#15606223
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MERIDEN SCHOOL

2004
YEAR 11
MATHEMATICS

YEARLY EXAMINATION

Total Marks: 120

Time allowed: 2 ½ hours.

Directions to Candidates.

- Attempt all questions
- Questions are not of equal value
- All necessary working should be shown. Marks may be deducted for carelessly or badly arranged work.
- Start a new page for EACH question.

OUTCOMES

P1- demonstrates confidence in using mathematics to obtain realistic solutions to problems.

P2 – provides reasoning to support conclusions which are appropriate in context.

P3 – performs routine arithmetic and algebraic manipulation involving surds, simple expressions and trigonometric identities.

P4 – chooses and applies appropriate arithmetic,algebraic, graphical, trigonometric and geometric techniques.

b) Find the coordinates of the point A, where the line $3x - 4y = 12$ meets the x axis, and determine the equation of the line through A, perpendicular to $3x - 4y = 12$. (4 marks)

c) Find the point(s) of intersection of $y = 2x + 1$ and $y = x^3 - 8x^2 + 14x + 1$. (4 marks)

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Question 3 (15 marks)

a) The earth revolves around the sun at a speed of approximately 106560 km/h. How far does the earth travel in a year (365 days)? Answer in scientific notation correct to 4 significant figures. (2 marks)

b) Express the decimal $0.\dot{3}\dot{4}$ in rational form. (2 marks)

c) Simplify $\dfrac{3}{x+5} - \dfrac{2}{x-5}$ (2 marks)

d) If $x = 7 - 4\sqrt{3}$, show that $x + x^{-1}$ is rational, and find its value. (2½ marks)

e) Express with positive indices $\dfrac{a^{-1} + b^{-1}}{(ab)^{-1}}$ (2½ marks)

f) Simplify $\dfrac{(a+b)^3 - a^3}{b}$ (4 marks)

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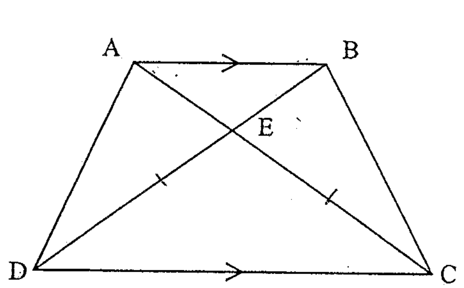
Question 4 (15 marks)

a)



AB || DC, DE = CE

Prove AE = EB (3 marks)

b) The sides of a triangular field are 84m, 93m and 108m. Calculate the size of the largest angle in this triangle correct to the nearest minute. (3 marks)

c) Find the exact value of

i) $\sec 405^0$ (2 marks)

ii) $\dfrac{\tan(-30^0)}{\tan 225^0}$ (2 marks)

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Question 7 (10 marks)

a) Find the value(s) of m for which the equation $4x^2 - mx + 9 = 0$ has

i) exactly one real root (2 marks)

ii) one as a root (1 mark)

b) If α and β are the roots of a quadratic equation $2x^2 + 5x - 8 = 0$, find the value of

i) $\alpha + \beta$ (1 mark)

ii) $\alpha\beta$ (1 mark)

iii) $\alpha^2 + \beta^2$ (2 marks)

c) Show that the equation $x^2 - (2a + b)x + ab = 0$ has real roots for all values of a and b. (3 marks)

