Year 12
Extension 1 (Additional) and
Extension 1/Extension 2 (Common)
Mathematics

March 2002

Half-Yearly Examination

Time Allowed: 2 hours
(+ 5 minutes reading time)

Assessment Weighting: 20%

Total Marks: 85

Instructions:
- All questions may be attempted
- Each question is not of equal value
- All necessary working should be shown in every question
- Marks may not be awarded for careless or badly arranged work
- Approved calculators may be used
- The paper should be returned in seven (7) separate bundles

Outcomes assessed:

A student:

PE1 - appreciates the role of mathematics in the solution of practical problems

PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

PE4 - uses the parametric representation together with differentiation to identify geometric properties of parabolas

PE5 - determines derivatives which require the application of more than one rule of differentiation

HE2 - uses inductive reasoning in the construction of proofs

HE6 - determines integrals by reduction to a standard form through a given substitution

HE7 - evaluates mathematical solutions to problem and communicates them in an appropriate form

Question 1: (9 marks)

a) Solve the inequality:

$$\frac{1}{x-3}>2$$ (1.5 marks)

b) Solve:

$$e^{3\ln x}=5\ln e^{x}$$ (1.5 marks)

c) Differentiate with respect to x

i. $3x+\frac{1}{x}+e^{-x}$ (1.5 marks)

ii. e^{x^2} (1.5 marks)

d) Evaluate correct to 3 significant figures

$$\int_0^1 x e^{5x^2+2}\,dx$$ (3 marks)

Question 2: START A NEW PAGE (10 marks)

a) Evaluate exactly:

$$\frac{\sin\frac{\pi}{18}\cos\frac{\pi}{9}+\cos\frac{\pi}{18}\sin\frac{\pi}{9}}{\cos^2\frac{5\pi}{12}-\sin^2\frac{5\pi}{12}}$$ (2 marks)

b) Find $\lim_{x\to 0}\dfrac{\sin\frac{x}{3}}{4x}$ (2 marks)

c) Calculate the derivative of 5^x (2.5 marks)

d) i. Sketch the region in the second quadrant bounded by the curve $y=\sqrt{x+4}$ and the coordinate axes (0.5 marks)

ii. Express the area as a definite integral with respect to ***x*** (DO NOT calculate the area) (1 mark)

iii. Express the area as a definite integral with respect to ***y*** and hence, calculate the exact area

(2 marks)

Question 3: START A NEW PAGE (16 marks)

a) Using $u = x^2 + 9$, evaluate $\int_0^4 x\sqrt{x^2+9}\,dx$ (2 marks)

b) OAC is a sector subtending an angle of $82°20'$ at the centre O of a circle of radius 8 cm.

i. Find the measure of the angle in radians correct to 3 significant figures (1 mark)

ii. Find the length of the arc AC using your approximation from part i. (1 mark)

iii. Find the area of the sector to 3 significant figures using your approximation from part i. (1 mark)

iv. Find the area of the segment AC to 3 significant figures using your approximation from part i. (1 mark)

v. The sector is rolled so that the two sides OA and OC are joined together to form a cone. Find the volume of the cone correct to 2 decimal places using your approximation from part i. (3 marks)

c) Let $f(x) = \frac{x^2}{x-2}$.

i. For what value(s) of x is f(x) undefined? (1 mark)

ii. Find any stationary points of the curve y = f(x) and determine their nature (4 marks)

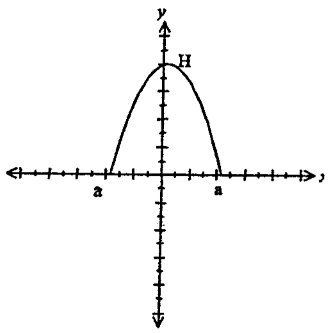
iii. Hence sketch y = f(x) (2 marks)

Question 4: START A NEW PAGE (10 marks)

a) The diagram below shows a symmetric parabolic arch of width 2a and height H.

i. Explain, using words, why Simpson's rule will give the exact area under the arch (1 mark)

ii. Use Simpson's Rule with 3 function values to show the area under the arch is $\frac{4}{3}aH$.



