YEARLY EXAMINATION 2003

YEAR 10 ADVANCED

MATHEMATICS

TIME ALLOWED: 1-1/2 hours

TOTAL MARKS 85

INSTRUCTIONS

You must answer all parts of the examination on separate sheets.
Answers must be written in biro except for diagrams which may be in pencil.
Show all working.
Start a new sheet of paper for each section.

Part A: Choose the best answer: All questions carry one mark each. (10 marks)

1. A television set is advertised as follows:

Cash Price: $760

Or

Terms: 15% deposit and $37·50 per month for two years.

How much extra is paid for the television if it is purchased on terms?

A) $26 B) $140 C) $254 D) $786

2. $1200 is invested for two years at 10% per annum compounded annually. The amount of interest earned in the second year is

A) $120 B) $126 C) $132 D) $252

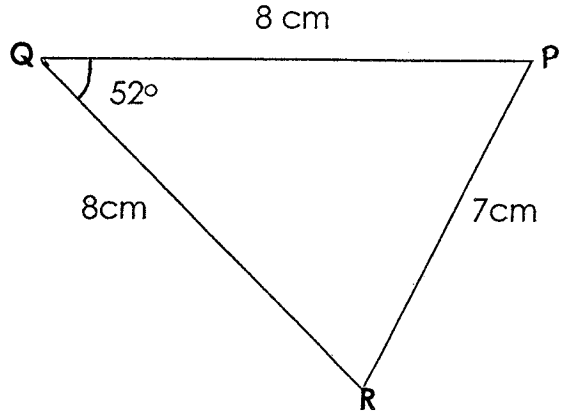
3. In ΔABC, if AB = 8 cm, BC = 12cm, AC = 15cm, then $\angle B$, to the nearest degree is

A) 5^0 B) 53^0 C) 81^0 D) 95^0

4. So far this year, Jane has been late for school on 25 days and has not been late on 75 days. Using this information, calculate the probability that Jane will be late on a particular day?

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$

5.

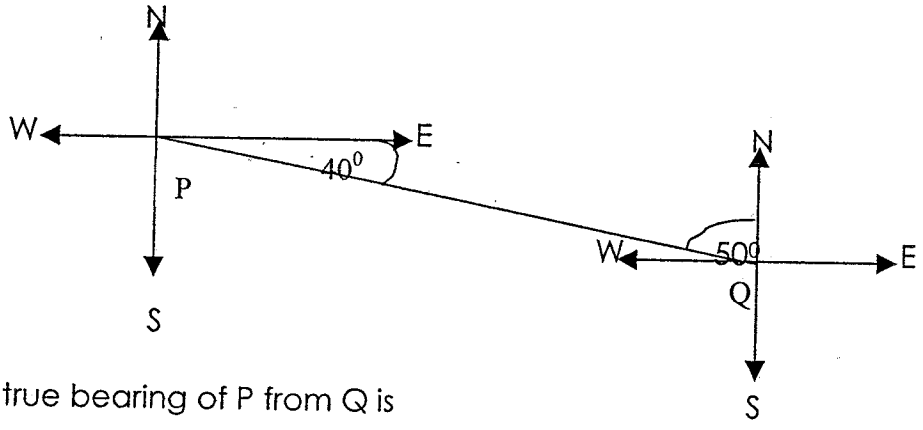


Which expression gives the area of Δ PQR in cm^2 ?

A) $\frac{1}{2} \times 8 \times 7 \times \sin 52^\circ$ B) $\frac{1}{2} \times 8 \times 8 \times \sin 52^\circ$ C) $\frac{1}{2} \times 8 \times 7 \times \cos 52^\circ$

D) $\frac{1}{2} \times 8 \times 8 \times \cos 52^\circ$

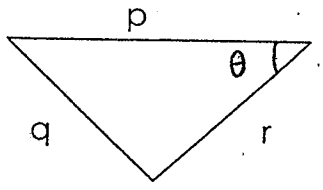
6.



