HALF YEARLY EXAMINATION 2004

YEAR 12

MATHEMATICS COURSE

MARCH, 2004

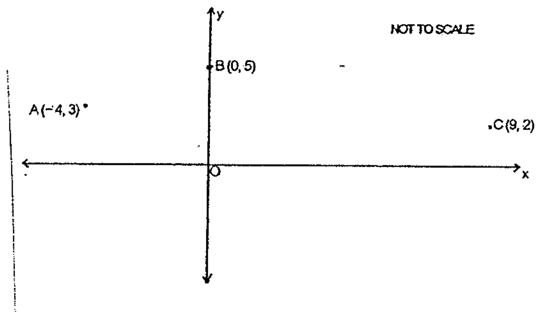
TIME ALLOWED: 3 HOURS
(plus 5 minutes reading time)

Instructions:

- Answer all questions on a new page
- STAPLE EACH QUESTION SEPARATELY
- Write your name and your teacher's name on the top of each question
- Show all necessary working.
- Write in pen.
- Approved calculators and geometrical instruments may be used.

QUESTION 1. (11 MARKS)

(a)



The diagram shows the origin O and the coordinates of the points A(-4, 3), B(0, 5) and C(9, 2).

(i) Find the exact length of the interval BC. 1.5

(ii) Show that the equation of the line k, drawn through A and parallel to BC is $x + 3y - 5 = 0$. 2

(iii) Find the coordinates of D, the point where the line k, meets the x axis. 1

(iv) Prove ABCD is a parallelogram. 2

(v) Find the perpendicular distance from the point B to the line k. 1.5

(vi) Hence, or otherwise find the area of ABCD. 1

(b) The first four terms of a series are 3, x, y and 192. Find the values of x and y if the series is arithmetic. 2

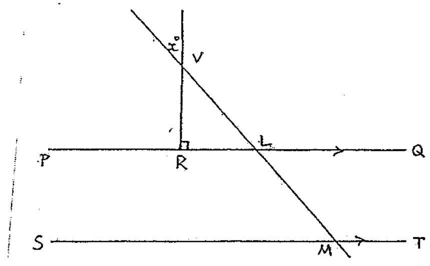
QUESTION 2 START A NEW PAGE (12 marks)

(a) Let $g(x) = \dfrac{1}{\sqrt{x+2}}$

(i) Find g(7). 1

(ii) What is the domain of g(x) ? 1

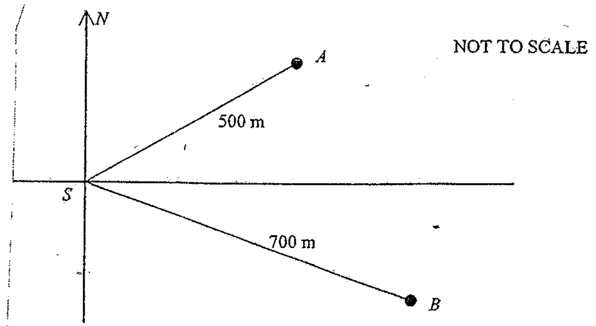
(b) In the diagram PQ || ST and RV ⊥ PQ.



(i) If $x = 25$ find the size of ∠VLR, giving reasons. 2

(ii) Hence or otherwise, find the size of ∠LMT, giving reasons. 2

(c)



From a ship at point S two buoys are observed, one at point A at a distance of 500 metres and a bearing of 043° T, the other at point B at a distance of 700 metres and a bearing of 118° T.

(i) Copy or trace the diagram into your writing booklet and mark on your diagram all the given information.

(ii) Show that ∠ASB is 75°. 1

(iii) Find the distance of buoy A from buoy B, correct to the nearest metre. 2

(iv) Find the bearing of buoy A from buoy B, correct to the nearest degree. 3

QUESTION 3 START A NEW PAGE (14 marks)

(a) The function $y = x^3 - 3x^2 - 9x + 1$ is defined in the domain $-2 \le x \le 5$.