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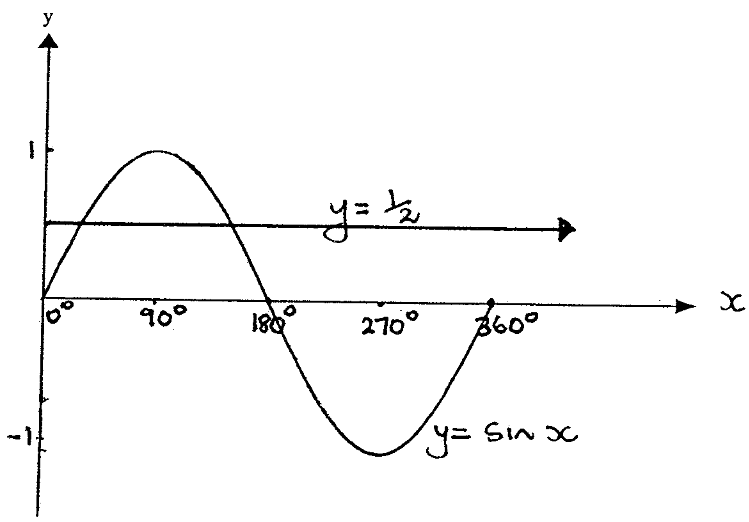
Question 8 (12 marks)

a)



Solve algebraically the intersection of the two graphs in the diagram above, for $0° \le x \le 360°$, (3 marks)

b) Solve $x^2 - 9x + 14 \le 0$ (2 marks)

c) Sketch the parabola $y = x^2 + 2x - 8$ showing x intercepts, y intercept, axis of symmetry and the vertex. (5 marks)

d) The parabola $y = 2x^2 + kx - 4$ is symmetrical about the line $x = 3$, find the value of k. (2 marks)

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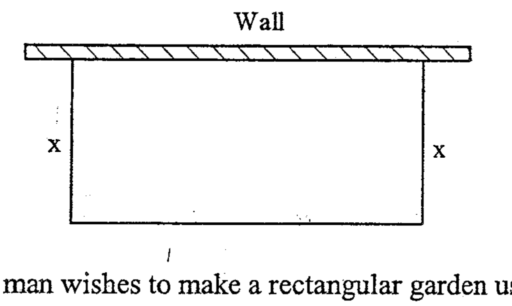
Question 9 (12 marks)

a) Solve for real x.

$$x^2 - x + \frac{24}{x^2 - x} = 14$$ (5 marks)

b)



A man wishes to make a rectangular garden using an existing wall as one side. He has 16 metres of wire netting.

i) If the width of the garden is x metres, find the length and show that the area is given by $A = 16x - 2x^2$. (3 marks)

ii) Sketch $A = 16x - 2x^2$ (2 marks)

iii) What is the maximum area of the garden? (1 mark)

iv) What dimensions give the maximum area? (1 mark)

d) Simplify $(1 - \cos^2\theta)(1 + \cot^2\theta)$ (2 marks)

e) The bearing of a ship from a lighthouse A is 075^0, and its bearing from a second lighthouse B, 44 km south of A is 040^0. Find the distance of the ship from B. (to the nearest km) (3 marks)

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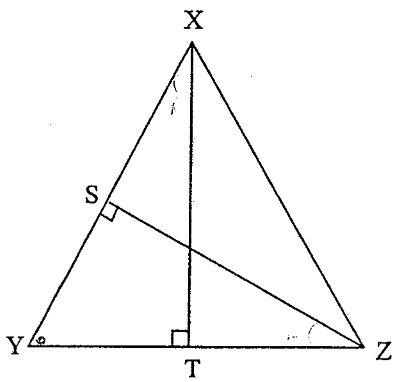
Question 5 (14 marks)

a) Solve $\frac{4 - 3x}{7} \leq 4$ and graph the solution on the number line. (3 mark)

b) Solve $x^2 + 4x - 2 = 0$, giving answer in exact form. (3 marks)

c)



ZS ⊥ XY and XT ⊥ YZ

i) Let $Y\hat{X}T = x^0$. Find the size of $S\hat{Z}Y$ in terms of x. (1½ marks)

ii) Prove that $\frac{XY}{YZ} = \frac{XT}{ZS}$ (3½ marks)

d) Factorise fully $a^2 - 9b^2 + 4a - 12b$ (3 marks)

