Meriden School

Year 11
Extension 1 Mathematics (Additional Paper)
Yearly Examinations
August, 2004

Total Marks: 84
Time Allowed: 2 hours

Instructions:

- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used
- Answer each question on a NEW PAGE
- Hand up in THREE (3) separate bundles stapled together
  - SECTION A – Questions 1, 2 and 3
  - SECTION B – Questions 4, 5 and 6
  - SECTION C – Questions 7 and 8

Outcomes Assessed:

PRELIMINARY

A student:

P1 - demonstrates confidence in using mathematics to obtain realistic solutions to problems

P2 - provides reasoning to support conclusions which are appropriate to the context

P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

P4 - chooses and applies appropriate arithmetic, algebraic, graphical, and trigonometric and geometric techniques

P5 - understands the concept of a function and the relationship between a function and its graph

P6 - relates the derivative of the function to the slope of its derivative

P7 - determines the derivative of a function through routine application of the rules of differentiation

P8 - understands and uses the language and notation of calculus

PE1 - appreciates the role of mathematics in the solution of practical problems

PE2 - uses multi-step deductive reasoning in a variety of contexts

PE3 - solves problems involving inequalities, polynomials and circle geometry

PE5 - determines derivatives which require the application of more than one rule of differentiation

PE6 - makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide context

NUMBER OF MARKS

SECTION A (36 marks):

START A NEW PAGE
Question 1 (12 marks):

(a) Find the (natural) domain for:

(i) $y=\sqrt{2x+6}$

2

(ii) $y=\dfrac{1}{x^2+2x-8}$

2

(b) Solve: $|x+3|+|x-1|\geq 3$

3

(c) (i) ***Show*** that $\dfrac{x+3}{x-2}$ can be written in the form of $1+\dfrac{C}{x-2}$, where C is a constant.

1

(ii) Sketch the curve $y=\dfrac{x+3}{x-2}$, clearly indicating any intercepts or asymptotes.

3

(d) Sketch the graph of $y = \tan x$ for $0^0 \leq x \leq 180^0$ showing all essential features.

1

START A NEW PAGE
Question 2 (12 marks):

(a) Find the derivative of:

(i) $y = 4x^4 + 3x^2 - 7$

1

(ii) $y = \dfrac{16x^2 + 1}{3x - 4}$

2

(iii) $y = \sqrt[3]{2x^4}$

2

(b) Show for $f(x) = (4x - 5)^2\sqrt{x^4 + 1}$ that

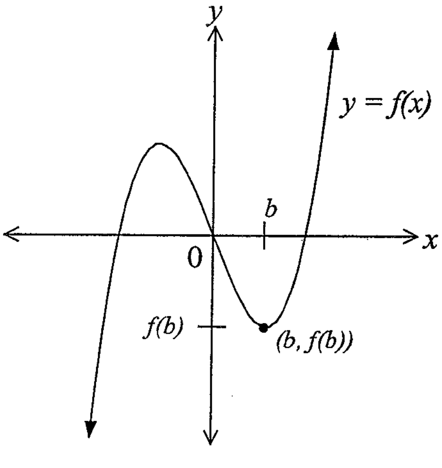
$$f'(x) = \frac{2(4x - 5)(8x^4 - 5x^3 + 4)}{\sqrt{x^4 + 1}}$$

3

(c) Find the coordinates of the point that divides the interval AB ***externally*** in the ratio of 1:3 where A(1, 2) and B(-1, 3).

2

(d) The equation of the tangent to the curve $y = f(x)$ at $(b, f(b))$ is $y = f(b)$.



(i) Copy the diagram onto your page.

(ii) Draw and label the equation of the ***normal*** to the curve at this point $(b, f(b))$.

2

START A NEW PAGE
Question 3 (12 marks):

(a) (i) Expand: $\cos(A+B)$.

1

(ii) Write an expression for $\cos 2A$ in terms of $\sin^2 A$.

1

(iii) ***Show*** that $\cos 3A = 4\cos^3 A - 3\cos A$.

2

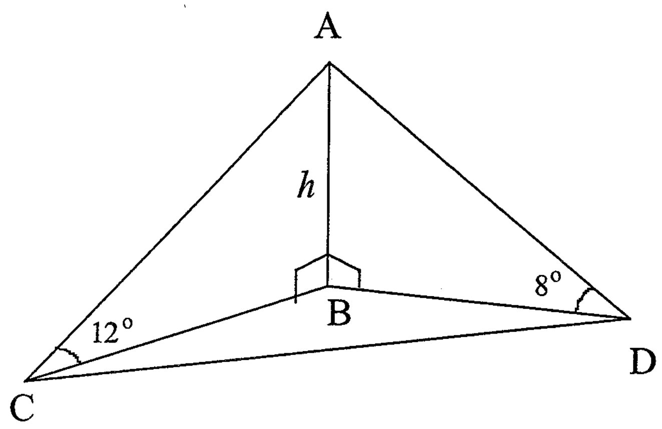
(b) (i) Write $x^2 + 4x + y^2 - 8y - 5 = 0$ in the form of $(x+a)^2 + (y+b)^2 = r^2$ where a, b and r are constants.