(i) Find the coordinates of any turning points and determine their nature. 5.5

(ii) Find the coordinates of any points of inflexion. 2

(iii) Determine the minimum value of the function in the domain $-2 \le x \le 5$. 1.5

(b) Find the amount of interest earned on an investment of $4 500 over 5 years which earns 8.99% p.a. with interest compounded monthly. 2

(c) Consider the parabola $(x-1)^2 = -8(y-1)$

(i) Find the vertex and focus of the parabola. 2

(ii) Find the equation of another parabola with the same focal length, focus and axis of symmetry. 1

QUESTION 4 START A NEW PAGE (11 MARKS)

(a) If $y = 2x^3 - 7x^2 - 3x + 1$

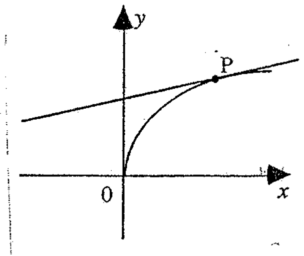
(i) Find $\dfrac{d^2y}{dx^2}$. 2

(ii) For what value of x is the curve $y = 2x^3 - 7x^2 - 3x + 1$ concave up ? 1.5

(b) A series $3y + 6y^2 + 12y^3$....... has a limiting sum of 12. Find the value of y. 2.5

(c) Evaluate $\displaystyle\sum_{n=1}^{4} \frac{1}{n(n+1)}$. 2

(d)



A sketch of $y = 2\sqrt{x}$ is shown with a tangent drawn at P.

(i) Find $\dfrac{dy}{dx}$. 1

(ii) Show that if the gradient at P is 3, then the coordinates of P are $\left(\dfrac{1}{9}, \dfrac{2}{3}\right)$. 2

QUESTION 5 START A NEW PAGE (12 Marks)

(a) If $\dfrac{d^2y}{dx^2} = 6x - 2$ and when $x = 1$, $y = 2$ and $\dfrac{dy}{dx} = 0$ find y in terms of x. 3.5

(b) Differentiate with respect to x :

(i) $y = (3x + 7)^5$ 1.5

(ii) $y = \dfrac{4 - x^2}{3x + 4}$ 3

(c) For this geometric series 2, -6, 18, ...-486 find :

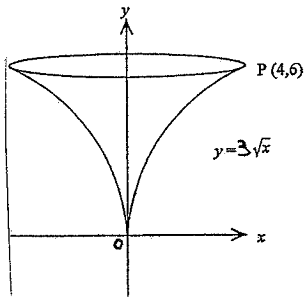
(i) how many terms are in the series. 2

(ii) the sum of the series. 2

QUESTION 6 START A NEW PAGE (11 Marks)

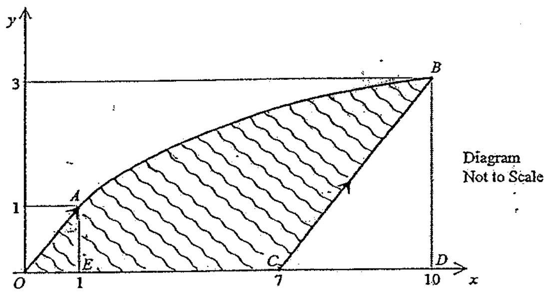
(a) Evaluate $\displaystyle\lim_{x \to 5} \frac{2x^2 - 9x - 5}{x - 5}$ 1.5

(b) The sketch represents a vase. It is generated by rotating the curve about the y axis. 3.5



Calculate the volume of water in the vase when it is filled to the top.

(c)



OA and CB are parallel segments, and AB is a segment of the curve $y = 1 + 2\log_{10} x$ between $x = 1$ and $x = 10$.

(i) Copy and complete the table for $y = 1 + 2\log_{10} x$, giving answers correct to 2 decimal places. 1

x	1	4	7	10
y	1			3

(ii) Hence use the Trapezoidal Rule with 4 function values to find an approximation to the area ABDE. 3

(iii) Calculate the area of the shaded region OABC correct to one decimal place. 2

QUESTION 7. START A NEW PAGE. (11.5 Marks)

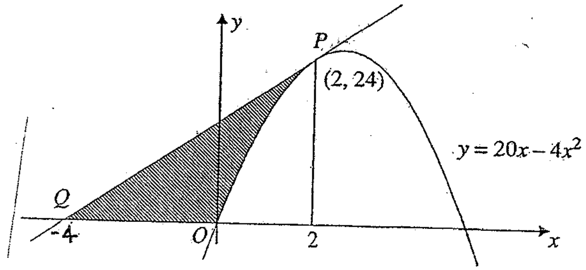
(a) Find:

(i) $\int (2x-3)^6 \, dx$ 2

(ii) $\int_1^4 x\sqrt{x} \, dx$ 2.5

(iii) $\int \frac{x^5 - 1}{x^2} \, dx$ 2

(b)



In the graph, a tangent is drawn at P(2, 24) on the parabola $y = 20x - 4x^2$. The tangent intersects the x axis at Q(-4, 0).