(1.5 marks)

b) Let $f(x) = \ln x - e^{-3x}$.

i. Write down the domain for f(x) (0.5 marks)

ii. Show that the curve y = f(x) cuts the x-axis between x = 1 and x = 1.5 (1.5 marks)

iii. Use Newton's method with a first approximation of x = 1 to find a second approximation to the root of $\ln x - e^{-3x} = 0$. Give your answer correct to 2 decimal places. (2 marks)

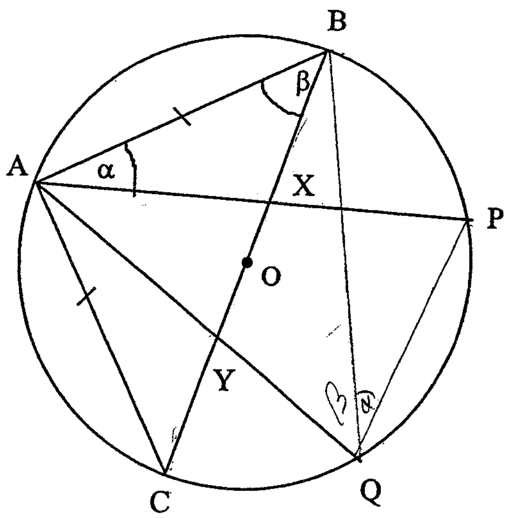
c) i. Solve $\sqrt{3}\sin\theta - \cos\theta = 1$ for θ , where $0 \le \theta \le 2\pi$ (1.5 marks)

ii. Find the general solution for $\sqrt{3}\sin\theta - \cos\theta = 1$ (2 marks)

Question 5: START A NEW PAGE (14 marks)

a) Using mathematical induction, show that $a^n - 1$ is divisible by $a - 1$, for n a positive integer (6 marks)

b)



Let ABPQC be a circle such that AB = AC, where BC is the diameter and O is the centre. AP meets BC at X and AQ meets BC at Y, as in the diagram. Let $\angle BAP = \alpha$ and $\angle ABC = \beta$.

i. Copy or trace the diagram and state why $\angle AXC = \alpha + \beta$. (0.5 marks)

ii. Show that $\angle BQP = \alpha$ (1 marks)

iii. Prove that $\angle BQA = \beta$ (1.5 marks)

iv. Prove that PQYX is a cyclic quadrilateral. (2 marks)

c) Consider $y = e^{kx}$ where k is a constant.

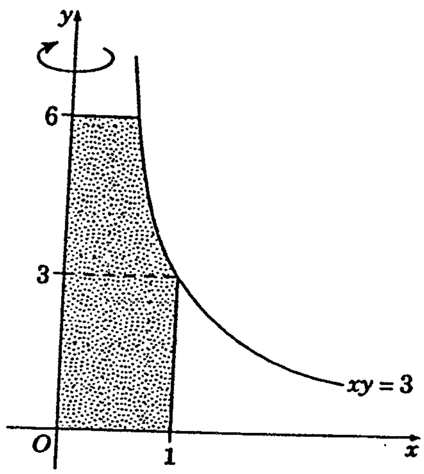
i. Find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ (1 mark)

ii. Determine the values of k for which $y = e^{kx}$ satisfies the equation

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 0$$ (2 marks)

Question 6: START A NEW PAGE (10.5 marks)

a) The shaded area is bounded by the curve xy = 3, the lines x = 1 and y = 6, and the two axes.



A solid is formed by rotating the shaded area about the y-axis. Find the volume of this solid by considering separately the regions above and below y = 3. (4 marks)

b) i. On the same axes, sketch the curves $y = \sin x$, $y = \cos x$, and $y = \sin x + \cos x$ for $0 \le x \le 2\pi$. (2 marks)

ii. From your graph, determine the number of values of x in the interval $0 \le x \le 2\pi$ for which sin x + cos x = 1 (1 mark)

iii. For what values of the constant k does sin x + cos x = k have exactly two solutions in the interval $0 \le x \le 2\pi$? (3.5 marks)

QUESTION 7: START A NEW PAGE (15.5 marks)

a) Points P(2ap, ap^2) and Q(2aq, aq^2) lie on the parabola $x^2 = 4ay$ and the chord PQ passes through (0,-4a).