The true bearing of P from Q is

A) 040⁰ B) 050⁰ C) 130⁰ D) 310⁰

7.



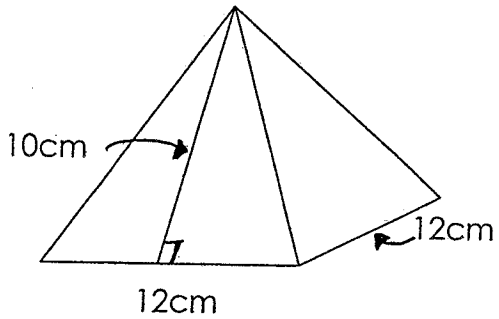
If in this diagram, $p^2 = q^2 + r^2$, then which of the following is correct?

A) $q = p\sin\theta$ B) $p = r\sin\theta$ C) $q = p\tan\theta$ D) None of these

8. The size of an interior angle of a polygon of twenty sides is equal to

A) 162° B) 140° C) 93° D) 170°

9. The surface area of the following pyramid is



A) 243cm² B) 384 cm² C) 214 cm² D)456 cm²

10. The only possible values of x in the following equation
$2x^2 - 3x - 2 = 0$ are

A)2,-1/2 B)3,4 C)2, 1 D)2,1/2

Part B: Start on a new sheet of paper **23.5 marks**

1. Solve $2x^2 + 13x - 24 = 0$ (2)

2. Solve for θ, $0^0 \leq \theta \leq 180^0$

$\sin \theta = 0.6$ (2)

3. A 3-digit number is formed from the digits 2, 3, 4 and 5. Each digit can only be used once.

a) Draw a tree diagram. (2)

b) List all possible combinations (1)

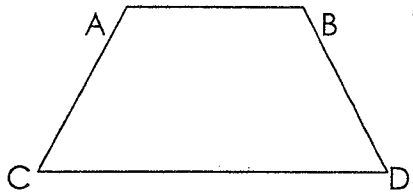
c) What is the probability of the number formed being (3)

I. Even

II. Divisible by 5

III. Less than 450

4.

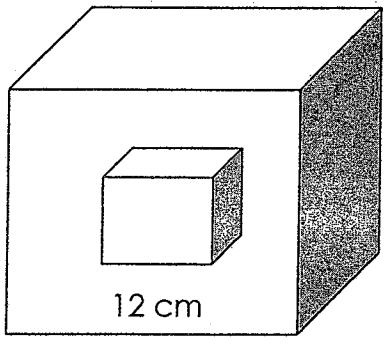


This diagram is not to scale.

ABCD is a quadrilateral. $\angle ADC = \angle BCD$, $AD = BC$.

Prove that $\Delta DAC \equiv \Delta CBD$. Give reasons and show all working. (2)

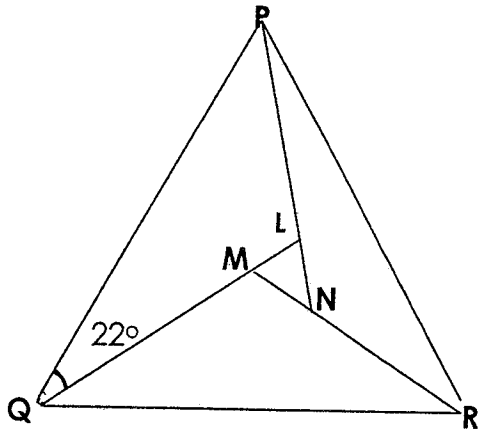
5. This solid has an inner prism removed. The prism is similar to the original solid. What fraction of the original solid has been removed? (2)



6. What is the simple interest on \$2400 at 8% p.a for 5 months? (2)

7. Solve $3x^2 = 2x + 2$, leaving your answer in the simplest surd form. (2)

8. Δ PQR is an equilateral triangle. Lines QL, RM, PN form a similar equilateral Δ LMN inside.