(i) Show that the equation of the tangent is $4x - y + 16 = 0$. 2

(ii) Find the area of the shaded region POQ. 3

QUESTION 8. START A NEW PAGE. (13 Marks)

(a) To prepare the Beach Völleyball courts on Bondi Beach, a truck is tipping loads of sand in the area.
For the first load of sand, the truck travels a total distance of 800 metres from the loading point to the courts and back.
For the second load the distance is 820 metres and each succeeding load is 20 metres further that the previous one.

(i) How far does the truck travel on the 8th load ? 1

(ii) On which trip would the truck travel 1.4 kilometres ? 2

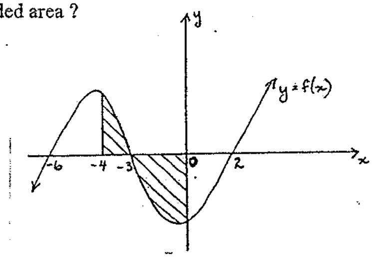
(iii) At lunchtime the driver found that the truck had travelled a total distance of 27.3 kilometres. How many loads had he tipped ? 4

(b) Consider the equation $x^2 + (m-3)x + m = 0$

(i) For what value of m does the equation have roots which are opposites of each other? 1

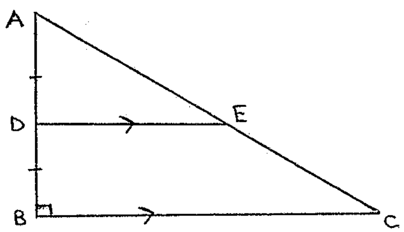
(ii) For what values of m does the equation have two distinct roots ? 3

(c) Write down the integral or integrals you would have to use to find the shaded area ? 2



QUESTION 9 START A NEW PAGE (15.5 marks)

(a) Triangle ABC has a right angle at B, D is the midpoint of AB and DE || BC.

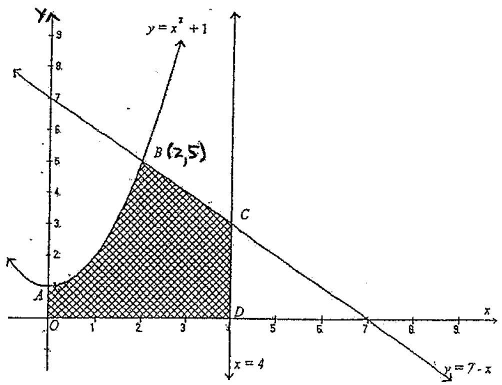


(i) Find the size of $\angle$ADE, giving a reason. 1

(ii) Prove $\Delta AED \equiv \Delta BED$. 2.5

(iii) Prove that BE = EC. 4.5

(b) In the diagram, the shaded region OABCD is bounded by $y = x^2 + 1$ the lines $y = 7 - x$, $x = 4$ and the x and y axes. 3.5



Use Simpson's rule with 4 strips to estimate the area of the shaded region.

(c) Miss Series wishes to buy a home unit. She obtained a loan of $120 000 from a bank which she agreed to repay by equal monthly repayments; compound interest is calculated at a fixed rate of 7.2% p.a.

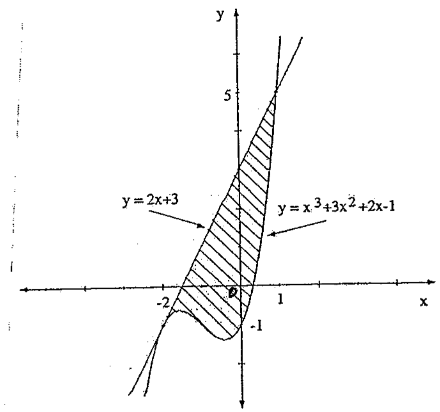
(i) If the monthly repayment is $M show that $A_1 = \$(120\,000 \times 1.006 - M)$ is the amount owing at the end of one month. 0.5

(ii) Show that $A_2 = \$(120\,000 \times 1.006^2 - M(1.006 + 1))$ is the amount owing at the end of two months. 1

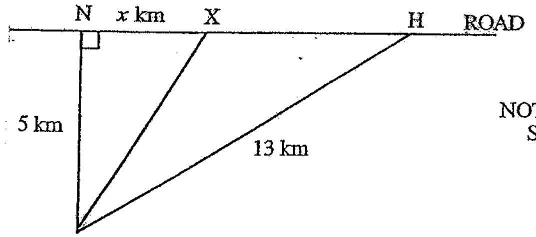
(iii) If the $120 000 loan (including interest charges) is exactly repaid at the end of 25 years, calculate the monthly repayment. 2.5

