Stef

YEAR 10 2003

ADVANCED MATHEMATICS

TIME ALLOWED: I hour and 40 minutes

INSTRUCTIONS

Answer all questions.
Show all working.
Start each section on a separate sheet of paper

TOPICS	MARKS
• TRIGONOMETRY	24.5
• SIMILARITY	22.5
• PROBABILITY	36
TOTAL	83

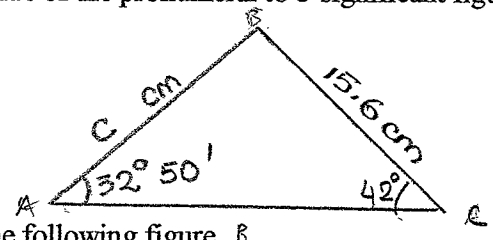
Area $= \frac{1}{2}ab\sin C$

$a^2 = b^2 + c^2 - 2bc\cos A$

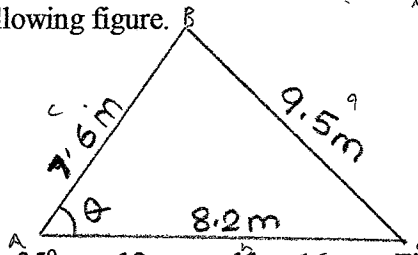
$\frac{\sin A}{a} = \frac{\sin B}{b}$

PART A (24.5 marks) TRIGONOMETRY

1. If $\sin\theta = \sin 140°$, find the value of the acute angle θ. (1)
2. Find the obtuse angle θ (correct to the nearest minute) if $\sin\theta = 0.065$. (2)
3. Find the value of the pronumeral to 3 significant figures. (2.5)

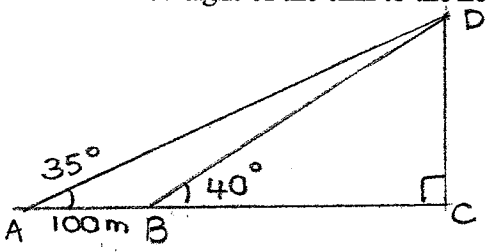


4. Find θ in the following figure.

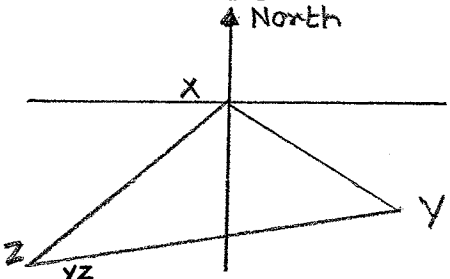


(2)

5. In Δ ABC, $\angle A = 35°$, a = 12 cm and b = 16 cm. Find $\angle B$ to the nearest degree if $\angle B$ is obtuse. (3.5)
6. Two sides of a triangular field are 60m and 50m and the included angle is 140°. Calculate to one decimal place. (6)
   a. the length of the third side and 5.5
   b. the area of the field.
7. In the diagram, CD is a cliff and A, B the positions of two boats at sea. The angles of elevation of D from A, B are 35° and 40° respectively and A, B, C and D are in the same vertical plane.
   a. Find the simplest expression for BD. (1.5)
   b. Hence determine the height of the cliff to the nearest metre. (2.5)



8. a. Copy the diagram onto your paper.

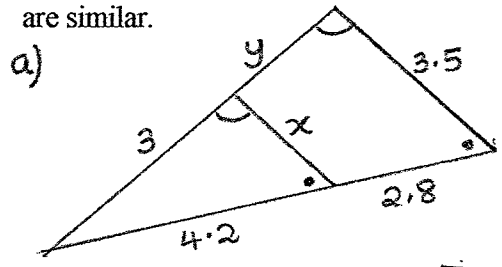


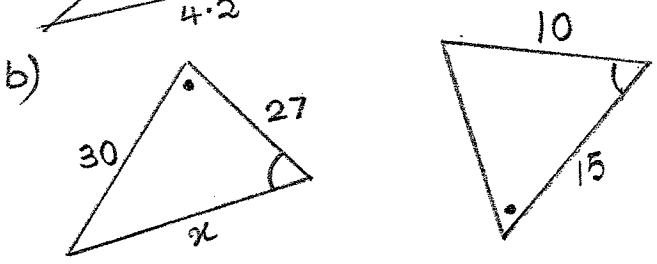
b. If XY = 5 km, YZ = 11 km and Y is 3 km South from X, find the bearing of X from Z, giving your answer to the nearest minute. Show all working. (5)

XZ = 8 km.