12 cm

a) Copy or trace diagram on your own paper.
b) Find the exact length of MQ. Show all working. (3.5)
c) By writing down all working, calculate the length of LM correct to decimal places. (2)

Part C Start on a new sheet of paper **24 marks**

1. Jenny plans to contribute to a superannuation fund. She will invest $600 on each birthday from age 25 to 64 inclusive. The fund pays interest on the investments at the rate of 8% p. a., compounded annually. How much money will be in Jenny's fund on her 65th birthday? Answer to the nearest dollar. (2)

2. The perimeter of a rectangle is 40cm and its area is 84cm^2.

I. If the width of the rectangle is x cm, express the length in terms of x. (1.5)

II. Write down the area of the rectangle in terms of x. (0.5)

III. Using a quadratic equation in x, solve it to find the length and width. (2.5)

3. Cliff and Dick are golfers. Each golfer played ten rounds of golf on the same course and their scores have been recorded below.

Cliff's Scores: 73, 81, 77, 85, 76, 76, 84, 73, 80, 75

Dick's Scores: 70, 84, 82, 78, 83, 73, 73, 74, 85, 78

For each set of scores calculate

a) the mean (1)

b)the standard deviation to two decimal places. (1)

c) Considering your results, who is the most consistent golfer? Give a reason. (1)

4. Solve by completing the square $x^2 - 3x - 5 = 0$. Leave your answer in surd form. (2)

5.Television shows receive a rating based on the percentage of people who are watching each particular program. The rating for a station's nightly news over a month is shown in the stem-and-leaf plot below.

0	9.
1	013445567889
2	0012222444566788
3	7

I. Find the median of the data. (1)

II. Find the range of the data. (1)

III. Find the interquartile range of the data. (1.5)

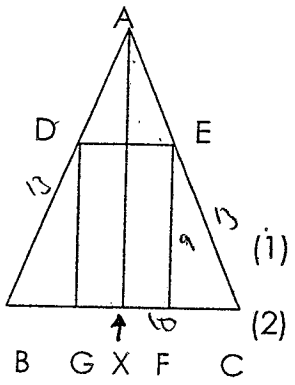
IV. Draw a box–and-whisker plot of the data, using a scale of 1 cm = 2 units. (2.5)

6. Δ ABC is isosceles with side lengths:

AB = 13cm AC = 13cm

BC = 10cm

DEFG is a rectangle with EF = 9cm.



a) Calculate the length of AX, given AX is a perpendicular bisector of BC. (1)

b) Prove that Δ AXC III Δ EFC. (2)

Give reasons.

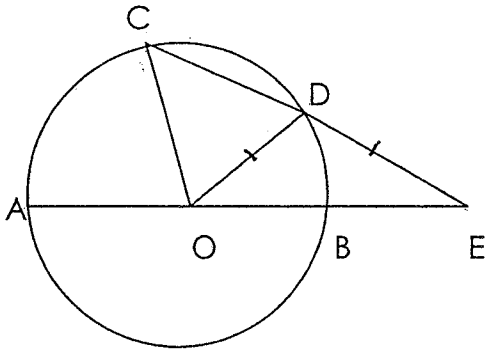
c) Calculate:

i) The length of FC, give a reason. (1.5)

ii) The width of the rectangle DEFG. (1.5)