QUESTION 10 START A NEW PAGE (11 Marks)

(a) In the diagram below, $y = 2x + 3$ is a tangent to the curve $y = x^3 + 3x^2 + 2x - 1$ at the point (-2, -1) and cuts the curve again at (1, 5). Find the area enclosed by these two curves. 4



(b)



A bushwalker is in a forest at B, 5 kilometres from N on a road with a house at H. The bushwalker intends to travel directly through the forest to X which is on the road between N and H and then walk along the road to the house at H which is 13 km from B.

(i) If X is x km from N along the road, show that $BX = \sqrt{25 + x^2}$ km and that $XH = (12 - x)$ km. 2

(ii) The bushwalker can travel at 3 km per hour through the forest and 5 km per hour on the road. The total time t hours taken to travel from B to X and then to H is given by : 2

$$t = \frac{\sqrt{25 + x^2}}{3} + \frac{12 - x}{5}$$

Find $\frac{dt}{dx}$.

(iii) Show that the least amount of time taken to reach the house will occur when $x = 3\frac{3}{4}$. 3

Yr 12 HALF YEARLY 2004

Question 1.

(a) $BC = \sqrt{(9-0)^2 + (2-5)^2}$
$= \sqrt{9^2 + (-3)^2}$
$= \sqrt{90}$
$= 3\sqrt{10}$

(b) $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2-5}{9-0}$
$= -\frac{3}{9}$
$= -\frac{1}{3}$

Eqn of k $y - y_1 = m(x - x_1)$
$m = -\frac{1}{3}$ $(-4, 3)$
$y - 3 = -\frac{1}{3}(x - -4)$
$y - 3 = -\frac{1}{3}(x + 4)$
$3y - 9 = -x - 4$
$x + 3y - 5 = 0$

(c) cuts x axis when $y = 0$
$x + 3 \times 0 - 5 = 0$
$x = 5$
D $(5, 0)$

(d) $AD = \sqrt{(-4-5)^2 + (3-0)^2}$
$= \sqrt{(-9)^2 + 3^2}$
$= \sqrt{90}$

$\therefore$ AD = BC
and since BC // AD (from b)
ABCD is a parallelogram.

(e) $x + 3y - 5 = 0$ $(0, 5)$
$d = \frac{|0 + 3 \times 5 - 5|}{\sqrt{1^2 + 3^2}}$
$= \frac{|10|}{\sqrt{10}}$
$= \frac{10}{\sqrt{10}}$ units

(f) $A = b \cdot h$
$= 3\sqrt{10} \times \frac{10}{\sqrt{10}}$
$= 30$ units^2

(b) $3, x, y, 192$
$x - 3 = y - x$
$2x - y = 3$ .... (i)

$y - x = 192 - y$
$-x + 2y = 192$ ....(ii)

(i) × 2
$4x - 2y = 6$ ...(iii)
(ii) + (iii)
$3x = 198$
$x = 66$
subst $x = 66$ into (i)
$2 \times 66 - y = 3$
$132 - y = 3$
$y = 129$

$\therefore x = 66, y = 129$

Question 2

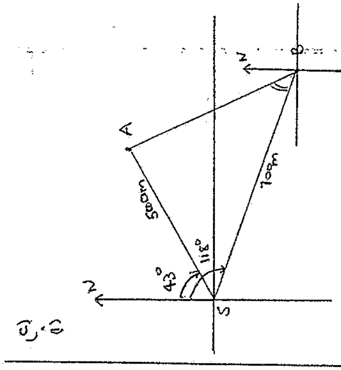
(a) (i) $g(7) = \frac{1}{\sqrt{7+2}}$
$= \frac{1}{\sqrt{9}}$
$= \frac{1}{3}$

(ii)
$x + 2 > 0$
$x > -2$

(b) (i) $\angle LVR = 25°$ (vert. opp. ∠s)
$\angle VLR = 180 - 90 - 25$ (∠ sum of Δ)
$= 65°$

(ii) $\angle RLM = 115°$ (adj supp ∠s) or (straight ∠)
$\angle LMT = 115°$ (alt ∠s PQ // ST)

(c) (i)