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Question 6 (11 marks)

a) Show that $\tan\theta = \frac{\sqrt{1 - a^2}}{a}$ if $\cos\theta = a$ (2 marks)

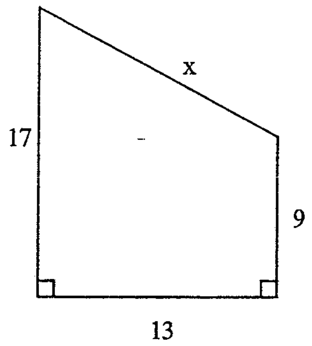
b) Show that $\frac{1}{\sin\theta} - \frac{1}{\sin^2\theta} = \frac{\sin\theta - 1}{(1 + \cos\theta)(1 - \cos\theta)}$ (3 marks)

c) Solve $2\cos^2\theta = 1$ for $0^0 \leq \theta \leq 360^0$ (3½ marks)

d) In $\triangle$ ABC, b = 4, c = 5 and the magnitude of angle BAC is $53^0\ 8'$. Calculate the perimeter of the triangle, given $\cos 53^0\ 8' = 0.6$, in exact form. (2½ marks)

Question 1 (14 marks)

a) Find x to 1 d.p. (2 marks)



b) Simplify $\dfrac{x^2 - x}{x - 3} \div \dfrac{x^2 - 5x}{x - 3}$ (2 marks)

c) Simplify $\sqrt{8} + \sqrt{18} - \sqrt{32}$ (2 marks)

d) Evaluate to 3 significant figures $\sqrt{\dfrac{(4.609)^5}{27.8 - 19.66}}$ (1 mark)

e) If $s = \dfrac{a}{1 - r}$ find r when $a = 15,\ s = 30$ (2 marks)

f) Solve $|3x - 1| = 8$ (3 marks)

g) The sale price of a TV set after 3½% discount is $1929. What is the original cost of the TV? (correct to the nearest dollar) (2 marks)

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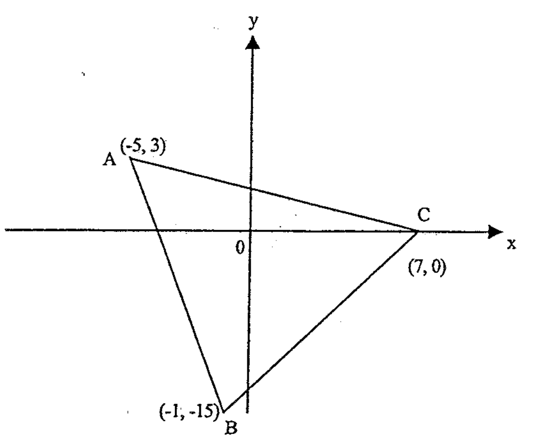
Question 2 (17 marks)

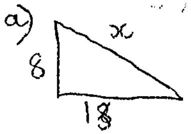
a)



i) Find the equation of AC, in general form. (3 marks)
ii) Find in exact form the perpendicular distance from B to AC. (2 marks)
iii) Find the length of AC, in exact form. (2 marks)
iv) Find the area of △ ABC. (2 marks)

(1)

a)



$x^2 = 8^2 + 13^2$

$= 233$

$x = \sqrt{233}$

$= 15.26...$

$= 15.3$

b) $\frac{x^2-x}{x-3} \times \frac{x-3}{x^2-5x}$

$= \frac{x(x-1)}{\cancel{x-3}} \times \frac{\cancel{x-3}}{x(x-5)}$

$= \frac{x-1}{x-5}$

c) $\sqrt{8} + \sqrt{18} - \sqrt{32}$

$= 2\sqrt{2} + 3\sqrt{2} - 4\sqrt{2}$

$= \sqrt{2}$

d) $\sqrt{255.5107...}$

$= 15.98...$

$= 16.0$ (3 sig figs)

e) $30 = \frac{15}{1-r}$

$30 - 30r = 15$

$-30r = -15$

$r = \frac{-15}{-30}$

$= \frac{1}{2}$

f) $|3x-1| = 8$

$3x - 1 = 8$ or $-(3x-1) = 8$

$3x = 9$ $\quad -3x + 1 = 8$

$x = 3$ $\quad -3x = 7$

$x = -\frac{7}{3}$

(g) $\frac{1929}{96.5} \times 100$

$= 1998.96...$

$= \$1999$ (nearest \$)