i. Show that PQ has the equation $(p+q)x - 2y = 2apq$ (2 marks)

ii. Prove that pq = 4 (2 marks)

iii. Show that the equation of the normal at P is $x + py - 2ap - ap^3 = 0$. Hence find the equation of the normal at Q. (4 marks)

iv. Hence prove that the locus of T, the point of intersection of the normals at P and Q, is also a parabola. (7.5 marks)

Extension 1 Half-Yearly
ANSWERS

Question 1 (9 marks)

a) $(x-3)^2 \times \frac{1}{x-3} > 2(x-3)^2$

$x-3 > 2(x-3)^2$

$x-3 > 2(x^2-6x+9)$

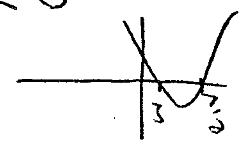
$x-3 > 2x^2-12x+18$

$0 > 2x^2-13x+21$

$0 > (2x-7)(x-3)$

$(2x-7)(x-3) < 0$

$3 < x < \frac{7}{2}$



b) $e^{3\ln x} = 5\ln e^x$

$x^3 = 5x$

$x^3 - 5x = 0$

$x(x^2-5) = 0$

$x=0$ no sol^n as $\ln 0$ is undefined

$x = +\sqrt{5}$ 1 sol^n

$x = -\sqrt{5}$ undefined as $\ln -\sqrt{5}$ is undefined

c) i. $3x + \frac{1}{x} + e^{-x}$

$\frac{dy}{dx} = 3 - x^{-2} - e^{-x}$

$= 3 - \frac{1}{x^2} - e^{-x}$

ii. e^{x^2}

$u = x^2$ $\quad y = e^u$

$\frac{du}{dx} = 2x$ $\quad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = e^u . 2x$

$= 2xe^{x^2}$

d) $\int_0^1 xe^{5x^2+2}\,dx$

$u = 5x^2+2$

$\frac{du}{dx} = 10x$

$dx = \frac{du}{10x}$

$\int_0^1 \cancel{x} e^u . \frac{du}{10\cancel{x}}$

$= \frac{1}{10}\int_0^1 e^u . du$

$= \frac{1}{10}\left[e^u\right]_0^1$

$= \frac{1}{10}\left[e^{5x^2+2}\right]_0^1$

$= \frac{1}{10}\left[e^{5.1^2+2} - e^{5.0+2}\right]$

$= \frac{1}{10}\left[e^7 - e^2\right]$

$= 108.9244\ldots$

$= 109$ (3 sig. figs)

OR

when $x=1$: $u = 5.1^2+2$
$= 7$

$x=0$ $\quad u = 5.0+2$
$= 2$

$\int_2^7 e^u\,du = \frac{1}{10}\left[e^u\right]_2^7$

$= \frac{1}{10}(e^7 - e^2)$

$= 108.9244\ldots$

$= 109$ (3 sig. figs)

(2)

Question 2 (10 marks)

a) $\dfrac{\sin\frac{\pi}{18}\cos\frac{\pi}{9} + \cos\frac{\pi}{18}\sin\frac{\pi}{9}}{\cos^2\frac{5\pi}{12} - \sin^2\frac{5\pi}{12}}$

$= \dfrac{\sin\left(\frac{\pi}{18}+\frac{\pi}{9}\right)}{\cos 2\left(\frac{5\pi}{12}\right)}$

$= \dfrac{\sin\frac{\pi}{6}}{\cos\frac{5\pi}{6}}$

$= \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= -\dfrac{1}{\sqrt{3}}$

$= -\dfrac{\sqrt{3}}{3}$

b) $\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{3}}{4x}$

$= \frac{1}{4}\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{3}}{x}$

$= \frac{1}{4}\lim\limits_{x\to 0} \dfrac{\frac{1}{3}\sin\frac{x}{3}}{x/3}$

$= \frac{1}{12}\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{3}}{x/3}$

$= \frac{1}{12}\times 1$

$= \frac{1}{12}$

c) $\dfrac{d}{dx} 5^x$

$5^x = e^{x\ln 5}$

$\dfrac{d}{dx}\left(e^{x\ln 5}\right)$?

