Further exponential growth and decay

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21 The number N of animals in a population at time t years is given by $N = 100 + Ae^{kt}$ for constants $A > 0$ and $k > 0$. Which of the following is the correct differential equation?

(A) $\dfrac{dN}{dt} = k(N - 100)$

(B) $\dfrac{dN}{dt} = -k(N + 100)$

(C) $\dfrac{dN}{dt} = -k(N - 100)$

(D) $\dfrac{dN}{dt} = k(N + 100)$

22 A bowl of soup at temperature T°C, when placed in a cooler environment, loses heat according to the law $\dfrac{dT}{dt} = k(T - T_0)$ where t is the time elapsed in minutes and T_0 is the temperature of the environment in degrees Celsius. A bowl of soup at 96°C is left to stand in a room at a temperature of 18°C. After 3 minutes the soup cools down to 75°C. What is the value of k correct to 4 decimal places?

(A) 0.0784

(B) 0.0856

(C) 0.1046

(D) 0.1236

23 A bottle of water has a temperature of 20°C and is placed in a refrigerator whose temperature is 2°C. The cooling rate of the bottle of water is proportional to the difference between the temperature of the refrigerator and the temperature T of the bottle of water. This is expressed by the equation $\dfrac{dT}{dt} = -k(T - 2)$ where k is a constant of proportionality and t is the number of minutes after the bottle of water is placed in the refrigerator. After 20 minutes in the refrigerator the temperature of the bottle of water is 10°C. What is the value of k in the above equation?

(A) $k = -\dfrac{1}{20}\log_e \dfrac{9}{4}$

(B) $k = -\dfrac{1}{10}\log_e \dfrac{4}{9}$

(C) $k = \dfrac{1}{20}\log_e \dfrac{9}{4}$

(D) $k = \dfrac{1}{10}\log_e \dfrac{4}{9}$

24 A piece of hot metal is placed in a room with a surrounding air temperature of 20°C and allowed to cool. It loses heat according to Newton's law of cooling, $\dfrac{dT}{dt} = -k(T - A)$ where T is the temperature of the metal in degrees Celsius at time t minutes, A is the surrounding air temperature and k is a positive constant. After 6 minutes the temperature of the metal is 80°C, and after a further 2 minutes it is 50°C. What is the value of k in the above equation?

(A) $k = 2\log_e 0.5$

(B) $k = 2\log_e 2$

(C) $k = \dfrac{\log_e 0.5}{2}$

(D) $k = \dfrac{\log_e 2}{2}$

25 Broccoli is cooked by immersing it in boiling water. On being removed, the broccoli cools in the air according to the equation $\dfrac{dT}{dt} = -k(T - T_0)$ where t is time in minutes, T is temperature in °C and T_O is the temperature of the air, while k is a positive constant. If the temperature of the boiling water is 100°C and that of the air is a constant 25°C, what are the values of A and k if a broccoli cools to 70°C in 3 minutes?

(A) $A = 25$ and $k = -\dfrac{1}{3}\log_e 0.6$

(B) $A = 25$ and $k = \dfrac{1}{3}\log_e 0.6$

(C) $A = 75$ and $k = -\dfrac{1}{3}\log_e 0.6$

(D) $A = 75$ and $k = \dfrac{1}{3}\log_e 0.6$

26 An ingot of aluminium is placed in a room with a surrounding air temperature of 25°C and allowed to cool. It loses heat according to Newton's law of cooling, $\dfrac{dT}{dt} = -k(T - A)$ where T is the temperature of the metal in degrees Celsius at time t minutes, A is the surrounding air temperature and k is a positive constant. The initial temperature of the aluminium is 1350°C. After 10 minutes the temperature of the aluminium is 720°C. What total time elapses for the aluminium to cool to 50°C?

(A) 60.35 minutes

(B) 61.53 minutes

(C) 64.52 minutes

(D) 65.28 minutes

27 The original temperature of a body is 100°C, the temperature of the surroundings is 20°C and the body cools to 70°C in 10 minutes. What is the approximate temperature after 20 minutes? Assume Newton's law of cooling.