2

(ii) Sketch $x^2 + 4x + y^2 - 8y - 5 = 0$ clearly indicating the domain and range of the graph.

2

(c) AB is a hill of height h metres. From the points C and D in the same plane as the base of the hill, the angles of elevation of the top of the hill are 12^o and 8^o respectively. From the base of the tower, the bearings of the points C and D are 229^o T and 187^o respectively. Find the height of the tower, if C is 400m from D (answer in metres correct to 3 significant figures).



4

SECTION B (36 marks):

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Question 4 (12 marks):

(a) Given that $t = tan\frac{\theta}{2}$, show the exact value for $tan\, 67^o\, 30'$ is $1 + \sqrt{2}$.

4

(b) (i) Divide $P(x) = x^4 + 2x^3 - 3x^2 + 7x - 9$ by $Q(x) = x - 2$ and write in the form of $P(x) = A(x)Q(x) + R(x)$, where $A(x)$ and $R(x)$ are polynomials.

3

(ii) ***Show*** that $P(2) = R(x)$, where $R(x)$ was found in part (i).

1

(c) (i) Factorise $-x^3 + 12x^2 - 17x - 90$ into three linear factors of the form $(a - x)(x - b)(x - c)$ where a, b and c are constants.

2

(ii) Sketch the curve $y = -x^3 + 12x^2 - 17x - 90$ showing all essential features.

2

START A NEW PAGE
Question 5 (12 marks):

(a) Solve for $-180^0 \leq \theta \leq 180^0$: $sin\ 2\theta = \frac{1}{\sqrt{2}}$.

3

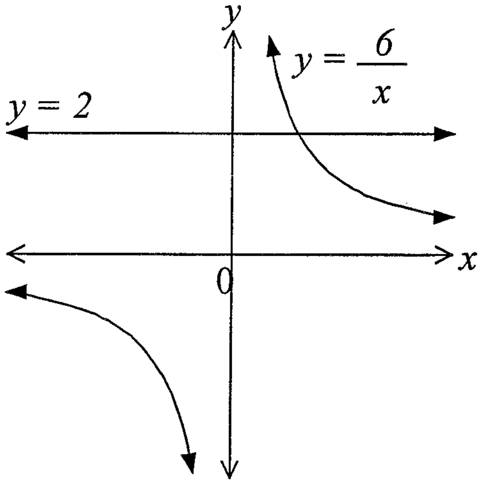
(b) Find the equation of the tangent to the curve $y = x^3 + 2x^2 - 4x - 6$ at the point where $x = 1$ (answer in gradient-intercept form).

3

(c) Find the angle between the tangents to the curves $y = x^2 + 4x - 3$ and $y = x^2 - 4x + 5$ at the point of intersection (answer to the nearest minute).

4

(d) Using the graph below, or otherwise, solve $\frac{6}{x} \leq 2$:



2

SECTION C (24 marks):

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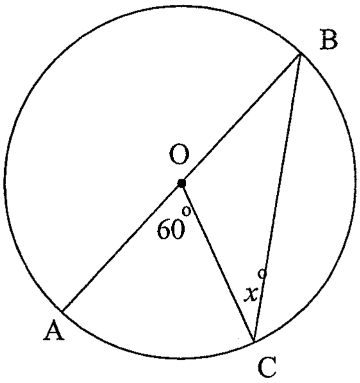
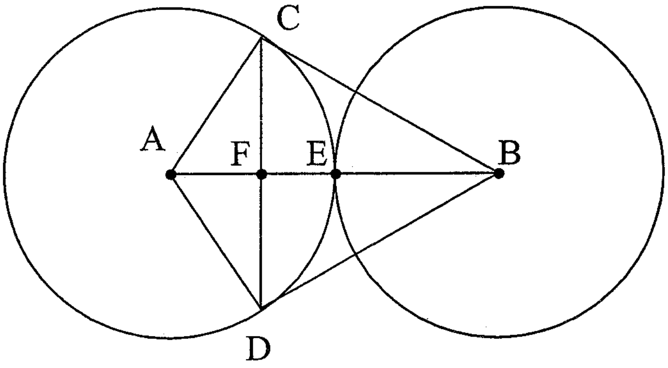
Question 6 (12 marks):

(a) A, B and C lie on the circumference of a circle of centre O. $\angle AOC = 60^o$. Find x and give reasons for your answer:



3

(b)



Two circles of equal radius and with centres A and B respectively touch each other externally at E. BC and BD are tangents from B to the circle with centre A.

(i) Copy the diagram onto your page.

(ii) Show that $BCAD$ is a cyclic quadrilateral.