$u = x\ln 5 \qquad y = e^u$

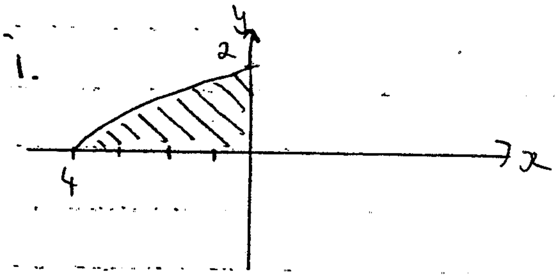
$\dfrac{du}{dx} = \ln 5 \qquad \dfrac{dy}{du} = e^u$

$\dfrac{dy}{dx} = e^u . \ln 5$

$= e^{x\ln 5} . \ln 5$

$= \ln 5 . 5^x$

d) i.



ii. $A = \displaystyle\int_{-4}^{0} (x+4)^{\frac{1}{2}}\, dx$

iii. $A = \left|\displaystyle\int_0^2 (y^2-4)\, dy\right|$

$= \left|\left[\dfrac{y^3}{3} - 4y\right]_0^2\right|$

$= \left|\dfrac{2^3}{3} - 4.2 - 0\right|$

$= \left|\dfrac{8}{3} - 8\right|$

$= \left|\dfrac{-16}{3}\right|$

$= \dfrac{16}{3}\ u^2$

Question 3 (16 marks)

a) $u = x^2 + 9$

$\dfrac{du}{dx} = 2x$

$dx = \dfrac{du}{2x}$

$\displaystyle\int_0^4 x . \sqrt{x^2+9}\, dx$

$= \displaystyle\int_0^4 \cancel{x} . u^{\frac{1}{2}} . \dfrac{du}{2\cancel{x}}$

$= \frac{1}{2}\int_0^4 u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left[\frac{2u^{3/2}}{3}\right]_0^4$

$= \frac{1}{3}\left[u^{3/2}\right]_0^4$

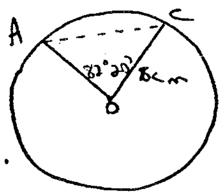
$= \frac{1}{3}\left[(x^2+9)^{3/2}\right]_0^4$

$= \frac{1}{3}\left[(4^2+9)^{3/2} - (0^2+9)^{3/2}\right]$

$= \frac{1}{3}\left[125 - 27\right]$

$= \frac{98}{3}$

b)



i. $82^\circ 20' \times \frac{\pi}{180} = 1.4369...$

$= 1.44$ rads (3 sig. figs)

ii. $l = r\theta$

$= 8 \times 1.44$

$= 11.52$cm

iii. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 8^2 \times 1.44$

$= 46.08$

$= 46.1\text{cm}^2$ (3 sig. figs)

iv. $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}\times 8^2(1.44 - \sin 1.44)$

$= 14.353...$

$= 14.4\text{cm}^2$ (3 sig figs)

v. Arc length is circumference of circular base

$11.52 = 2\pi R$

$R = \frac{11.52}{2\pi}$

$= 1.833$cm

$h^2 = 8^2 - 1.83^2$

$h = \sqrt{60.6511}$

$= 7.78788$

$= 7.79$cm

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\pi\times 1.833 \times 7.7878...$

$= 27.4153$

$= 27.42\text{cm}^3$

c) $f(x) = \frac{x^2}{x-2}$

i. $f(x)$ undefined at $x=2$

ii. $f(x) = \frac{x^2}{x-2}$

$u = x^2$ $\quad v = x-2$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 1$

$\frac{dy}{dx} = \frac{2x(x-2) - x^2}{(x-2)^2}$

$= \frac{2x^2 - 4x - x^2}{(x-2)^2}$

$= \dfrac{x^2-4x}{(x-2)^2}$

$f'(x) = 0$

$\dfrac{x^2-4x}{(x-2)^2} = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, 4$

$x = 0, \ y = 0$

$x = 4, \ y = 8$

$f''(x) = \dfrac{(2x-4)(x-2)^2 - 2(x-2)(x^2-4x)}{(x-2)^4}$

$= \dfrac{\cancel{(x-2)}\left[(2x-4)(x-2) - 2(x^2-4x)\right]}{(x-2)^{\cancel{4}\,3}}$