Question (2)

a) (i) $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\frac{y-3}{x+5} = \frac{0-3}{7+5}$

$\frac{y-3}{x+5} = -\frac{3}{12} = -\frac{1}{4}$

$4(y-3) = -1(x+5)$

$4y - 12 = -x - 5$

$x + 4y - 7 = 0$

(ii) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ $\quad \left.\begin{array}{l} x_1 = -1 \\ y_1 = -15 \end{array}\right\}$

$= \frac{|-1 - 60 - 7|}{\sqrt{1^2 + 4^2}}$ $\quad a = 1$, $b = 4$, $c = -7$

$= \frac{|-68|}{\sqrt{17}}$

$= \frac{68}{\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}}$

$= \frac{68\sqrt{17}}{17}$

$= 4\sqrt{17}$ units.

(iii) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(-5-7)^2 + 3^2}$

$= \sqrt{144+9}$

$= \sqrt{153}$

$= 3\sqrt{17}$ units.

(iv) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3\sqrt{17} \times 4\sqrt{17}$

$= 6 \times 17$

$= 102$ units^2

b) $3x - 4y = 12$

$y = 0$, $\quad 3x = 12$

$x = 4$

$A(4, 0)$

Grad. $3x - 4y = 12$

$4y = 3x - 12$

$y = \frac{3}{4}x - 3$

$m_1 = \frac{3}{4}$

lines perp. $m_1 \times m_2 = -1$

$\frac{3}{4} \times m_2 = -1$

$m_2 = -\frac{4}{3}$

Eqn. of line.

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{4}{3}(x-4)$

$3y = -4x + 16$

$4x + 3y - 16 = 0$

Question 3

a)