2

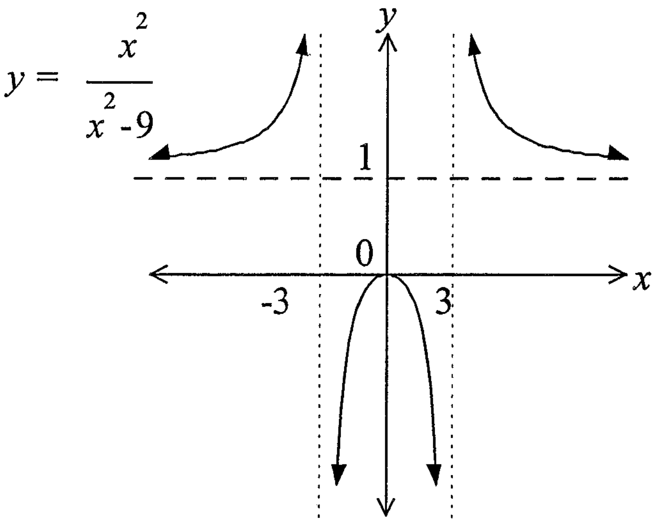
(iii) Show that E is the centre of the circle, which passes through B, C, A, and D.

2

(iv) Prove that $\angle CBA = \angle DBA$.

2

(c)



Consider the graph above and clearly evaluate:

(i) y when $x \to 3^-$. **1**

(ii) y when $x \to 3^+$. **1**

(iii) y when $x \to \infty$. **1**

START A NEW PAGE
Question 7 (12 marks):

(a) It is known that the polynomial $P(x) = x^3 + x^2 - 8x - 12$ has a double root.

(i) Solve for $P(x) = 0$. **2**

(ii) Show that for the double root, $P(x) = P'(x) = 0$. **2**

(b) Consider the polynomial $P(x) = x^5 + ax^4 - 14x^2 + bx - 6$ with a triple root at $x = -1$. Find the values for a and b. **3**

(c) Solve $4\sin\theta + 3\cos\theta = 1$ for $0^o \leq \theta \leq 360^o$ (answer to the nearest minute). **3**

(d) Find the general solution for $\tan\theta = -1$. **2**

End of Examination

Ext 1. Mathematics Yearly Solutions 2004

Q1(a) (i) $2x+6 \geqslant 0$

$2x \geqslant -6$

$\therefore \underline{\underline{x \geqslant -3}}$

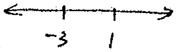
(ii) $x^2+2x-8 \neq 0$

$(x+4)(x-2) \neq 0$

$\therefore \underline{\underline{x \neq -4, 2}}$

(b) $|x+3| + |x-1| \geqslant 3$

⊕ $x+3 \geqslant 0$ $x \geqslant -3$	⊕ $x-1 \geqslant 0$ $x \geqslant 1$
⊖ $x<-3$	⊖ $x<1$



Case 1 $x<-3$

$-(x+3) + -(x-1) \geqslant 3$

$-x-3 - x+1 \geqslant 3$

$-2x-2 \geqslant 3$

$-2x \geqslant 5$

$x \leq -2.5$

$\therefore x \leq -2.5$ and $x<-3 = \underline{\underline{x<-3}}$

Case 2: $-3 \leq x < 1$

$+(x+3) + -(x-1) \geqslant 3$

$\cancel{x}+3 - \cancel{x}+1 \geqslant 3$

$4 \geqslant 3$, true

$\therefore$ true for all $\underline{\underline{-3 \leq x < 1}}$

Case 3: $x \geqslant 1$

$+(x+3) + +(x-1) \geqslant 3$

$x+3 + x-1 \geqslant 3$

$2x + 2 \geqslant 3$

$2x \geqslant 1$

$x \geqslant \frac{1}{2}$

$\therefore x \geqslant \frac{1}{2}$ and $x \geqslant 1 = \underline{\underline{x \geqslant 1}}$

FINAL SOLUTION: $x<-3$ or $-3 \leq x < 1$ or $x \geqslant 1$ = all real x

b) Alternatively (Graphical Solutions)

$|x+3| + |x-1| \geqslant 3$

$\therefore |x+3| \geqslant 3 - |x-1|$

let $\left.\begin{array}{l} y = |x+3| \\ y = 3-|x-1| \end{array}\right\}$

$y = |x+3|$

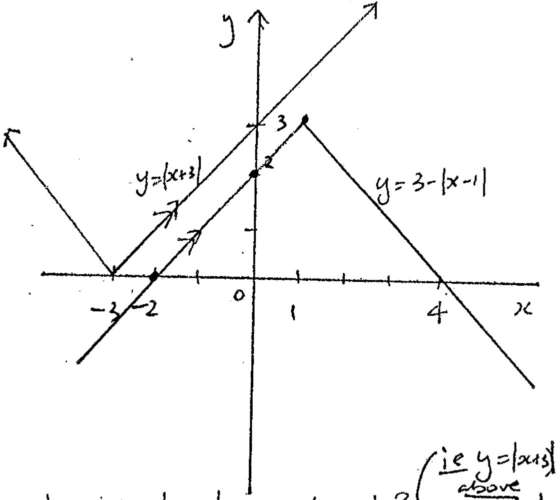
$= \begin{cases} x+3, & x \geqslant -3 \\ -x-3, & x<-3 \end{cases}$

$y = 3-|x-1|$

$= \begin{cases} 3-(x-1), & x \geqslant 1 \\ 3-(-(x-1)), & x<1 \end{cases}$

$= \begin{cases} 3-x+1, & x \geqslant 1 \\ 3+x-1, & x<1 \end{cases}$

$= \begin{cases} 2-x, & x \geqslant 1 \\ 2+x, & x<1 \end{cases}$



when is $|x+3| \geqslant 3-|x-1|$? (ie $y=|x+3|$ above $y=3-|x-1|$)