(A) 50°C

(B) 51°C

(C) 52°C

(D) 53°C

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	Solution	**Criteria**
21	$N=100+Ae^{kt}$ or $Ae^{kt}=N-100$ $\frac{dN}{dt}=Ake^{kt}$ $=kAe^{kt}$ $=k(N-100)$	1 Mark: A
22	$T=T_0+Ae^{-kt}$ satisfies the equation $\frac{dT}{dt}=k(T-T_0)$ When $t=0$, $T=96$, $T_0=18$ so $A=78$ $T=18+78e^{-kt}$ Also $t=3$ and $T=75$ $75=18+78e^{-3k}$ $e^{-3k}=\frac{57}{78}$ $-3k=\log_e\frac{57}{78}$ $k=-\frac{\log_e\frac{57}{78}}{3}$ $=0.1045525196\approx 0.1046$	1 Mark: C
23	$T=2+Ae^{-kt}$ satisfies the equation $\frac{dT}{dt}=-k(T-2)$ Initially $t=0$ and $T=20$ $T=2+Ae^{-kt}$ $20=2+Ae^{-k\times 0}$ $A=18$ Also $t=20$ and $T=10$ $T=2+18e^{-kt}$ $10=2+18e^{-k\times 20}$ $e^{-k\times 20}=\frac{8}{18}$ $-20k=\log_e\frac{4}{9}$ $k=-\frac{1}{20}\log_e\frac{4}{9}$ $=\frac{1}{20}\log_e\frac{9}{4}$	1 Mark: C

24	$T=A+Be^{-kt}$ satisfies the equation $\frac{dT}{dt}=-k(T-A)$ $A=20$ (surrounding air temperature). Also $t=6$ and $T=80$ $T=20+Be^{-kt}$ $80=20+Be^{-k\times 6}$ $Be^{-6k}=60$ ....(1) Also $t=8$ and $T=50$ $T=20+Be^{-kt}$ $50=20+Be^{-k\times 8}$ $Be^{-8k}=30$...........(2) Eqn (1) ÷ (2) $\frac{Be^{-6k}}{Be^{-8k}}=\frac{60}{30}$ $e^{2k}=2$ $2k=\log_e 2$ $k=\frac{\log_e 2}{2}$	1 Mark: D
25	$T=T_0+Ae^{-kt}$ satisfies the equation $\frac{dT}{dt}=-k(T-T_0)$ with $T_0=25$ Initially $t=0$ and $T=100$ $100=25+Ae^{-k\times 0}$ $A=75$ Also $t=3$ and $T=70$ $70=25+75e^{-k\times 3}$ $e^{-k\times 3}=\frac{45}{75}$ $-3k=\log_e 0.6$ $k=-\frac{1}{3}\log_e 0.6$	1 Mark: C

26	$T=A+Be^{-kt}$ satisfies the equation $\dfrac{dT}{dt}=-k(T-A)$ $A=25$ (surrounding air temperature). Also $t=0$ and $T=1350$ $T=20+Be^{-kt}$ $1350=25+Be^{-k\times 0}$ $B=1325$ Also $t=10$ and $T=720$ $720=25+1325e^{-k\times 10}$ $e^{-10k}=\dfrac{695}{1325}$ $-10k=\log_e\dfrac{139}{265}$ $k=-\dfrac{1}{10}\log_e\dfrac{139}{265}=0.064525589...$ $50=25+1325e^{-0.064525589...\times t}$ $e^{-0.064525589...\times t}=\dfrac{25}{1325}$ $-0.064525589...\times t=\log_e\dfrac{25}{1325}$ $t=\log_e\dfrac{25}{1325}\div -0.064525589...=61.53$ minutes	1 Mark: B
27	$T=20+Ae^{-kt}$ satisfies the equation $\dfrac{dT}{dt}=-k(T-20)$ Initially $t=0$ and $T=100$ $T=20+Ae^{-kt}$ $100=20+Ae^{-k\times 0}$ $A=80$ Also $t=10$ and $T=70$ $T=20+80e^{-kt}$ $70=20+80e^{-k\times 10}$ $e^{-k\times 10}=\dfrac{50}{80}$ $-10k=\log_e\dfrac{5}{8}$ $k=-\dfrac{1}{10}\log_e\dfrac{5}{8}=\dfrac{1}{10}\log_e\dfrac{8}{5}\approx 0.047$ When $t=20$ $T=20+80e^{-0.047\times 10}$ $=51.25\approx 51$	1 Mark: B