$D = S \times T$

$= 106560 \times 365 \times 24$

$= 933465600$

$= 9.335 \times 10^8$ km

b) $0.\dot{3}\dot{4}$

$x = 0.343434\ldots$

$100x = 34.3434\ldots$

$99x = 34$

$x = \frac{34}{99}$

c) $\frac{3}{x+5} - \frac{2}{x-5}$

$= \frac{3(x-5) - 2(x+5)}{x^2 - 25}$

$= \frac{3x - 15 - 2x - 10}{x^2 - 25}$

$= \frac{x - 25}{(x+5)(x-5)}$ OR $\frac{x-25}{x^2-25}$

d) $x + x^{-1}$

$7 - 4\sqrt{3} + \frac{1}{7 - 4\sqrt{3}}$

$= 7 - 4\sqrt{3} + \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}$

$7 - 4\sqrt{3} + \frac{7+4\sqrt{3}}{49 - 16 \times 3}$

$= 7 - 4\sqrt{3} + 7 + 4\sqrt{3}$

$= 14$

e) $\left(\frac{1}{a} + \frac{1}{b}\right) \div \frac{1}{ab}$

$= \frac{b+a}{ab} \times \frac{ab}{1}$

$= b + a$

f) $\frac{(a+b)^3 - a^3}{b}$

$= [(a+b) - a][(a+b)^2 + a(a+b) + a^2]$

$= [a + b - a][\cancel{a^2} + 2ab + b^2 + \cancel{a^2} + ab + \cancel{a^2}]$

$= b[3a^2 + 3ab + b^2]$

$= 3a^2b + 3ab^2 + b^3$

(c). $y = 2x + 1$ — ①

$y = x^3 - 8x^2 + 14x + 1$ — ②

① = ②

$2x + \cancel{1} = x^3 - 8x^2 + 14x + \cancel{1}$

$x^3 - 8x^2 + 12x = 0$

$x(x^2 - 8x + 12) = 0$

$x(x-6)(x-2) = 0$

$x = 0, 6, 2$

Sub in ①

$x = 0, y = 1$

$x = 6, y = 13$

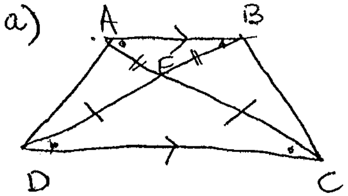
$x = 2, y = 5$

pts. of intersection

$(0,1)$ $(6,13)$ and $(2,5)$

Question 4

a)



In $\triangle DEC$

$E\hat{D}C = E\hat{C}D$ (equal $\angle$'s opp equal sides, isos $\triangle$)

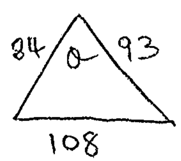
$A\hat{B}E = E\hat{D}C$ (alt. $\angle$'s $AB \parallel DC$)

$B\hat{A}E = D\hat{C}E$ (" ")

$\therefore \triangle AEB$ is isos (2 = angles)

$\therefore AE = EB$.

b)



$$\cos\theta = \frac{84^2 + 93^2 - 108^2}{2 \times 84 \times 93}$$

$$= 0{\cdot}2586\ldots$$

$$\theta = 75^\circ 1'$$

c) $\sec 405^\circ$

$$\theta = 405^\circ - 360^\circ$$
$$= 45^\circ$$

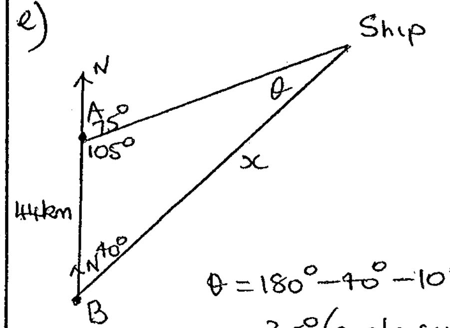
$$\frac{1}{\cos 405} = \frac{1}{\cos 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

ii)
$$\frac{\tan(-30^\circ)}{\tan 225^\circ}$$
$$= \frac{\tan 330^\circ}{\tan 225}$$
$$= \frac{-\tan 30^\circ}{\tan 45^\circ}$$
$$= \frac{-\frac{1}{\sqrt{3}}}{1}$$
$$= -\frac{1}{\sqrt{3}} \quad \text{or} \quad -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= -\frac{\sqrt{3}}{3}$$

d) $(1 - \cos^2\theta)(1 + \cot^2\theta)$

$$= \sin^2\theta \ \operatorname{cosec}^2\theta$$
$$= \frac{\sin^2\theta}{\sin^2\theta}$$
$$= 1$$

e)



$$\theta = 180^\circ - 40^\circ - 105^\circ$$
$$= 35^\circ \text{ (angle sum of } \triangle)$$

$$\frac{x}{\sin 105^\circ} = \frac{44}{\sin 35^\circ}$$

$$x = \frac{44 \sin 105^\circ}{\sin 35^\circ} = 74{\cdot}09$$

Question 5

a)
$$\frac{4 - 3x}{7} \leq 4$$

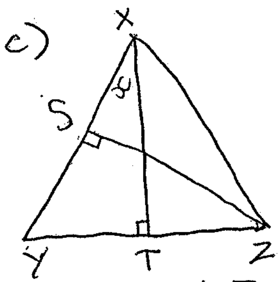
$$4 - 3x \leq 28$$
$$-3x \leq 24$$
$$x \geq \frac{24}{-3}$$
$$x \geq -8$$