$\underline{\underline{\text{All real values of } x}}$

Q1(c)(i) $\dfrac{x+3}{x-2} = \dfrac{x-2}{x-2} + \dfrac{5}{x-2}$

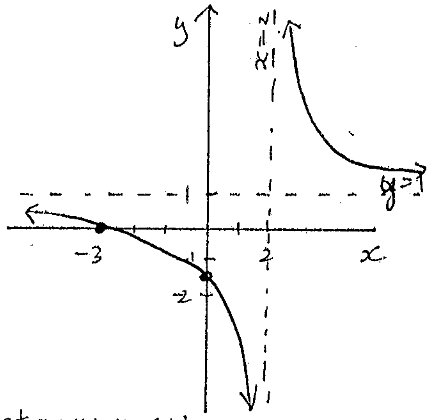
$= \underline{\underline{1 + \dfrac{5}{x-2}}}$

Alternatively:

$$\begin{array}{r} 1 \\ x-2 \overline{)\, x+3} \\ \underline{-(x-2)} \\ \underline{5} \end{array}$$

$\therefore \dfrac{x+3}{x-2} = 1 + \dfrac{5}{x-2}$

(ii)

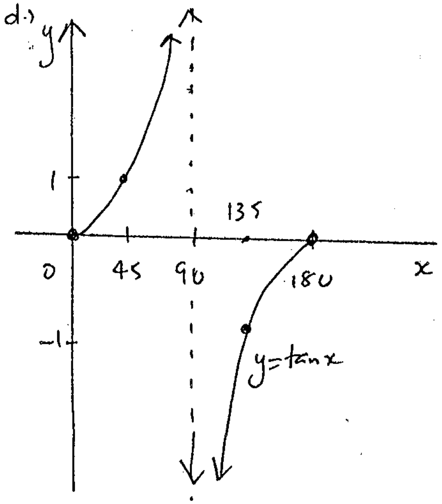


let $x=0$: $y = \dfrac{0+3}{0-2}$

$= -1.5$

let $y=0$: $0 = x+3 \Rightarrow x = -3$

(d)



Q2(a)(i) $y = 4x^4 + 3x^2 - 7$

$\underline{\underline{\dfrac{dy}{dx} = 16x^3 + 6x}}$

(ii) $y = \dfrac{16x^2+1}{3x-4}\ \begin{array}{l} u \\ v \end{array}$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(3x-4) \times 32x - (16x^2+1) \times 3}{(3x-4)^2}$

$= \dfrac{96x^2 - 128x - 48x^2 - 3}{(3x-4)^2}$

$= \underline{\underline{\dfrac{48x^2 - 128x - 3}{(3x-4)^2}}}$

(iii) $y = \sqrt[3]{2x^4}$

$= (2x^4)^{\frac{1}{3}}$

$= \frac{1}{3}(2x^4)^{-\frac{2}{3}} \cdot 8x^3$

$= \underline{\underline{\dfrac{8x^3}{3(2x^4)^{\frac{2}{3}}}}}$

b) $f(x) = (4x-5)^2\sqrt{x^4+1}$

$= \underset{u}{(4x-5)^2}\underset{v}{(x^4+1)^{\frac{1}{2}}}$

$f'(x) = vu' + uv'$

$= (x^4+1)^{\frac{1}{2}} \cdot 2(4x-5) \cdot 4$

$+ (4x-5)^2 \cdot \frac{1}{2}(x^4+1)^{-\frac{1}{2}} \cdot 4x^3$

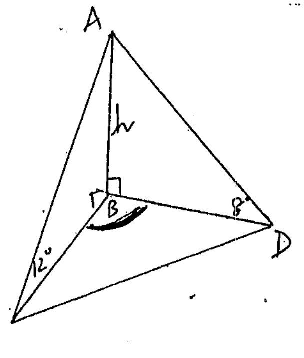
$= 8(x^4+1)^{\frac{1}{2}}(4x-5) + 2x^3(4x-5)^2(x^4+1)^{-\frac{1}{2}}$

$= 2(x^4+1)^{-\frac{1}{2}}(4x-5)^1[4(x^4+1) + x^3(4x-5)]$

$= 2(x^4+1)^{-\frac{1}{2}}(4x-5)(\underline{4x^4}+4+\underline{4x^4}-5x^3)$

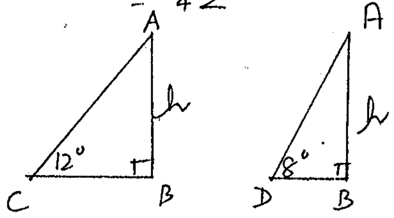
$= \underline{2(4x-5)(8x^4-5x^3+4)}$, as required

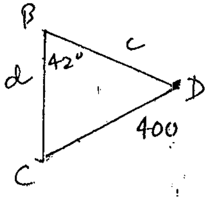
(c)