Part D Start on a new sheet of paper. **27.5 marks**

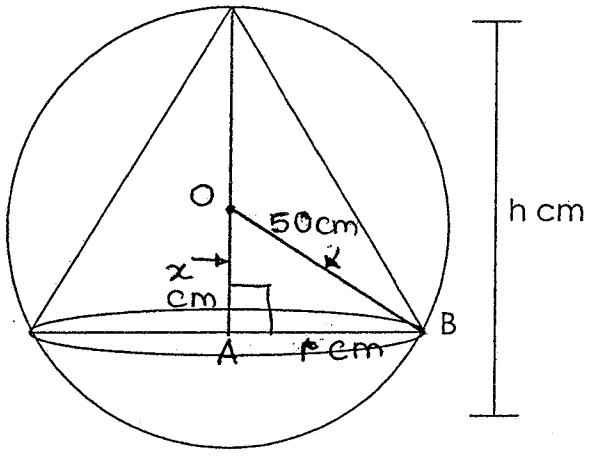
1. AB is a diameter and CD is a chord of a circle which has centre O. CD produced meets AB produced at E and DE = OD. Prove that $\angle AOC = 3\angle DOB$ (3)



2. A survey of a certain district showed that 6% of families have 1 child, 38% have 2 children, 42% have 3 children, and 10% have more than 3 children. A family is selected at random. Express as a percentage the probability that the family will have:

i) Some children (1)
ii) No children (1)
iii) At least 2 children (1)
iv) Not more than 2 children (1)

3.



The diagram shows a cone of a base radius r cm and height of h cm inscribed in a sphere of radius 50cm. The centre of the sphere is O and $\angle OAB = 90^0$.

Let OA be x cm. Show that

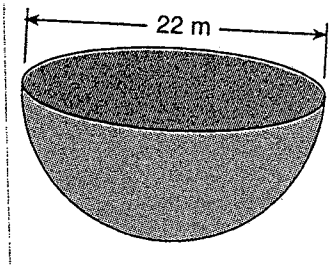
a) $r = \sqrt{2500 - x^2}$, all working is to be shown. (1.5)

b) Volume, $V cm^3$ of the cone

$V = \frac{\pi}{3}(2500 - x^2)(50 + x)$. All working is to be shown. (1)

c) If x =30, what is the volume of the cone in exact form? (1)

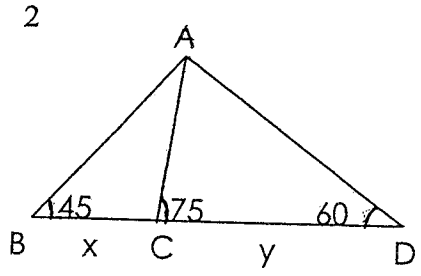
6. Calculate the surface area of the following solid correct to two decimal places. (3)



7. a) In the diagram below, find AC in terms of (i) x and (ii) y. (3)

b) Show that $\frac{x}{y} = \frac{\sqrt{3}}{2}$ and (3)

c) $\frac{area\Delta ABC}{area\Delta ADC} = \frac{\sqrt{3}}{2}$ (4)



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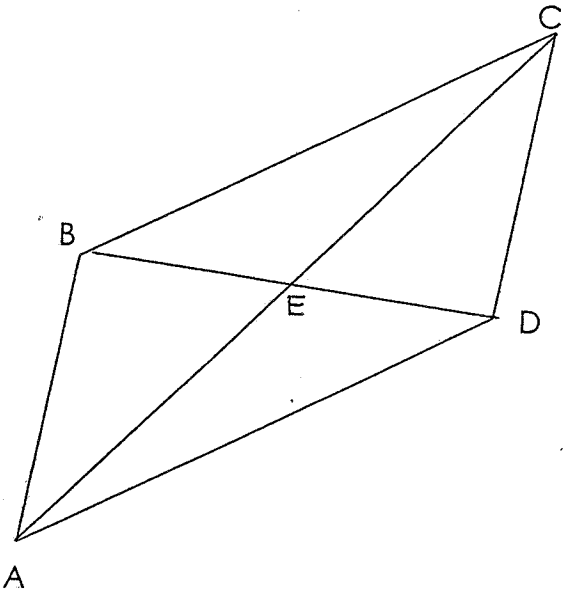


In the parallelogram ABCD, the diagonal BD is perpendicular to AB. If the diagonals intersect at E, prove that

$$AD^2 = AE^2 + 3ED^2$$ (4)

Show all working and give reasons where necessary.