-8 -6 -4 -2 0 2 4

b) $x^2 + 4x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{16 - 4 \times 1 \times -2}}{2}$$
$$= \frac{-4 \pm \sqrt{24}}{2}$$
$$= \frac{-4 \pm 2\sqrt{6}}{2}$$
$$= -2 \pm \sqrt{6}$$

c)



In $\triangle XYT$

$X\hat{Y}Z = 90^\circ - x^\circ$ (angle sum of $\triangle$)

In $\triangle SZY$

$S\hat{Z}Y = 90^\circ - (90^\circ - x)$

$= x^\circ$ (angle sum

(ii) In $\triangle XYT$ and $\triangle SYZ$

$Z\hat{S}Y = X\hat{T}Y$ (both 90°)

$X\hat{Y}Z = 90^\circ - x$ (common proved above)

$Y\hat{X}T = S\hat{Z}Y = x^\circ$ (proved above)

$\therefore \triangle XYT \parallel\!\mid \triangle SYZ$ (equiangular

$\therefore \frac{XY}{YZ} = \frac{XT}{ZS}$ (corresp. sides similar $\triangle$s)

d) $a^2 - 9b^2 + 4a - 12b$

$$= (a - 3b)(a + 3b) + 4(a - 3b)$$
$$= (a - 3b)(a + 3b + 4)$$

Question 6

a) $\tan\theta = \dfrac{\sqrt{1-a^2}}{a}$

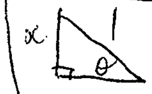
R.HS $= \dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

$= \dfrac{\sqrt{\sin^2\theta}}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$=$ L.HS.



$x^2 = 1^2 - a^2$

$x = \sqrt{1-a^2}$

$\therefore \tan\theta = \dfrac{\sqrt{1-a^2}}{a}$

b) R.H.S

$= \dfrac{\sin\theta - 1}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{\sin\theta - 1}{\sin^2\theta}$

$= \dfrac{\sin\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{1}{\sin^2\theta}$

$=$ L.HS.

c) $\cos^2\theta = \dfrac{1}{2}$

$\cos\theta = \pm\dfrac{1}{\sqrt{2}}$

acute$\theta = 45^\circ$

Quad ① $\theta = 45^\circ$

② $180^\circ - \theta$

$= 135^\circ$

③ $180^\circ + \theta$

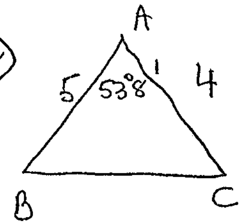
$= 225^\circ$

④ $360^\circ - \theta$

$= 315^\circ$

$\therefore \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$.

d)



Using Cosine Rule

$BC^2 = 5^2 + 4^2 - 2\times5\times4\times0\cdot6$

$= 17$

$BC = \sqrt{17}$.

$P = 5 + 4 + \sqrt{17}$

$= (9+\sqrt{17})$ units.

Question 7 (10 marks)

a) $4x^2 - mx + 9 = 0$

(i) $\Delta = 0$

$b^2 - 4ac = 0$

$m^2 - 4\times4\times9 = 0$

$m^2 = 16\times9$

$m = \pm12$

(ii) $x = 1$,

$4\times1^2 - m + 9 = 0$

$4 - m + 9 = 0$

$13 - m = 0$

$m = 13$

b) (i) $\alpha + \beta = -\dfrac{b}{a}$

$= \dfrac{-5}{2}$

(ii) $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{-8}{2}$

$= -4$

(iii) $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\dfrac{-5}{2}\right)^2 - 2\times-4$

$= 14\frac{1}{4}$

(c) $x^2 - (2a+b)x + ab = 0$

$\Delta \geq 0$

$b^2 - 4ac$

$= (2a+b)^2 - 4\times1\times ab$

$= 4a^2 + 4ab + b^2 - 4ab$

$= 4a^2 + b^2$

$4a^2 + b^2 \geq 0$

will always be ≥ 0

as it is a square $\therefore$

(d) $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{9}{4}$

$\alpha = 1, \quad \beta = \dfrac{9}{4}$

$\alpha + \beta = -\dfrac{b}{a}$

$1 + \dfrac{9}{4} = m$

Question (8)

a) $y = \sin x$

$y = \frac{1}{2}$

$\sin x = \frac{1}{2}$.