$\angle C\ BD = 229^\circ - 187^\circ$

$= 42^\circ$





In $\triangle ABC$: $\tan 12^\circ = \frac{h}{BC} \Rightarrow h = (BC)\tan 12^\circ$

$\Rightarrow BC = \frac{h}{\tan 12}$

In $\triangle ABD$: $\tan 8^\circ = \frac{h}{BD} \Rightarrow h = (BD)\tan 8^\circ$

$\Rightarrow BD = \frac{h}{\tan 8}$

In $\triangle BCD$: Use cosine Rule:

$b^2 = d^2 + c^2 - 2dc\cos B$

$400^2 = BC^2 + BD^2 - 2 \times BC \times BD \times \cos 42^\circ$

$400^2 = \frac{h^2}{\tan^2 12^\circ} + \frac{h^2}{\tan^2 8^\circ} - 2 \times \frac{h}{\tan 12^\circ} \times \frac{h}{\tan 8^\circ} \times \cos 42^\circ$

$160000 = \frac{h^2}{\tan^2 12} + \frac{h^2}{\tan^2 8^\circ} - \frac{2h^2\cos 42^\circ}{\tan 12^\circ \tan 8^\circ}$

$160000 = h^2\left\{\frac{1}{\tan^2 12^\circ} + \frac{1}{\tan^2 8^\circ} - \frac{2\cos 42^\circ}{\tan 12^\circ \tan 8^\circ}\right\}$

$160000 = h^2 \times 23.0082......$

$h^2 = 6954.038471....$

$h = 83.3908.....$

$\underline{\underline{h = 83.4 \text{ (to 3 s.f.)}}}$

Q4(c) $t = \tan\left(\frac{\theta}{2}\right)$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A} \Rightarrow \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$\tan\theta = \frac{2t}{1-t^2}$

let $\theta = 135^\circ \Longrightarrow \frac{\theta}{2} = 67^\circ 30'$

$\Longrightarrow t = \tan\frac{\theta}{2} = \tan 67^\circ 30$

$\therefore \tan 135^\circ = \frac{2t}{1-t^2}$

$-1 = \frac{2t}{1-t^2}$

$-1(1-t^2) = 2t$

$-1 + t^2 = 2t$

$t^2 - 2t - 1 = 0$

$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{2 \pm \sqrt{4-4.1.-1}}{2}$

$= \frac{2 \pm \sqrt{8}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2}$

$= 1 \pm \sqrt{2}$

$t = 1+\sqrt{2}\,(>0)$ or $t = 1-\sqrt{2}\,(<0)$

but $\tan 67^\circ 30^\circ$ (1st quad) > 0 $\therefore \underline{t = 1+\sqrt{2}}$

Q2(c) $A(\overset{x_1}{1}, \overset{y_1}{2})$ $k : l = 1 : -3$

$B(\overset{x_2}{-1}, \overset{y_2}{3})$

$x = \frac{kx_2 + lx_1}{k+l}$

$= \frac{1(-1)+(-3)(1)}{1+-3}$

$= \frac{-1-3}{-2}$

$= \frac{-4}{-2}$

$= 2$

$y = \frac{ky_2 + ly_1}{k+l}$

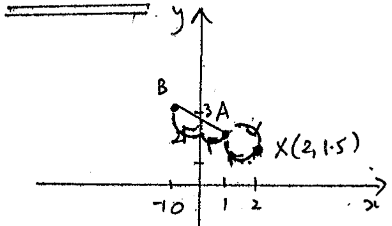
$= \frac{1(3)+(-3)(2)}{1+(-3)}$

$= \frac{3-6}{-2}$

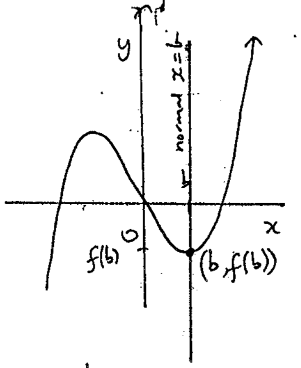
$= \frac{-3}{-2}$

$= 1.5$

$\underline{\underline{(2, 1.5)}}$



(d)



normal: $x = b$

Q3(a)

i) $\cos(A+B)$

$\underline{= \cos A\cos B - \sin A\sin B}$

(ii) $\cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$= \cos^2 A - \sin^2 A$

$= (1-\sin^2 A) - \sin^2 A$

$\underline{= 1 - 2\sin^2 A}$

(iii) $\cos(3A)$

$= \cos(2A + A)$

$= \cos 2A\cos A - \sin 2A\sin A$

$= (1-2\sin^2 A)\cos A - (2\sin A\cos A)\sin A$

$= \cos A - 2\sin^2 A\cos A - 2\sin^2 A\cos A$

$= \cos A - 4\sin^2 A\cos A$

$= \cos A - 4(1-\cos^2 A)\cos A$

$= \cos A - 4\cos A + 4\cos^3 A$

$\underline{\underline{= 4\cos^3 A - 3\cos A}}$, as required

b)(i) $x^2 + 4x + y^2 - 8y - 5 = 0$

$(x^2+4x+4) + (y^2-8y+16) = 5+4+16$

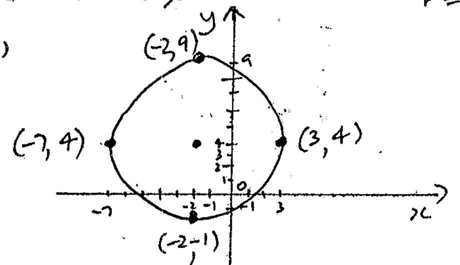
$\therefore (x+2)^2 + (y-4)^2 = 25$

which is of the form

$(x+a)^2 + (y+b)^2 = r^2$ where $a = 2$, $b = -4$

$r = 5$

(ii)



