(D) $\sum_{r=0}^{n}{}^{2n}C_r=2^{2n-1}+\frac{(2n-1)!}{2(n!)^2}$

Mathematical induction		Main Menu
	Solution	**Criteria**
68	Step 2: Assume the result true for $n=k$ $1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{1}{3}k(2k+1)(2k-1)$	1 Mark: B
69	$\sum_{r=1}^{n}\frac{r^2}{(2r-1)(2r+1)}=\frac{1^2}{1\times3}+\frac{2^2}{3\times5}+\ldots+\frac{n^2}{(2n-1)(2n+1)}$ Step 1: To prove the statement true for $n=1$ $\text{LHS}=\frac{1^2}{1\times3}=\frac{1}{3}$ $\quad\text{RHS}=\frac{1(2)}{2(3)}=\frac{1}{3}$ Result is true for $n=1$	1 Mark: A
70	Step 2: Assume the result true for $n=k$ $\frac{2}{1\times2}+\frac{2}{2\times3}+\frac{2}{3\times4}+\ldots+\frac{2}{k\times(k+1)}=\frac{2k}{k+1}$ To prove the result is true for $n=k+1$ $\frac{2}{1\times2}+\frac{2}{2\times3}+\frac{2}{3\times4}+\ldots+\frac{2}{k\times(k+1)}+\frac{2}{(k+1)(k+2)}=\frac{2(k+1)}{(k+2)}$ $\text{LHS}=\frac{2}{1\times2}+\frac{2}{2\times3}+\frac{2}{3\times4}+\ldots+\frac{2}{k\times(k+1)}+\frac{2}{(k+1)(k+2)}$ $=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$	1 Mark: C
71	$\text{LHS}=3^{2(k+1)}-1$ $=3^{2k+2}-1$ $=3^{2k}\times3^2-1$ $=(8P+1)\times3^2-1$ $=72P+9-1$ $=72P+8$ $=8(9P+1)$ $=8Q=\text{ RHS}$ Line 4 contains the error	1 Mark: D
72	$\text{LHS}=(k+1)^3+(k+2)^3+(k+3)^3$ $=9P-(k)^3+(k+3)^3$ from (1) $=9P-k^3+(k+3)(k^2+6k+9)$ $=9P-k^3+k^3+6k^2+9k+3k^2+18k+27$ $=9(P+k^2+3k+3)$ $=9Q=\text{ RHS}$	1 Mark: D

Binomial theorem		Main Menu
	Solution	**Criteria**
73	$T_6={}^9C_5(2x)^{9-5}(-3y)^5$ $={}^9C_5\times2^4\times x^4(-3)^5y^5$ $={}^9C_5\times2^4\times(-3)^5x^4y^5$	1 Mark: C
74	$T_{r+1}={}^9C_r(x^2)^{9-r}\left(-\frac{2}{x}\right)^r$ $={}^9C_rx^{18-2r}(-2)^rx^{-r}$ $={}^9C_r(-2)^rx^{18-3r}$ Term independent of x $18-3r=0$ $r=6$ $T_7={}^9C_6(-2)^6x^{18-3\times6}={}^9C_6(-2)^6$	1 Mark: B
75	$T_{r+1}={}^{12}C_r(2x^3)^{12-r}\left(-\frac{1}{x}\right)^r$ $={}^{12}C_r\times2^{12-r}\times(-1)^r\times x^{36-3r}\times x^{-r}$ $={}^{12}C_r\times2^{12-r}\times(-1)^r\times x^{36-4r}$ Term with a coefficient of x^8 $36-4r=8$ $4r=28$ $r=7$ $T_8={}^{12}C_7\times2^{12-7}\times(-1)^7\times x^{36-4\times7}$ $=-{}^{12}C_7\times2^5\times x^8$ Coefficient of x^8 is $-{}^{12}C_7\times2^5$	1 Mark: D
76	$T_{r+1}={}^{20}C_r(2x^2)^{20-r}\left(-\frac{1}{x}\right)^r$ $={}^{20}C_r\times2^{20-r}\times(-1)^r\times x^{40-2r}\times x^{-r}$ $={}^{20}C_r\times2^{20-r}\times(-1)^r\times x^{40-3r}$ Term with a coefficient of x^{-5} $40-3r=-5$ $3r=45$ $r=15$ $T_{16}={}^{20}C_{15}\times2^{20-15}\times(-1)^{15}\times x^{40-3\times15}$ $=-{}^{20}C_{15}\times2^5\times x^{-5}$ Coefficient of x^8 is $-{}^{20}C_{15}\times2^5$	1 Mark: D