Quad (1) + (2)

acute $x = 30°$

$\therefore x° = 30°$

(2) $180° - \cancel{30°}$

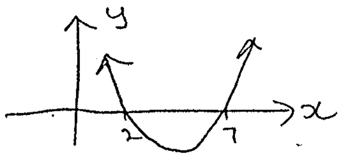
$= 150°$

$\therefore x° = 30°, 150°$

Soln. $(30°, \frac{1}{2})$ and $(150°, \frac{1}{2})$

b) $x^2 - 9x + 14 \le 0$

$(x-7)(x-2) \le 0$



$2 \le x \le 7$

c) $y = x^2 + 2x - 8$

$y=0, \quad x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = 2, -4$

$x=0, \ y = -8$

Axis of symm: $x = -\frac{b}{2a}$

$= -\frac{2}{2}$

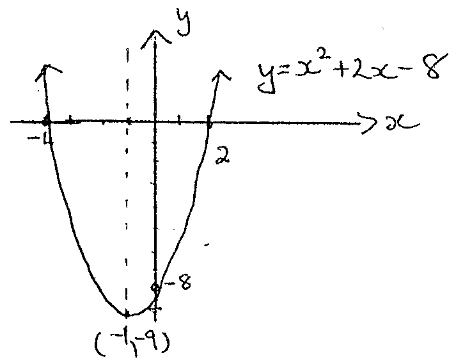
$= -1$

Vertex $x = -1$

$y = (-1)^2 + 2\times -1 - 8$

$= -9$

$(-1, -9)$



d) $y = 2x^2 + kx - 4$

$x = \frac{-b}{2a}$

$3 = \frac{-k}{4}$

$-k = 12$

$k = -12$

Ques (9) 12 marks

a) $x^2 - x + \frac{24}{x^2 - x} = 14$

Let $X = x^2 - x$

$X + \frac{24}{X} = 14$

$X^2 + 24 = 14X$

$X^2 - 14X + 24 = 0$

$(X-12)(X-2) = 0$

$X = 12, 2$

$\therefore x^2 - x = 12$ or $x^2 - x = 2$

$x^2 - x - 12 = 0$; $x^2 - x - 2 = 0$

$(x-4)(x+3) = 0$; $(x-2)(x+1) = 0$

$x = 4, -3$; $x = 2, -1$

$x = -1, -3, 2, 4$

b) i) $2x + l = 16$

$l = 16 - 2x$

$A = LB$

$= (16 - 2x)(x)$

$= 16x - 2x^2$

(ii) $y = 0, \quad 16x - 2x^2 = 0$

$2x(8 - x) = 0$

$x = 0, 8$. $\quad y = -2x^2 + 16x$

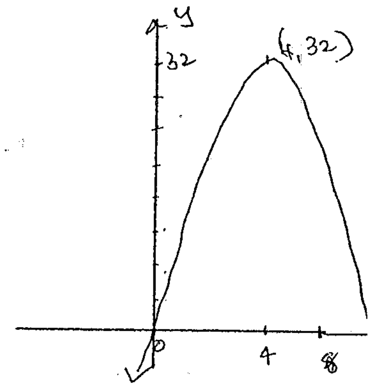
Axis of symm, $x = \frac{-b}{2a}$

$= \frac{+16}{4} = 4$

Vertex $x = 4$

$y = 16\times 4 - 2\times 4^2$

$= 32$



(iii) Max $= 32\,m^2$

(iv) $x = 4$

width $= 4m$

(length) $l = 16 - 2\times 4$

$= 8m$