Q5(b) $y = x^3 + 2x^2 - 4x - 6$

$y' = 3x^2 + 4x - 4$

[at $x = 1$]

Find gradient:

$m_T = 3(1)^2 + \cancel{4(1)} - \cancel{4}$

$m_T = 3$

Find y:

$y = 1^3 + 2(1)^2 - 4(1) - 6$

$= 1 + 2 - 4 - 6$

$= -7$

Find equation:

$y - y_1 = m(x - x_1)$

$y + 7 = 3(x - 1)$

$y + 7 = 3x - 3$

$\underline{\underline{y = 3x - 10}}$

(c) Point of Intersection:

$\left.\begin{array}{l} y = x^2 + 4x - 3 \\ y = x^2 - 4x + 5 \end{array}\right\}$

$\cancel{x^2} + 4x - 3 = \cancel{x^2} - 4x + 5$

$8x = 8$

$\underline{\underline{x = 1}}$

$y = 1^2 + 4(1) - 3$

$\underline{\underline{= 2}}$

For $y = x^2 + 4x - 3$

$y' = 2x + 4$

[$x = 1$]

$m_T = 2(1) + 4$

$m_1 = 6$

For $y = x^2 - 4x + 5$

$y' = 2x - 4$

[at $x = 1$]

$m_T = 2(1) - 4$

$m_2 = -2$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\dfrac{6 - (-2)}{1 + 6 \times -2}\right|$

$= |\underline{8}|$

$\tan\theta = \dfrac{8}{11}$

$\theta = 36^\circ 01' 38''$

$\underline{\underline{\theta = 36^\circ 02'}}$ (to nearest minute)

(d) Pt. of Intersection (x-value):

$\dfrac{6}{x} = 2$

$6 = 2x$

$\underline{x = 2}$

$\left.\begin{array}{l} \text{let } y = \frac{6}{x} \\ y = 2 \end{array}\right\}$

When is $\dfrac{6}{x} \leq 2$?

["hyperbola" below "the line"]

$\therefore \underline{\underline{x < 0, \quad x \geqslant 2}}$

Section C:

Q6

(a) $\angle ABC = 30^\circ$ (angle at the circumference is half the angle at the centre of a circle subtended from the same arc AC)

$x^\circ = 30^\circ$ (equal angles of an isosceles $\triangle OBC$, equal radii $OC = OB$)

Q4(b)(i)

$$\begin{array}{r} x^3 + 4x^2 + 5x + 17 \\ x-2 \overline{\smash{\big)} x^4 + 2x^3 - 3x^2 + 7x - 9} \\ \underline{-(x^4 - 2x^3)} \qquad\qquad\quad \\ 4x^3 - 3x^2 \qquad\quad \\ \underline{-(4x^3 - 8x^2)} \qquad\quad \\ 5x^2 + 7x \quad \\ \underline{-(5x^2 - 10x)} \quad \\ 17x - 9 \\ \underline{-(17x - 34)} \\ \underline{25} \end{array}$$

$\therefore x^4 + 2x^3 - 3x^2 + 7x - 9$

$\underline{\underline{= (x^3 + 4x^2 + 5x + 17)(x - 2) + 25}}$

(ii) $P(x) = x^4 + 2x^3 - 3x^2 + 7x - 9$

$\therefore P(2) = 2^4 + 2(2)^3 - 3(2)^2 + 7(2) - 9$

$= 16 + 16 - 12 + 14 - 9$

$= 25$ which is $R(x)$ as found above in (i)

(c)(i) $P(x) = -x^3 + 12x^2 - 17x - 90$

$P(5) = -125 + 12 \times 25 - 17 \times 5 - 90$

$= 0 \therefore (x-5)$ is a factor

$P(9) = -729 + 12 \times 81 - 17 \times 9 - 90$

$= 0 \therefore (x-9)$ is a factor

$P(-2) = +8 + 12 \times 4 + 17 \times 2 - 90$

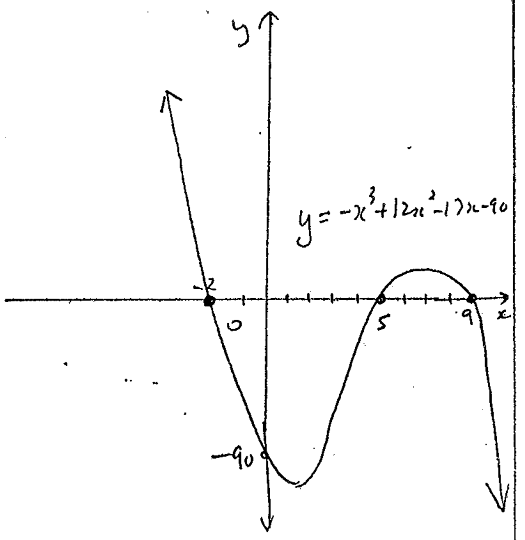
$= 0 \quad \therefore (x+2)$ is a factor)