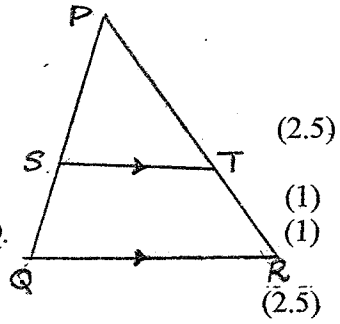
PART B (22.5 marks) **SIMILARITY**

1. Find the value of the pronumeral in each case below, given that the triangles are similar. (6.5)

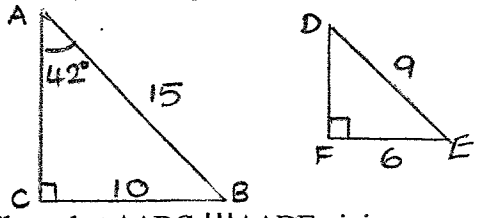




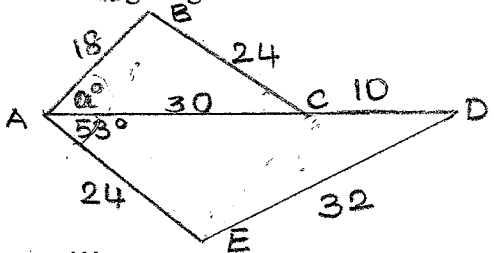
2. In ΔPQR, if ST || QR,
   a. Prove that ΔPST ||| ΔPQR. (2.5)
   b. Complete the ratios $\frac{PS}{PQ} = \frac{?}{QR} = \frac{?}{PR}$ (1)
   c. If QR = 5cm, ST = 2cm, SP = 1cm, find PQ. (1)



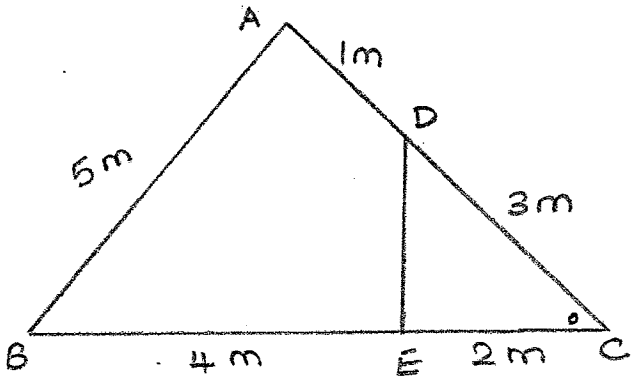
3. Show ΔABC |||ΔDEF giving reasons. (2.5)



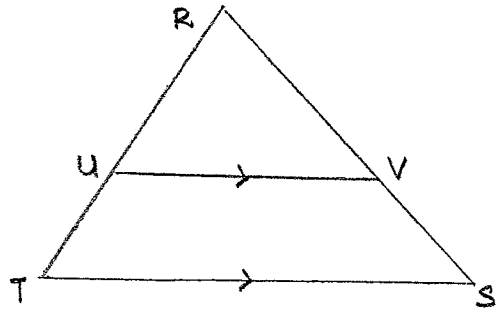
4. a. Show that ΔABC |||ΔADE giving reasons. (2)
   b. Hence find ∠a giving reasons. (1)



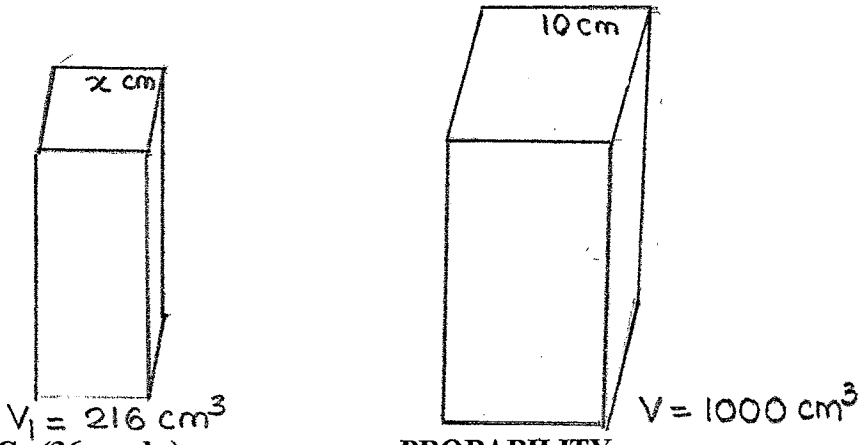
5. Show that ΔABC |||ΔDCE giving reasons. (2)