(ii) $\angle ASB = 118° - 43°$
$= 75°$ ← given

(iii)
$AB^2 = 500^2 + 700^2 - 2 \times 500 \times 700 \cos 75$
$= 558826.6684...$
$AB = \sqrt{558826.6684...}$
$= 747.547...$
$= 748$ m

(iv) $\frac{\sin \angle ABS}{500} = \frac{\sin 75}{747.547...}$

$\sin \angle ABS = \frac{500 \sin 75}{747.547...}$
$= 0.6406....$
$\angle ABS = 40°$

$\therefore$ Bearing $= 270 + 28 + 40$
$= 338°$

Question 3

(a) (i) $y = x^3 - 3x^2 - 9x + 1$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

For stat pts $\frac{dy}{dx} = 0$

$3x^2 - 6x - 9 = 0$
$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = 3, -1$

when $x = 3$ $\quad y = 3^3 - 3(3)^2 - 9(3) + 1$
$= -26$

$(3, -26)$

when $x = -1$ $\quad y = (-1)^3 - 3(-1) - 9(-1) + 1$
$= 6$

$(-1, 6)$

Test $\quad \frac{d^2y}{dx^2} = 6x - 6$

at $x = 3$
$\frac{d^2y}{dx^2} = 6 \times 3 - 6$
$= 12$
> 0
concave up

$\therefore$ Minimum stat pt at $(3, -26)$

Test at $x = -1$
$\frac{d^2y}{dx^2} = 6 \times -1 - 6$
$= -12$
< 0
concave down

$\therefore$ Maximum stat pt $(-1, 6)$

(ii) points of inflexion occur when $\frac{d^2y}{dx^2} = 0$

$6x - 6 = 0$
$6x = 6$
$x = 1$

Test

x	0	1	2
$\frac{d^2y}{dx^2}$	$-$	0	$+$

$\therefore$ change in concavity

when $x = 1$ $\quad y = 1^3 - 3(1)^2 - 9(1) + 1$
$= -10$

$\therefore$ point of inflexion at $(1, -10)$

(iii) when $x = -2$
$y = (-2)^3 - 3(-2)^2 - 9(-2) + 1$
$= -1$

when $x = 5$
$y = 5^3 - 3(5)^2 - 9(5) + 1$
$= 6$

$\therefore$ Minimum value $= -26$

(b) $5 \times 12 = 60$

$\frac{8.99}{12}\%$

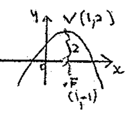
$A = \$4500\left(1 + \frac{8.99}{1200}\right)^{60}$