$\therefore P(x) = -x^3 + 12x^2 - 17x - 90$

$= (2 + x)(x - 5)(x - 9)$

$= (5 - x)(x + 2)(x - 9)$

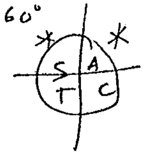
is of the form $(a - x)(x - b)(x - c)$



Q5(a) $-180^\circ \leq \theta \leq 180^\circ$

$\downarrow \qquad \downarrow \times 2 \qquad \downarrow$

$-360^\circ \leq 2\theta \leq 360^\circ$



$\sin 2\theta = \dfrac{1}{\sqrt{2}}$

(1st, 2nd)

$2\theta = 45^\circ, 180 - 45^\circ, 45^\circ - 360^\circ, 135^\circ - 360^\circ$

$2\theta = 45^\circ, 135^\circ, -315^\circ, -225^\circ$

$\theta = -157.5^\circ, -112.5^\circ, 22.5^\circ, 67.5^\circ$

or

$\underline{\underline{\theta = -157^\circ 30', -112^\circ 30', 22^\circ 30', 67^\circ 30'}}$

Q7

(c) $4\sin\theta + 3\cos\theta = 1$

Consider $R(\sin(\theta+\alpha))$

$= R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$

$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

let $4\sin\theta + 3\cos\theta = R\sin(\theta+\alpha)$

$\Rightarrow 4\sin\theta + 3\cos\theta$

$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

(equate both sides of equation)

$\therefore 4 = R\cos\alpha$ ①

$3 = R\sin\alpha$ ②

$② \div ① \Rightarrow \frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \frac{3}{4}$

$\tan\alpha = \frac{3}{4} \Rightarrow \alpha \doteqdot 36°52'$

$①^2 + ②^2$: $R^2\cos^2\alpha + R^2\sin^2\alpha$

$= 4^2 + 3^2$

$R^2(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}) = 16 + 9$

$R^2 = 25$

$R = 5 \ (R > 0)$

$\therefore 4\sin\theta + 3\cos\theta$

$= 5\sin(\theta+\alpha)$

Solve for $0 \le \theta \le 360° \xrightarrow{+36°52'} 36°52' \le \theta + 36°52' \le 396°52'$

$\therefore 5\sin(\theta + 36°52') = 1$

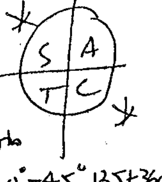
$\sin(\theta + 36°52') = \frac{1}{5}$

$\therefore \theta + 36°52' = \overset{1st}{11°32'}, \overset{2nd}{180° - 11°32'}$

$\theta + 36°52' = \cancel{11°32'}, 168°28', 371°32', \ldots$

$\therefore \theta = 131°36', 334°40'$

(d) $\tan\theta = -1$



$\theta = \overset{2nd}{135°}, \overset{4th}{360° - 45°}, 135 \pm 360°, 315° \pm 360°, \ldots$

$\theta = 135°, 315°, \ldots$

$\theta = 180°n - 45°$ where

n is an integer.

Alternate solution to Q7(c):

Consider: $R\cos(\theta - \alpha) = 4\sin\theta + 3\cos\theta$

LHS $= R\cos(\theta - \alpha)$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

RHS $= 3\cos\theta \quad + 4\sin\theta$

$\therefore R\cos\alpha = 3$ ① $\Rightarrow ①^2: R^2\cos^2\alpha = 9$

$R\sin\alpha = 4$ ② $\Rightarrow ②^2: R^2\sin^2\alpha = 16$

$R^2\sin^2\alpha + R^2\cos^2\alpha = 9 + 16$

$R^2[\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}] = 25$

$\therefore R^2 = 25$

$\therefore R = 5 \ (R > 0)$

$② \div ①$: $\frac{R\sin\alpha}{R\cos\alpha} = \frac{4}{3} \Rightarrow \tan\alpha = \frac{4}{3}$

$\therefore \alpha = 53°8'$

So:

$5\cos(\theta - 53°8') = 4\sin\theta + 3\cos\theta$

$\therefore 5\cos(\theta - 53°8') = 1$

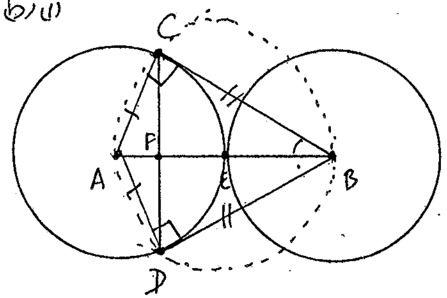
$\cos(\theta - 53°8') = \frac{1}{5}$

$\therefore \theta - 53°8' = 78°28', 360 - 78°28'$

$\theta - 53°8' = 78°28', 281°32'$

$\therefore \theta = 131°36', 334°40'$

Q6b)(i)



(ii) $\angle ACB = 90°$ $\angle ADB = 90°$ {the tangent to a circle ⊥ radius drawn to the point of contact}

Hence $\angle ACB + \angle ADB = 180°$

so $\angle CAD + \angle CBD = 180°$ (angle sum of a quad.)