6. In the figure given below, find RV: VS given that the two triangles are similar and that area Δ RUV: ΔRST= 4:25. (1.5)



7. For the figures given below,
   a. Find the value of x. (1.5)
   b. Find the ratio of the areas of the corresponding faces. (1)



PART C (36 marks) **PROBABILITY**

1. Two coins are tossed 40 times. The results are given below. (3)

RESULT	OUTCOME
HH	9
HT	12
TH	8
TT	11

Calculate the relative probability for obtaining
   (a) Exactly one head
   (b) Two heads
   (c) Two tails.

2. If the probability of rain occurring on a particular day is 0.4, calculate the probability that it will not rain. (1)

3. A group of students recorded the colours of 100 cars passing the school in a one-hour period. The results are shown below: (2)

COLOUR	NUMBER OF CARS
RED	16
BLUE	27
WHITE	46
YELLOW	3
OTHER	8
TOTAL	100

a. Calculate the experimental probability of a white car passing the school.

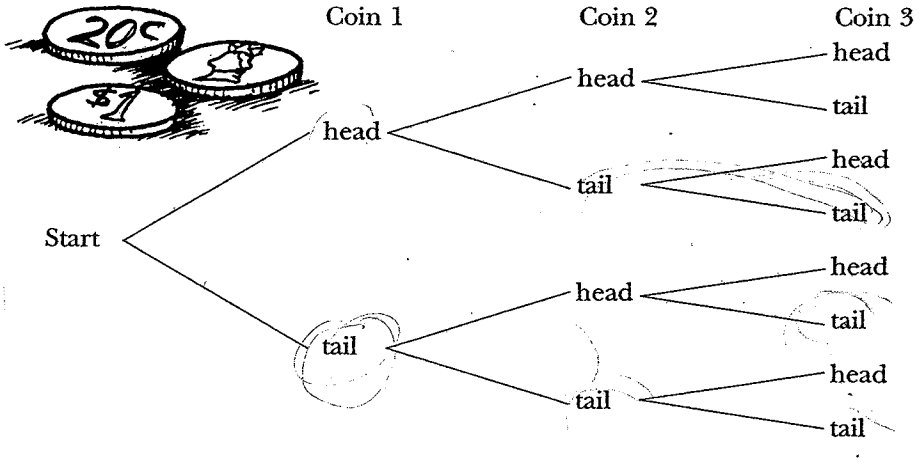
b. Calculate the probability that a white or a yellow car passing the school.

4. A bag contains 4 red, 2 white and 3 blue marbles. A marble is drawn from the bag, its colour noted. Find the probability that the marble is (4)
   a. Red
   b. White or blue
   c. Pink
   d. Not blue

5. A pack of cards contains 52 cards. A card is drawn at random. What is the probability that it is (10)
   a. the 10 of hearts?
   b. black?
   c. a jack?
   d. a 7, 8 or 9?
   e. e. an ace or a king
   f. a red queen?
   g. an black 10 or a red 8?
   h. a club?
   i. not a 5?
   j. a picture card?

6. Use the tree diagram below to find the probability of tossing: (7)
   a. exactly one head
   b. exactly two tails
   c. more tails than heads
   d. three heads
   e. at least one head
   f. either two heads or two tails
   g. either one tail or more than one head

Three coins are tossed. The tree diagram below shows all possible outcomes.



7. Into a barrel are placed 100 blue tickets numbered 1 to 100, 50 red tickets numbered 1 to 50 and 50 green tickets numbered 1 to 50. If one ticket is drawn at random from the barrel, what is the probability that the ticket
   a. is green
   b. is a 36
   c. is less than 51
   d. is not a 50
   e. is green or red.
   f. is a 72
   g. is less than 60
   h. is not less than a 60. (9)
   i. either a 36 or a 72.