Multiple Choice

1. C
2. C
3. D
4. A
5. B
6. D
7. A
8. A
9. B
10. A

Part B.

1. $2x^2 + 13x - 24 = 0$

$2x^2 + 16x - 3x - 24 = 0$

$2x(x+8) - 3(x+8) = 0$

$(2x-3)(x+8) = 0$

$2x-3 = 0$ or $x = \frac{3}{2}$

$x+8 = 0$ or $x = -8$

2. $\sin\theta = 0.6$

$\theta = \sin^{-1} 0.6$

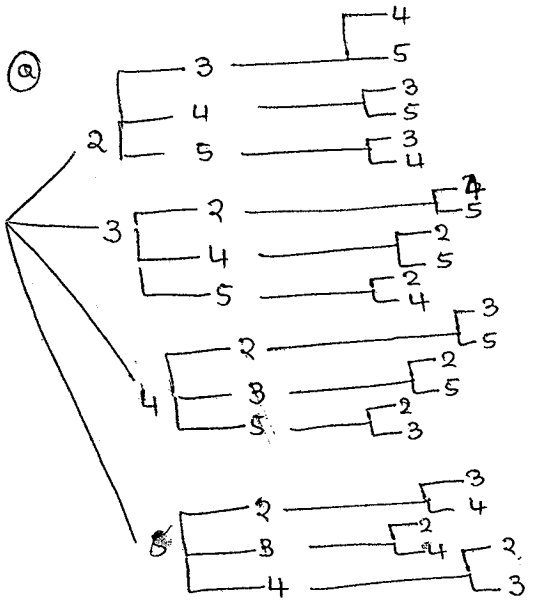
$=$

$\sin(180-\theta) = \sin\theta$

$\therefore \theta$ for $0° \leq \theta \leq 180°$ $= 36°52', 143°8'$

$-\frac{1}{2}$ if rogej degree
$\sqrt{}$ correct

3. (a)



(3) (b)

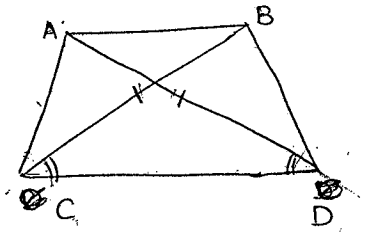
234	324	423	523
235	325	425	524
243	342	432	532
245	345	435	534
253	352	452	542
254	354	453	543

(9) (i) $P(\text{even}) = \frac{12}{24} = \frac{1}{2}$

(ii) $P(\text{Divisible by 5}) = \frac{6}{24} = \frac{1}{4}$

(iii) $P(\text{Less than 450}) = \frac{16}{24} = \frac{2}{3}$

4.



$\angle ADC = \angle BCD$ (given)
$AD = BC$ (given)
CD is common.
(SAS) (two sides & included angle are equal)
$\therefore \Delta DAC \equiv \Delta BDC$.

5. Volume of original cube $= 20^3 \text{ cm}^3$

Volume of solid removed $= 12^3 \text{ cm}^3$

$\therefore$ fraction of solid removed $= \frac{12^3}{20^3}$

$= \frac{27}{125}$

6. $I = \frac{PTR}{100}$

$P = \not{\text{₦}}\,2400$ $\quad T = \frac{5\frac{1}{2}}{12}$ years $\quad R = 8\%$

$I = \frac{2400 \times 5 \times 8}{12 \times 100}$

$= \not{\text{₦}}\,80$

7. $3x^2 = 2x + 2$

$3x^2 - 2x - 2 = 0$

using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{2 \pm \sqrt{4 + 4 \times 3 \times 2}}{2 \times 3}$

$= \frac{2 \pm \sqrt{28}}{6}$

$= \frac{2 \pm 2\sqrt{7}}{6}$ or $\frac{1 \pm \sqrt{7}}{3}$

$x = \frac{2 - 2\sqrt{7}}{6}$ or $x = \frac{2 + 2\sqrt{7}}{6}$

$x = \frac{1 - \sqrt{7}}{3}$ or $x = \frac{1 + \sqrt{7}}{3}$.

