Simple harmonic motion

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46 A particle moving in a straight line obeys $v^2 = -x^2 + 2x + 8$ where x is its displacement from the origin in metres and v is its velocity in ms^{-1}. The motion is simple harmonic. What is the amplitude?

(A) 2π metres

(B) 3 metres

(C) 8 metres

(D) 9 metres

47 The speed v (cm/s) of a particle moving in a straight line is given by $v^2 = 6 + 4x - 2x^2$, where the magnitude of its displacement from a fixed point O is x cm. The motion is simple harmonic. What is the centre of the motion?

(A) $x = -2$

(B) $x = -1$

(C) $x = 1$

(D) $x = 2$

48 A particle is moving with simple harmonic motion in a straight line so that its displacement x cm from a fixed point O in the line at time t is defined by $x = 4\sin 2t$. Which of the following is the correct equation for v as a function of x?

(A) $v = \pm\sqrt{4(1-4x^2)}$

(B) $v = \pm\sqrt{4(16-x^2)}$

(C) $v = \pm\sqrt{4(1-4x^2)}$

(D) $v = \pm\sqrt{16(4-x^2)}$

49 A particle is moving in a straight line with $v^2 = 36 - 4x^2$ and undergoing simple harmonic notion. If the particle is initially at the origin, which of the following is the correct equation for its displacement in terms of t?

(A) $x = 2\sin(3t)$

(B) $x = 3\sin(2t)$

(C) $x = 2\sin(9t)$

(D) $x = 3\sin(4t)$

50 The rise and fall of tides is approximated to simple harmonic motion. In a tidal river the depth of water of low tide is 2 m at 11.00 a.m. The following high tide is at 5.20 p.m. and the depth of water is 6 m. What times could a boat safely cross the river, if a minimum depth of water is 3.5 m?

(A) Between 12.40 p.m. and 8.00 p.m.

(B) Between 1.40 p.m. and 8.00 p.m.

(C) Between 12.40 p.m. and 9.00 p.m.

(D) Between 1.40 p.m. and 9.00 p.m.

51 A particle moving in simple harmonic motion starts from rest at a distance 6 metres from the centre of oscillation. The period is 4π seconds. What is the time taken to move to a point 3 metres from the origin?

(A) $t = \frac{\pi}{3}$

(B) $t = \frac{\pi}{6}$

(C) $t = \frac{2\pi}{3}$

(D) $t = \frac{5\pi}{6}$

52 A point moving with simple harmonic motion starts from a point 5 cm from the centre of the motion with a speed of 1 cm/s. The period is 8 seconds. What is the maximum acceleration?

(A) 4.9 ms^{-2} (B) 5.2 ms^{-2}

(C) 24.4 ms^{-2} (D) 25.6 ms^{-2}

53 A particle is moving in a straight line so that its acceleration at any time is given by $\ddot{x} = -4x$. What is the period and amplitude given that $t = 0$, $x = 3$ and $v = -6\sqrt{3}$?

(A) $T = \frac{\pi}{2}$ and $a = 3$

(B) $T = \frac{\pi}{2}$ and $a = 6$

(C) $T = \pi$ and $a = 3$

(D) $T = \pi$ and $a = 6$

54 A particle moves in a straight line and its position at any time t is given by $x = 3\cos 2t + 4\sin 2t$. The motion is simple harmonic. What is the greatest speed?

(A) 6 (B) 10

(C) 12 (D) 20

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	Solution	**Criteria**
46	$v^2 = -x^2 + 2x + 8$ $= 1^2(8 + 2x - x^2)$ $= 1^2(9 - (x-1)^2)$ $= n^2(a^2 - x^2)$ $a^2 = 9$ $a = 3$ Amplitude is 3 metres	1 Mark: B
47	$v^2 = 6 + 4x - 2x^2$ $\frac{1}{2}v^2 = \frac{6 + 4x - 2x^2}{2}$ $a = \frac{d}{dx}(3 + 2x - x^2)$ $= 2 - 2x$ $\ddot{x} = -2(x-1)$ Centre of motion is $x = 1$	1 Mark: C
48	$x = 4\sin 2t$ $v = \frac{dx}{dt} = 8\cos 2t$ $v^2 = 64\cos^2 2t$ $= 64(1 - \sin^2 2t)$ $= 64\left(1 - \frac{x^2}{16}\right)$ $= 4(16 - x^2)$ $v = \pm\sqrt{4(16 - x^2)}$	1 Mark: B
49	$v^2 = 36 - 4x^2$ $= 2^2(9 - x^2)$ $= n^2(a^2 - x^2)$ $a^2 = 9$ or $a = 3$, $n = 2$ and $\alpha = 0$ (initially at the origin) $x = a\sin(nt + \alpha)$ $= 3\sin(2t)$	1 Mark: B