$x^2 = 50^2 + 60^2 - 2 \times 50 \times 60 \cos 140^\circ$

$= 10696.26...$

$x = 103.422...$

$= 103.4 \text{ m}$

(b)

$\text{Area} = \frac{1}{2} \times 60 \times 50 \times \sin 140^\circ$

$= 964.18...$

$= 964.2 \text{ m}^2$

)

(a)

$\angle ADB = 40 - 35$

$= 5^\circ$

$\frac{BD}{\sin 35^\circ} = \frac{100}{\sin 5^\circ}$

$BD = \frac{100 \sin 35^\circ}{\sin 5^\circ}$

(b)

$\sin 40 = \frac{CD}{BD}$

$CD = BD \sin 40$

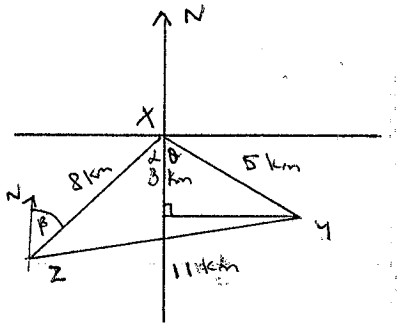
$= \frac{100 \sin 35^\circ}{\sin 5^\circ} \sin 40^\circ$

$= 658.105..... \times \sin 40^\circ$

$= 423.02...$

$= 423 \text{ m}$

8(a)



YEAR 10

PART A

① $\theta = 40^\circ$

② acute $\theta = 3^\circ 44'$

obtuse $\theta = 180 - 3^\circ 44'$

$= 176^\circ 16'$

③

$\frac{c}{\sin 42^\circ} = \frac{15.6}{\sin 32^\circ 50'}$

$c = \frac{15.6 \sin 42^\circ}{\sin 32^\circ 50'}$

$= 19.252...$

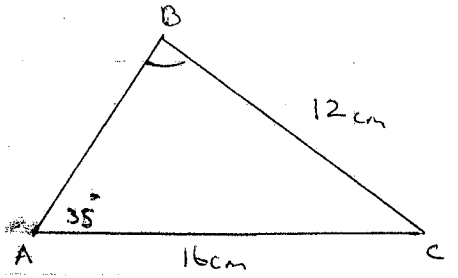
$= 19.3 \text{ cm}$

④ $\cos \theta = \frac{7.6^2 + 8.2^2 - 9.5^2}{2 \times 7.6 \times 8.2}$

$= 0.278...$

$\theta = 73^\circ 49'$

5



$\frac{\sin B}{16} = \frac{\sin 35^\circ}{12}$

$\sin B = \frac{16 \sin 35^\circ}{12}$

$= 0.76...$

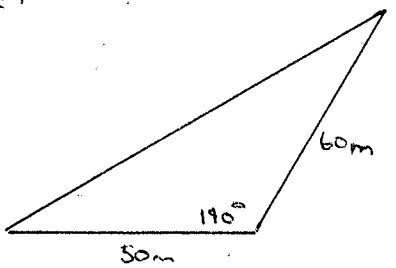
$\angle B = 49^\circ 53'$

obtuse $\angle B = 180 - 49^\circ 53'$

$= 130^\circ 7'$

$= 130^\circ$

6 (a)



PART B

(a) $\frac{x}{3.5} = \frac{4.2}{4.2+2.8}$

$\frac{x}{3.5} = \frac{4.2}{7}$

$x = \frac{4.2}{7} \times 3.5$

$= 2.1$

$\frac{y+3}{3} = \frac{7}{4.2}$

$y+3 = 3 \times \frac{7}{4.2}$

$y = 3 \times \frac{7}{4.2} - 3$

$= 2$

(b) $\frac{x}{10} = \frac{27}{15}$

$x = \frac{27}{15} \times 10$

$= 18$

(a) In ΔPST, ΔPQR

$\angle P$ is common

$\angle PST = \angle PQR$ (corresp $\angle$s, $ST // QR$)

[OR $\angle PTS = \angle PRQ$ (corresp $\angle$s, $ST // QR$)]

$\therefore \Delta PST \sim \Delta PQR$ (equiangular)

(b) $\frac{PS}{PQ} = \frac{ST}{QR} = \frac{PT}{PR}$

(c) $\frac{PQ}{1} = \frac{5}{2}$

(corresp sides in sim Δs in same ratio)