Hence ACBD is a cyclic quadrilateral (opp. ∠s are supplementary)

(iii) A, C, B, D all lie on the circumference of the cyclic quadrilateral

$\angle ACB = 90°$, such that AB is the diameter (∠ in semicircle = 90°)

hence AE = EB are equal radii

hence E is centre of circle ACBD.

(iv) BC = BD (tangents to a circle from an external point are equal)

AC = AD (equal radii)

Hence ACBD is a kite, so AB is the axis of symmetry of ACBD.

OR AC = AD (equal radii)

BC = BD (reason as above)

AB is common

$\therefore \Delta ACB \equiv \Delta ADB$ (SSS)

$\therefore \angle CBA = \angle DBA$ (corresponding ∠ of ≡ Δ)

(c) (i) $y \rightarrow -\infty$

(ii) $y \rightarrow +\infty$

(iii) $y \rightarrow 1^+$

Q7(a)(i) $P(x) = x^3 + x^2 - 8x - 12$

$P(-2) = (-2)^3 + (-2)^2 - 8(-2) - 12$

$= -8 + 4 + 16 - 12$

$= 0 \ \therefore (x+2)$ is a factor

$P(3) = 3^3 + 3^2 - 8(3) - 12$

$= 27 + 9 - 24 - 12 \ \therefore (x-3)$ is a factor

$\therefore P(x) = x^3 + x^2 - 8x - 12$

$= (x+2)^2(x-3)$ NB double root at $x = -2$

$\therefore P(x) = 0 \Rightarrow x = -2, 3$

(ii) $P(x) = x^3 + x^2 - 8x - 12$

$P'(x) = 3x^2 + 2x - 8$

$P'(-2) = 3(-2)^2 + 2(-2) - 8$

$= 12 - 4 - 8$

$= 0$ and $P(2) = 0$ (shown above)

$\therefore P(-2) = P'(-2) = 0$

b) $P(x) = x^5 + ax^4 - 14x^2 + bx - 6$

$P'(x) = 5x^4 + 4ax^3 - 28x + b$

$P(-1) = P'(-1) = 0$ for triple root

$P(-1) = 0 \Rightarrow (-1)^5 + a(-1)^4 - 14(-1)^2 + b(-1) - 6 = 0$

$-1 + a - 14 - b - 6 = 0$

$a - b = 21$ ①

$P'(-1) = 0 \Rightarrow 5(-1)^4 + 4a(-1)^3 - 28(-1) + b = 0$

$5 - 4a + 28 + b = 0$

$-4a + b = -33$ ②

$① + ②: -3a = -12 \Rightarrow a = 4$

$① \Rightarrow 4 - b = 21 \Rightarrow b = -17$

Alternate solution to Q7(c):

Q7(c) Solve: $4\sin\theta + 3\cos\theta = 1$

let $t = \tan\left(\frac{\theta}{2}\right)$ $\quad 0 \le \theta \le 360°$

$\Downarrow \div 2$

$0 \le \frac{\theta}{2} \le 180°$

$\therefore \sin\theta = \frac{2t}{t^2+1}$

$\cos\theta = \frac{1-t^2}{t^2+1}$

$$4\left(\frac{2t}{t^2+1}\right) + 3\left(\frac{1-t^2}{1+t^2}\right) = 1$$

$$4(2t) + 3(1-t^2) = 1+t^2$$

$$8t + 3 - 3t^2 = 1+t^2$$

$$-4t^2 + 8t + 2 = 0$$

$(\div -2) \qquad (\div -2)$

$$2t^2 - 4t - 1 = 0$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{+4 \pm \sqrt{(-4)^2 - 4.2.-1}}{2.2}$$

$$= \frac{4 \pm \sqrt{16+8}}{4}$$

$$= \frac{4 \pm \sqrt{24}}{4}$$

$$= \frac{4 \pm \sqrt{4 \times 6}}{4}$$

$$= \frac{4 \pm 2\sqrt{6}}{4}$$

$$= \frac{2 \pm \sqrt{6}}{2}$$

for $t = \frac{2+\sqrt{6}}{2}$

$\therefore \tan\left(\frac{\theta}{2}\right) = \frac{2+\sqrt{6}}{2}$

$\therefore \frac{\theta}{2} = 6\overset{1st}{5}.79....$

$\therefore \theta = 131.59...$

$\therefore \underline{\underline{\theta = 131°36'}}$

and $t = \frac{2-\sqrt{6}}{2}$

$\tan\left(\frac{\theta}{2}\right) = \frac{2-\sqrt{6}}{2}$

$\therefore \frac{\theta}{2} = 180° - 12°40'$

$\frac{\theta}{2} = 167.33......$

$\therefore \underline{\underline{\theta = 334\ 40'}}$

$$\therefore \underline{\underline{\theta = 131°36',\ 334\ 40'}}$$