8.

8

(c) In Δ PQL,

$\angle PLQ = 120^\circ$, $\angle PQL = 22^\circ$ $\therefore \angle QPL = 38^\circ$

$$\therefore \frac{PQ}{\sin 120^\circ} = \frac{QL}{\sin 38^\circ}$$

$$\frac{12}{\sin 120^\circ} = \frac{QL}{\sin 38^\circ}$$

$$QL = \frac{12 \sin 38^\circ}{\sin 120^\circ}$$

$\therefore$ LM = QL − MQ

$$= \frac{12 \sin 38^\circ}{\sin 120^\circ} - \frac{12 \sin 22^\circ}{\sin 120^\circ}$$

$$= \frac{12}{\sin 120^\circ} (\sin 38^\circ - \sin 22^\circ)$$

$$= 3.340\,15\ldots$$

$$= 3.3 \text{ (1 dec pl)}$$

Part C

1. P = $600

r = 0.08 = 8%

n = 40

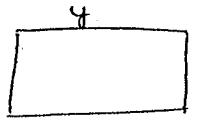
$A = P(1+r)^n$

$= 600 (1+0.08)^{40}$

$= 13034.71$

= 13035 $

2. $(x+y)2 = 40$



(i) $2x + 2y = 40$

$2y = 40 - 2x$

$y = \frac{40-2x}{2} = 20 - x$

(ii) Area $= xy$

$y = 20 - x$

$= x(20-x) = 20x - x^2$

(iii) $-x^2 + 20x - 84 = 0$

$x^2 - 20x + 84 = 0$

$x^2 - 14x - 6x + 84 = 0$

$x(x-14) - 6(x-14) = 0$

$(x-6) = 0$ or $x - 14 = 0$

$x = 6$ or $x = 14$

$y = 14$ or $y = 6$

3. (a) Cliff: mean 78 Dick: 78

(b) Cliff SD: 4.07 Dick: ~~4.56~~ 5.06.

(c) Cliff → less deviation from the mean

4. $x^2 - 3x = 5$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 5 + \left(\frac{3}{2}\right)^2$

$x^2 - 3x + \frac{9}{4} = 5 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$

$x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$

$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$

$x = \frac{3+\sqrt{29}}{2}$ $x = \frac{3-\sqrt{29}}{2}$

5. (i) median $\frac{20+21}{2} = \frac{41}{2} = 20.5$

(ii) Range 9 – 37

(iii) Interquartile range:

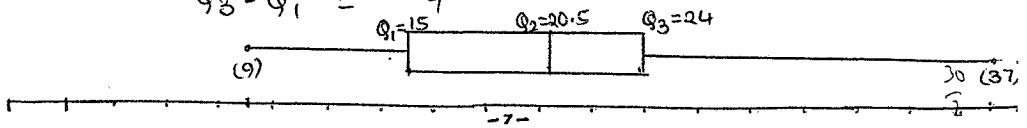
$Q_2 = 20.5$

$Q_1 = 15$

$Q_3 = 24$

$Q_3 - Q_1 = 9$

9, 10, 11, 13, 14, 14,
15, 15, 16, 17, 18, 18, 19
20, 20, 21, 22, 22, 22, 22,
24, 24, 24, 25, 26, 26, 27,
28, 28, 37.