$= \dfrac{2x^2-4x-4x+8-2x^2+8x}{(x-2)^3}$

$= \dfrac{8}{(x-2)^3}$

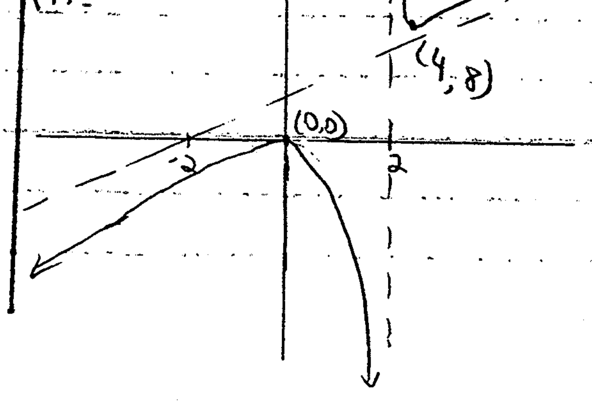
$f''(0) = \dfrac{8}{(-2)^3} = -1 < 0$

$\therefore$ max. t.p.

$f''(4) = \dfrac{8}{(4-2)^3} = 1 > 0$

$\therefore$ min t.p.

iii.



Question 4 (11 marks)

a) i. Simpson's rule is exact for quadratics

ii. $h = \dfrac{a-(-a)}{2}$

$= \dfrac{2a}{2}$

$= a$

$A = \dfrac{a}{3}\left[(0+0) + 4(H)\right]$

$= \dfrac{a}{3}\left[4H\right]$

$= \dfrac{4}{3}aH$, as req'd

b) $f(x) = \ln x - e^{-3x}$

i. D: $x > 0$, all real x

ii. $f(1) = \ln 1 - e^{-3}$

$= -0.0498\ldots$

$f(1.5) = \ln 1.5 - e^{-9/2}$

$= 0.3943\ldots$

$f(1)$ -ve $\quad f(1.5)$ +ve

$\therefore$ curve cuts x-axis between $x=1$ and $x=1.5$

iii. $x_1 = 1 \qquad x_2 = ?$

$f'(x) = \dfrac{1}{x} + 3e^{-3x}$

$x_2 = x_1 - \dfrac{f(x)}{f'(x)}$

$= 1 - \dfrac{\ln 1 - e^{-3}}{\frac{1}{1} + 3e^{-3\times 1}}$

$= 1.0433\ldots$

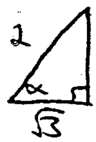
$= 1.04$ (2 dec pl)

c) i. $\sqrt{3}\sin\theta - \cos\theta = 1$

$\cos\alpha\sin\theta - \cos\theta\sin\alpha = 1$

$\cos\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \frac{1}{2}$



$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{2}$

$0 \le \theta \le 2\pi,\ 0 \le \alpha \le \frac{\pi}{2}$

$\sin\theta\cos\alpha - \cos\theta\sin\alpha = \frac{1}{2}$

$2\sin(\theta - \alpha) = 1$

$\sin(\theta - \alpha) = \frac{1}{2},\ \sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$

$\theta - \alpha = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$

$\theta - \frac{\pi}{6} = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{3},\ \pi$

ii. $\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\theta - \frac{\pi}{6} = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{3},\ \pi$

$\therefore x = n\pi + (-1)^n \cdot \frac{\pi}{3}$

$\theta = \pi, 2\pi + \pi, 4\pi + \pi, \ldots$

$\therefore \theta = (2n-1)\pi$

Question 5 (14 marks)

a) $a^n - 1$ ÷ by $a-1$ $n \ge 0$

Test $n=1$

$a^n - 1 = a - 1$ which is divisible by $a-1$

$\therefore$ result true for $n=1$

Assume result true for $n=k$

$a^k - 1 = (a-1)M$ where M is a constant

Hence show result true for $n=k+1$

$a^{k+1} - 1 = (a-1)P$

$a^{k+1} - 1 = a^{k+1} - a + a - 1$

$= a(a^k - 1) + 1(a-1)$

$= a(a-1)M + 1(a-1)$

$= (a-1)(aM+1)$

$= (a-1)P$ where $P = aM+1$

Hence if result true for $n=k$ then it is true for $n=k+1$

Since result true for $n=1$, then true for $n=2$, $n=3$ and so on for all +ve integers of n.