$PQ = 2\frac{1}{2}$ cm

$\cos \theta = \frac{3}{5}$

$\theta = 53^\circ 8'$

$\cos \angle ZXY = \frac{8^2 + 5^2 - 11^2}{2 \times 8 \times 5}$

$= -0.4$

$\angle ZXY = 113^\circ 35'$

$\alpha = 113^\circ 35' - 53^\circ 8'$

$= 60^\circ 27'$

$\beta = 60^\circ 27'$

bearing of X from Z = N $60^\circ 27'$ E

⑤ In ΔABC, ΔDCE

$\angle C$ is common

$$\left.\begin{aligned}\frac{AC}{EC} &= \frac{4}{2}\\ &= 2\end{aligned}\right\}$$

$$\left.\begin{aligned}\frac{BC}{DC} &= \frac{6}{3}\\ &= 2\end{aligned}\right\}$$

$\therefore \Delta ABC \;///\; \Delta DCE$

(2 pairs of corresp sides in same ratio, included $\angle$ =)

⑥

$$\frac{RV^2}{RS^2} = \frac{4}{25}$$

$$\frac{RV}{RS} = \frac{2}{5}$$

$RV - RS = VS = 3$

$RV : VS = 2 : 3$

⑦ (a)

$$\frac{x^3}{10^3} = \frac{216}{1000}$$

$$x^3 = 216$$

$$x = 6$$

(b) $6^2 : 10^2$

$= 36 : 100$

③ In ΔABC, ΔDEF

$\angle C = \angle F$ (given 90°)

$$\left.\begin{aligned}\frac{AB}{BC} &= \frac{15}{10}\\ &= \frac{3}{2}\end{aligned}\right\}$$

$$\left.\begin{aligned}\frac{DE}{EF} &= \frac{9}{6}\\ &= \frac{3}{2}\end{aligned}\right\}$$

$\therefore \Delta ABC \;///\; \Delta DEF$

(both Δs are right angled and ratio of hyp to one side in one Δ = ratio of hyp. to one side in other Δ)

④ (a) In ΔABC, ΔADE

$$\left.\begin{aligned}\frac{AB}{AE} &= \frac{18}{24}\\ &= \frac{3}{4}\end{aligned}\right\}$$

$$\left.\begin{aligned}\frac{BC}{DE} &= \frac{24}{32}\\ &= \frac{3}{4}\end{aligned}\right\}$$

$$\left.\begin{aligned}\frac{AC}{AD} &= \frac{30}{40}\\ &= \frac{3}{4}\end{aligned}\right\}$$

$\therefore \Delta ABC \;///\; \Delta ADE$

(corresponding sides are in same ratio)

(b) $a = 53^\circ$

(corresp $\angle$s in sim Δs =)

5) (g) $\frac{4}{8}$

$= \frac{1}{2}$

7) 100 B 1-100

50 R 1-50

50 G 1-50

(a) $\frac{50}{200}$

$= \frac{1}{4}$

(b) $\frac{3}{200}$

(c) $\frac{150}{200}$

$= \frac{3}{4}$

(d) $\frac{197}{200}$

(e) $\frac{100}{200}$

$= \frac{1}{2}$

(f) $\frac{1}{200}$

(g) $\frac{159}{200}$

(h) $\frac{41}{200}$

(i) $\frac{4}{200}$

$= \frac{1}{50}$

PART C (1 each)

1 (a) $\frac{12}{40}$

$= \frac{1}{2}$

(b) $\frac{9}{40}$

(c) $\frac{11}{40}$

2 $1 - 0.4$

$= 0.6$

3 (a) $\frac{46}{100}$

$= \frac{23}{50}$

(b) $\frac{46+3}{100}$

$= \frac{49}{100}$

4 (a) $\frac{4}{9}$

(b) $\frac{5}{9}$

(c) 0

(d) $\frac{6}{9}$

$= \frac{2}{3}$

5 (a) $\frac{1}{52}$

(b) $\frac{1}{2}$

(c) $\frac{4}{52}$

$= \frac{1}{13}$

(d) $\frac{12}{52}$

$= \frac{3}{13}$

(e) $\frac{8}{52}$

$= \frac{2}{13}$

(f) $\frac{2}{52}$

$= \frac{1}{26}$

(g) $\frac{4}{52}$

$= \frac{1}{13}$

(h) $\frac{1}{4}$

(i) $\frac{48}{52}$

$= \frac{12}{13}$

(j) $\frac{12}{52}$

$= \frac{3}{13}$

6 (a) $\frac{3}{8}$

(b) $\frac{3}{8}$

(c) $\frac{4}{8} = \frac{1}{2}$

(d) $\frac{1}{8}$

(e) $\frac{7}{8}$

(f) $\frac{6}{8}$

$= \frac{3}{4}$