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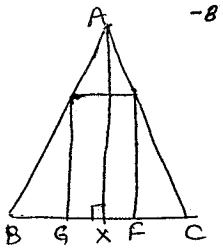


a) $AX^2 = AB^2 - BX^2$ (Pythagoras)

$BX = \frac{BC}{2}$ (given)

$\therefore AX^2 = AB^2 - \frac{BC^2}{4}$

$= 169 - \frac{100}{4}$

$= 169 - 25 = 144$

$\therefore AX = 12\text{cm}$

(b) $\triangle AXC$ and $\triangle EFC$

$\angle CAX = \angle$

$\angle AXC = \angle EFC = 90^\circ$

$\angle ACX = \angle ECF$ (common)

$\therefore \angle CAX = \angle CEF$

$\therefore \triangle AXC \equiv \triangle EFC$ (three angles of the triangles equal)

(c) (i) $\triangle AXC$ and $\triangle EFC$

$\frac{AX}{EF} = \frac{CX}{FC}$ ($\triangle AXC \equiv \triangle EFC$ ratio corresponding sides equal)

$\therefore FC = \frac{CX \cdot EF}{AX}$

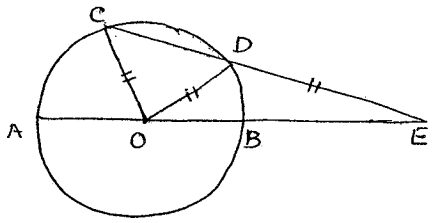
$= \frac{5 \times 9}{12} = \frac{45}{12} = \frac{15}{4}$

(ii) $GF = 2 \times XF$

$XF = XC - FC = 5 - 15 - 5/11$ $\therefore GF = \frac{5}{2} = 2.5\text{cm}$

Part D

1.



(a) Δ DOE,

$\angle CDO = \angle DOB + \angle DEB$

(external angle is equal to the sum of the two opposite internal angles)

$\angle DOB = \angle DEB$ (DO = DE (given) $\therefore$ Δ DOE is isoceles)

$\therefore \angle CDO = 2\angle DOB.$

Δ COE,

$\angle AOC = \angle OCD + \angle DEB$

(external angle is equal to the sum of two opposite internal angles)

and $\angle DEB = \angle DOB$ and $\angle DCO = \angle CDO$

($OC = OD \rightarrow$ radii of the circle)

$\therefore$

$\angle AOC = \angle CDO + \angle DOB$

$= 2\angle DOB + \angle DOB$

$= 3\angle DOB$

2.

a) P(some children) $= \frac{96}{100} = \frac{48}{50} = \frac{24}{25}$

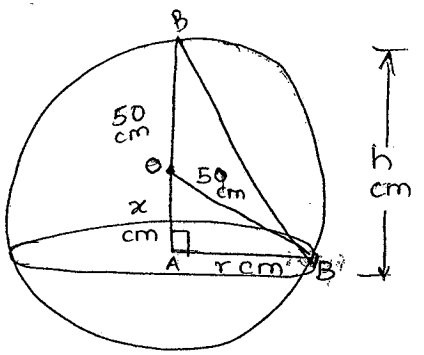
b) P(no children) $= 1 - P(\text{children})$

$= 1 - \frac{96}{100} = \frac{4}{100} = \frac{1}{25}$

c) P(at least 2 children) $= \frac{90}{100} = \frac{9}{10}$

d) P(not more than 2 children) $= \frac{44}{100} = \frac{22}{50} = \frac{11}{25}$

3.



(a) Δ OAB is right angled at A.

$\therefore$ using Pythagoras theorem

$OB^2 = OA^2 + AB^2$

$(50)^2 = x^2 + r^2$

or $r^2 = 50^2 - x^2$

$= 2500 - x^2$

$\therefore r = \sqrt{2500 - x^2}$

3. (b) Volume of a cone $= \frac{1}{3}\pi r^2 h$.