77	${}^6C_1 \times (2x)^1 \times k^5 = 12k^5 x$ ${}^6C_2 \times (2x)^2 \times k^4 = 60k^4 x^2$ Therefore $12k^5 = 60k^4$ $k = 5$	1 Mark: A
78	$(1 + x + kx^2)^n = 1 + {}^9C_1(x + kx^2) + {}^9C_2(x + kx^2)^2 + \ldots$ $= 1 + 9x + 9kx^2 + 36(x^2 + 2kx^3 + k^2x^4) + \ldots$ Therefore $9k + 36 = 0$ $k = -4$	1 Mark: A
79	$(1 - 3x + 2x^3)(1 - 2x)^6$ $= (1 - 3x + 2x^3)[1 + {}^6C_1(-2x) + {}^6C_2(-2x)^2$ $+ {}^6C_3(-2x)^3 + {}^6C_4(-2x)^4 + {}^6C_5(-2x)^5 + {}^6C_6(-2x)^6]$ The terms for x^5 ${}^6C_5(-2x)^5 - 3x \times {}^6C_4(-2x)^4 + 2x^3 \times {}^6C_3(-2x)^3) = -792x^5$	1 Mark: A
80	$T_{r+1} = {}^{12}C_r (5)^{12-r} (2x)^r$ $\quad T_r = {}^{12}C_{r-1} (5)^{13-r} (2x)^{r-1}$ $= {}^{12}C_r (5)^{12-r} \times 2^r \times x^r$ $\quad = {}^{12}C_{r-1} (5)^{13-r} \times 2^{r-1} \times x^{r-1}$ $\frac{T_{r+1}}{T_r} = \frac{{}^{12}C_r (5)^{12-r} \times 2^r \times x^r}{{}^{12}C_{r-1} (5)^{13-r} \times 2^{r-1} \times x^{r-1}}$ Ratio of terms $= \frac{(13 - r) \times 2}{r \times 5}$ $= \frac{26 - 2r}{5r}$ Greatest coefficient: $26 - 2r \ge 5r$ $r \le 3\frac{5}{7}$ Greatest coefficient when $r = 3$ $T_4 = {}^{12}C_3 \times 5^9 \times 2^3 \times x^3$ hence greatest coefficient is ${}^{12}C_3 \times 5^9 \times 2^3$	1 Mark: A
81	Differentiate both sides of the identity $n(1 + x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + \ldots + n\,{}^nC_n x^{n-1}$ Substitute $x = 1$ $n(1 + 1)^{n-1} = {}^nC_1 + 2\,{}^nC_2 1 + \ldots + n\,{}^nC_n 1^{n-1}$ $n2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n$ ${}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n = n2^{n-1}$	1 Mark: A

82	$(1 + x)^{2n} = {}^{2n}C_0 x + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots + {}^{2n-1}C_{2n-1} x^n + {}^{2n}C_{2n} x^n$ Substitute $x = 1$ $(2)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots + {}^{2n}C_n \ldots + {}^{2n-1}C_{2n-1} + {}^{2n}C_{2n}$ $= 2\,{}^{2n}C_0 + 2\,{}^{2n}C_1 + 2\,{}^{2n}C_2 + \ldots + {}^{2n}C_n$ (since ${}^nC_r = {}^nC_{n-r}$) $= 2\,{}^{2n}C_0 + 2\,{}^{2n}C_1 + 2\,{}^{2n}C_2 + \ldots + 2\,{}^{2n}C_n - {}^{2n}C_n$ $2^{2n} + {}^{2n}C_n = 2 \times \sum_{r=0}^{n} {}^{2n}C_r$ $\sum_{r=0}^{n} {}^{2n}C_r = \frac{2^{2n} + {}^{2n}C_n}{2} = 2^{2n-1} + \frac{(2n)!}{2n!n!} = 2^{2n-1} + \frac{(2n)!}{2(n!)^2}$	1 Mark: C