50	Time between low and high tide is 6 hours and 20 minutes ($6\frac{1}{3}$ h) Period $T = 2 \times 6\frac{1}{3} = 12\frac{2}{3}$ h and $n = \frac{2\pi}{T} = \frac{2\pi}{12\frac{2}{3}} = \frac{3\pi}{19}$ Depth of water. Low tide = 2m and High tide = 6m Mean tide = 4m and amplitude = 2m Therefore $x = -2\cos\frac{3\pi t}{19}$ Boat can safely travel when $x \geq -0.5$ $-0.5 = -2\cos\frac{3\pi t}{19}$ $\cos\frac{3\pi t}{19} = 0.25$ $\frac{3\pi t}{19} = 1.318$ $\frac{3\pi t}{19} = 2\pi - 1.318$ $t = 2.66 = 2$ h 40 m $t = 10.00$ h Between 1.40 p.m. and 9.00 p.m.	1 Mark: D
51	$T = \frac{2\pi}{n}$ $4\pi = \frac{2\pi}{n}$ or $n = \frac{1}{2}$ Amplitude is 6 metres $x = a\cos(nt + \alpha)$ $= 6\cos(\frac{1}{2}t + \alpha)$ $= 6\cos(\frac{1}{2}t)$ when $t = 0$, $x = 6$ implies $\alpha = 0$ When $x = 3$ $3 = 6\cos\frac{1}{2}t$ $\cos\frac{1}{2}t = \frac{1}{2}$ $\frac{1}{2}t = \frac{\pi}{3}$ or $t = \frac{2\pi}{3}$	1 Mark: C

52	$T=\frac{2\pi}{n}$ $8=\frac{2\pi}{n}$ or $n=\frac{\pi}{4}$ when $v=1$, $x=5$, $n=\frac{\pi}{4}$ $v^2=n^2(a^2-x^2)$ $1^2=\frac{\pi^2}{16}(a^2-5^2)$ $a^2=\frac{16}{\pi^2}+25$ $a=\pm 5.1595677...\approx 5.2\text{ ms}^{-2}$ (maximum acceleration)	1 Mark: B
53	$\ddot{x}=-n^2x$ Period $T=\frac{2\pi}{n}=\frac{2\pi}{2}=\pi$ $=-4x$ or $n=2$ $x=a\sin(nt+\alpha)$ $=a\sin(2t+\alpha)$ When $t=0$, $x=3$ $3=a\sin(2\times 0+\alpha)$ $3=a\sin\alpha$ (1) $v=2a\cos(2t+\alpha)$ $-6\sqrt{3}=2a\cos(2\times 0+\alpha)$ $-3\sqrt{3}=a\cos\alpha$ (2) Dividing eqn (1) by eqn (2) $\tan\alpha=-\frac{1}{\sqrt{3}}$ $\alpha=\frac{5\pi}{6}$ ($\sin\alpha>0$ and $\cos\alpha<0$) From eqn (1) $3=a\sin\frac{5\pi}{6}$ $a=6$ Period is π and amplitude is 6.	1 Mark: D

54	$x=3\cos 2t+4\sin 2t$ $=5\cos(2t-\alpha)$ Here $0<\alpha<\frac{\pi}{2}$ and $\tan\alpha=\frac{4}{3}$ $\dot{x}=-10\sin(2t-\alpha)$ $\ddot{x}=-20\cos(2t-\alpha)$ Greatest speed occurs when $\ddot{x}=0$ $-20\cos(2t-\alpha)=0$ $2t-\alpha=\frac{\pi}{2},\frac{3\pi}{2},...$ Therefore $\dot{x}=-10\sin(\frac{\pi}{2})$ $=-10$ Magnitude of maximum speed is 10	1 Mark: B