$r = \sqrt{2500 - x^2}$

$h = OA + OB$

$= x + 50$

$\therefore V = \frac{1}{3}\pi \left(\sqrt{2500-x^2}\right)^2 . (50+x)$

$= \frac{1}{3}\pi . (2500-x^2)(50+x)$

(c) $x = 30$ cm

$V = \frac{1}{3}\pi (2500-x^2)(50+x)$

$= \frac{1}{3}\pi (2500 - 900)(80)$

$= \frac{1}{3}\pi (\quad 1600)(80)$

$=$

b. Surface area of the solid hemisphere

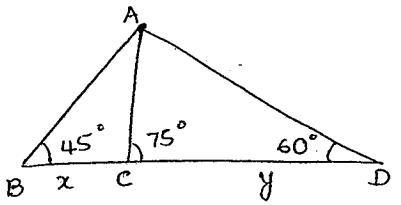
$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3 \times \frac{22}{7} \times 22^2$

$=$ cm^2

7.



(i) ΔABC, $\angle ACB = 180 - 75^\circ = 105^\circ$ (BD is a straight line)

$\angle BAC = 180 - (105+45)$ (angle sum of a Δ)

$= 180 - (150)$

$= 30^\circ$

$\frac{AC}{\sin 45^\circ} = \frac{x}{\sin 30^\circ}$

$AC = x \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{2x}{\sqrt{2}}$

(ii) ΔACD, $\angle CAD = 180 - (75+60)$

$= 180 - (135)$

$= 45^\circ$

$\frac{AC}{\sin 60^\circ} = \frac{y}{\sin 45^\circ}$

$AC = y \frac{\sin 60^\circ}{\sin 45^\circ} = \frac{y \cdot \sqrt{3}}{2} \times \sqrt{2}$.

(b) AC from (i) = AC from (ii)

$\frac{2x}{\sqrt{2}} = y \frac{\sqrt{3} \cdot \sqrt{2}}{2}$ rearranging this, we get

$\frac{x}{y} = \frac{2\sqrt{3}}{42} = \frac{\sqrt{3}}{2}$

(c) Area ΔABC

$= \frac{1}{2} BC \times AC \times \sin 105^\circ$

$\sin 105^\circ = \sin(180 - 105) = \sin 75^\circ$

and $AC = \frac{2x}{\sqrt{2}}$

$\therefore$ Area $\Delta ABC = \frac{1}{2} \cdot x \cdot \frac{2x}{\sqrt{2}} \cdot \sin 75$

$= \frac{x^2}{\sqrt{2}} \sin 75^\circ$

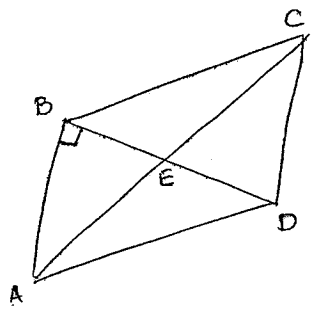
Area $\Delta ADC = \frac{1}{2} \cdot CD \cdot AC \sin 75^\circ$

$= \frac{1}{2} \cdot y \cdot \frac{2x}{\sqrt{2}} \cdot \sin 75^\circ$

$= \frac{2xy}{\sqrt{2}} \sin 75^\circ$

$\frac{\text{Area } \Delta ABC}{\text{Area } \Delta ADC} = \frac{\frac{x^2}{\sqrt{2}} \sin 75^\circ}{\frac{xy}{\sqrt{2}} \sin 75^\circ} = \frac{x}{y} = \frac{\sqrt{3}}{2}$.

8.



ABCD is a parallelogram.

In ΔAEB,

$AE^2 = AB^2 + BE^2$ (Pythagoras theorem)

$BE = ED$ (diagonals bisect each other)

$\therefore AE^2 = AB^2 + ED^2$

or $AB^2 = AE^2 - ED^2$

In ΔABD, $AD^2 = AB^2 + BD^2$

$= AE^2 - ED^2 + BD^2$ (Pythagoras theorem)

$BD = 2ED$ (diagonals bisect each other)

$AD^2 = AE^2 - ED^2 + 4ED^2$.