b) i. $\angle AXC = \alpha + \beta$ (exterior angles of Δ equals sum of interior opposite $\angle$s)

ii. $\angle BQP = \angle BAP$ ($\angle$s at circumference on same arc equal)

$\therefore \angle BQP = \alpha$

iii. Since $AC = AB$

ΔABC is isosceles

6

$\angle ACB = \angle ABC = \beta$

Also, $\angle BQA = \angle ACB$

($\angle$'s at circumference on same arc equal)

$\therefore \angle BQA = \beta$

iv. $\angle PXY + \angle AXY = 180°$

(straight line)

ie $\angle PXY + (\alpha + \beta) = 180°$

$\angle PXY + \angle PQY = 180°$

since $\angle PQY = \angle BQP + \angle BQA$

$= \alpha + \beta$

$\therefore PQYX$ is a cyclic quadrilateral (opp $\angle$'s of a cyclic quad are supplementary).

c) $y = e^{kx}$

i. $\frac{dy}{dx} = ke^{kx}$

$\frac{d^2y}{dx^2} = k.ke^{kx}$

$= k^2e^{kx}$

ii. $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 0$

$k^2e^{kx} + 7ke^{kx} + 12e^{kx} = 0$

$e^{kx}(k^2 + 7k + 12) = 0$

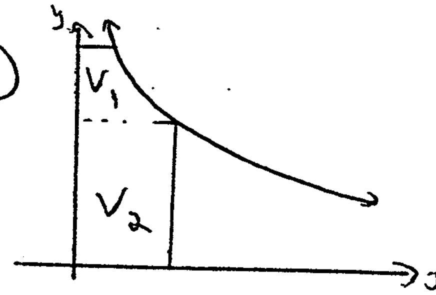
$k^2 + 7k + 12 = 0$

$(k+3)(k+4) = 0$

$k = -3, -4$

Question 6 (10.5 marks)

a)



$V_1 = \pi \int x^2 \, dy$

$= \pi \int_3^6 \left(\frac{3}{y}\right)^2 dy$

$= \pi \int_3^6 \frac{9}{y^2} dy$

$= 9\pi \int_3^6 \frac{1}{y^2} dy$

$= 9\pi \left[-\frac{1}{y}\right]_3^6$

$= 9\pi \left[-\frac{1}{6} + \frac{1}{3}\right]$

$= 9\pi \times \frac{1}{6}$

$= \frac{3\pi}{2}$ units^3

V_2 is a cylinder when rotated

$r = 1$, $h = 3$,

$V_2 = \pi r^2 h$

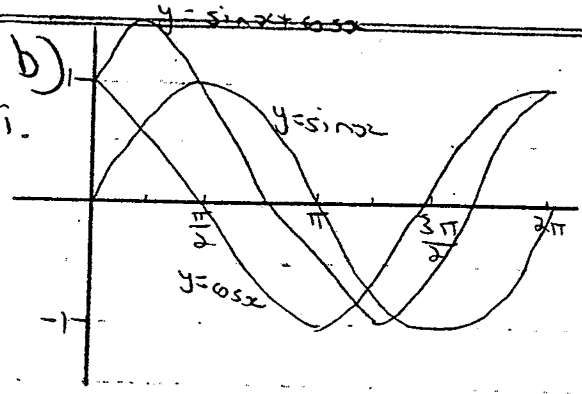
$= \pi \times 1^2 \times 3$

$= 3\pi$ units^3

Total $V = 3\pi + \frac{3\pi}{2}$

$= \frac{9\pi}{2}$ units^3

b) i.



ii. 3 times at $x = 0, \frac{\pi}{2}, 2\pi$

iii. minimum value at $x = \frac{5\pi}{4}$

$y = \sin x + \cos x$

$= \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4}$

$= -\frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)$

$= -\frac{2}{\sqrt{2}}$

$= -\sqrt{2}$

Maximum value at $x = \frac{\pi}{4}$

$y = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

$x = 0, \frac{\pi}{2}, 2\pi$

$y = 1$

∴ 2 sol^ns exactly

$1 < k < \sqrt{2}$

$-\sqrt{2} < k < 1$

Question 7 (15.5 marks)

i. $P(2ap, ap^2)$ $Q(2aq, aq^2)$

$x^2 = 4ay$

$m = \frac{aq^2 - ap^2}{2aq - 2ap}$

$= \frac{\cancel{a}(\cancel{q-p})(q+p)}{2\cancel{a}(\cancel{q-p})}$

$= \frac{p+q}{2}$

$y - y_1 = m(x - x_1)$

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$2y - 2ap^2 = (p+q)x - 2ap^2 - 2apq$

$\therefore (p+q)x - 2y = 2apq$ as req'd.

ii. $x^2 = 4ay$

focus $(0, -4a)$

$(p+q) \cdot 0 - 2(-4a) = 2apq$

$8a = 2apq$

$pq = \frac{8\cancel{a}}{2\cancel{a}}$

$\therefore pq = 4$ as req'd

iii. $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$

subs $x = 2ap$

$m = \frac{2ap}{\;} - p$

m of normal: $m_1 m_2 = -1$

$p \times m_2 = -1$

$m_2 = -\frac{1}{p}$

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$\therefore x + py - 2ap - ap^3 = 0$ as req'd

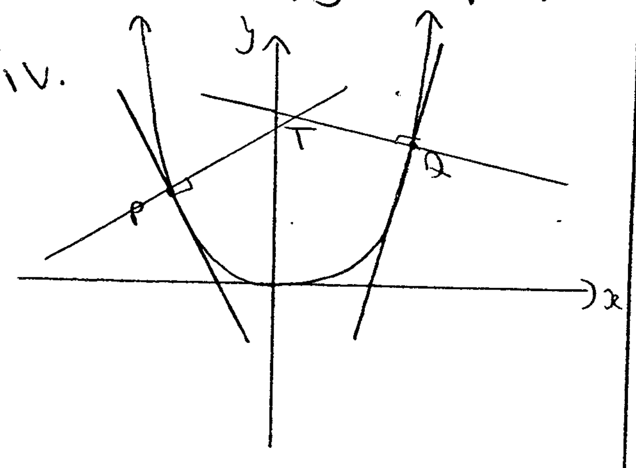
Similarly, gradient of normal at Q is $-\frac{1}{q}$

$y - aq^2 = -\frac{1}{q}(x - 2aq)$

$qy - aq^3 = -x + 2aq$

$\therefore$ equation of normal at Q is: $x + qy - 2aq - aq^3 = 0$

iv.



$x + py - 2ap - ap^3 = x + qy - 2aq - aq^3$

$py - qy - 2ap + 2aq - ap^3 + aq^3 = 0$

$y(p-q) - 2a(p-q) - a(p^3 - q^3) = 0$

$y(p-q) = 2a(p-q) + a(p-q)(p^2 + pq + q^2)$

$y = 2a + a(p^2 + pq + q^2)$

$= a(p^2 + pq + q^2 + 2)$

Subs y into $x + py = 2ap + ap^3$

$x = 2ap + ap^3 - ap(p^2 + pq + q^2 + 2)$

$= 2ap + ap^3 - ap^3 - ap^2q - apq^2 - 2ap$

$= -ap^2q - apq^2$

$= -apq(p+q)$

Now $pq = 4$

$x = -4a(p+q)$

$p + q = -\frac{x}{4a}$

So, $y = a[p^2 + pq + q^2 + 2]$

$= a[(p+q)^2 - pq + 2]$

$= a\left[\left(\frac{-x}{4a}\right)^2 - 4 + 2\right]$

$= a\left[\frac{x^2}{16a^2} - 2\right]$

$y = \frac{x^2}{16a} - 2a$

$\therefore$ Locus of T is a parabola because y is in form of $\frac{x^2}{4a} = y$ with vertex $(0, -2a)$ and focal length $4a$