$= \$7042.068\ldots$
$= \$7042.07$

Interest $= \$7042.07 - \$4500$
$= \$2542.07$

(c) (i) Vertex $(1, 1)$

$4a = 8$
$a = 2$



Focus $(1, -1)$

(ii) $(x-1)^2 = 8(y+3)$

Question 4

(a) $y = 2x^3 - 7x^2 - 3x + 1$

(i) $\frac{dy}{dx} = 6x^2 - 14x - 3$

$\frac{d^2y}{dx^2} = 12x - 14$

(ii) For concave up $\frac{d^2y}{dx^2} > 0$

$12x - 14 > 0$
$12x > 14$
$x > \frac{7}{6}$

(b) $S_\infty = \frac{a}{1-r}$

$a = 3y$

$r = \frac{6y^2}{3y}$
$= 2y$

$12 = \frac{3y}{1-2y}$

$12(1-2y) = 3y$
$12 - 24y = 3y$
$27y = 12$
$y = \frac{4}{9}$

(c) $\sum_{n=1}^{4} \frac{1}{n(n+1)}$

$= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}$

$= \frac{4}{5}$

(d) (i) $y = 2\sqrt{x}$
$= 2x^{\frac{1}{2}}$

$\frac{dy}{dx} = x^{-\frac{1}{2}}$

(ii) $x^{-\frac{1}{2}} = 3$

$\frac{1}{\sqrt{x}} = 3$

$\sqrt{x} = \frac{1}{3}$

$x = \frac{1}{9}$ $\leftarrow$ given

Subst $x = \frac{1}{9}$ into y

$y = 2 \times \sqrt{\frac{1}{9}}$

$= \frac{2}{3}$ $\leftarrow$ given

$\therefore P\left(\frac{1}{9}, \frac{2}{3}\right)$ $\leftarrow$ given

Question 5

(a) $\frac{d^2y}{dx^2} = 6x - 2$

$\frac{dy}{dx} = 3x^2 - 2x + c$

subst $x = 1$ $\frac{dy}{dx} = 0$

$0 = 3 - 2 + c$
$c = -1$

$\therefore \frac{dy}{dx} = 3x^2 - 2x - 1$

$y = x^3 - x^2 - x + k$

subst $x = 1$ $y = 2$

$2 = 1 - 1 - 1 + k$
$k = 3$

$\therefore y = x^3 - x^2 - x + 3$

(b) (i) $\frac{dy}{dx} = 5(3x+7)^4 \times 3$

$= 15(3x+7)^4$

(ii) $u = 4 - x^2$ $\quad v = 3x + 4$
$u' = -2x$ $\quad v' = 3$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{(3x+4) \times -2x - (4 - x^2)3}{(3x+4)^2}$

$= \frac{-6x^2 - 8x - 12 + 3x^2}{(3x+4)^2}$

$= \frac{-3x^2 - 8x - 12}{(3x+4)^2}$

(c) (i) $T_n = ar^{n-1}$

$a = 2$ $\quad T_n = -486$

$r = \frac{-6}{2}$
$= -3$

$2 \times (-3)^{n-1} = -486$

$(-3)^{n-1} = -243$

$(-3)^{n-1} = (-3)^5$

$n - 1 = 5$
$n = 6$

$\therefore$ 6 terms in series.

(ii) $S_n = \frac{a(1-r^n)}{1-r}$

$S_6 = \frac{2(1-(-3)^6)}{1--3}$

$= \frac{2 \times -728}{4}$

$= -364$

Question 6

(a) $\lim_{x \to 5} \frac{(2x+1)(x-5)}{x-5}$

$= \lim_{x \to 5} 2x + 1$

$= 2 \times 5 + 1$

$= 11$

(b) $y = 3\sqrt{x}$

$\frac{y}{3} = \sqrt{x}$

$x = \frac{y^2}{9}$

$x^2 = \frac{y^4}{81}$

$$\text{Volume} = \pi\int_0^6 \frac{y^4}{81}\,dy$$

$$= \pi\left[\frac{y^5}{81\times5}\right]_0^6$$

$$= \pi\left[\frac{y^5}{405}\right]_0^6$$

$$= \pi\left[\frac{6^5}{405} - 0\right]$$

$$= \frac{96\pi}{5} \text{ cubic units}$$

(c) (i)

x	1	4	7	10
y	1	2.20	2.69	3

(ii) 4 function values = 3 strips

$$\text{Area} = \int_1^{10} 1 + 2\log_{10} x\,dx$$

$$= \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + y_3\right]$$

$$\doteq \frac{3}{2}\left[1 + 2\times2.2 + 2\times2.69 + 3\right]$$

$$= 20.67 \text{ units}^2.$$

(iii)

Area DABC

= Area $\triangle$OAE + Area ABCD − Area $\triangle$BCD

$= \frac{1}{2}\times1\times1 + 20.67 - \frac{1}{2}\times3\times3$

$= 16.66\ldots \text{ units}^2$

$= 16.7 \text{ units}^2$

Question 7

(a) (i) $\int (2x-3)^6\,dx$

$$= \frac{(2x-3)^7}{2\times7} + c$$

$$= \frac{(2x-3)^7}{14} + c$$

(ii) $\int_1^4 x\sqrt{x}\,dx$

$$= \int_1^4 x^{\frac{3}{2}}\,dx$$

$$= \left[\frac{2\cdot x^{5/2}}{5}\right]_1^4$$

$$= \frac{2}{5}\left[4^{5/2} - 1^{\frac{5}{2}}\right]$$

$$= 12\tfrac{2}{5}$$

(iii) $\int \frac{x^5 - 1}{x^2}\,dx$

$$= \int x^3 - x^{-2}\,dx$$

$$= \frac{x^4}{4} - \frac{x^{-1}}{-1} + c$$

$$= \frac{x^4}{4} + x^{-1} + c$$

(b) (i) $y = 20x - 4x^2$

$y' = 20 - 8x$

at $x = 2$

$y' = 20 - 8\times2$

$= 4$

Eqn of tangent

$m = 4 \quad (2, 24)$

$y - y_1 = m(x - x_1)$

$y - 24 = 4(x - 2)$

$y - 24 = 4x - 8$

$4x - y + 16 = 0$ (given)

(ii)

$$\text{Area} = \text{Area of } \Delta - \int_0^2 20x - 4x^2\,dx$$

$$= \frac{1}{2}\times6\times24 - \left[10x^2 - \frac{4x^3}{3}\right]_0^2$$

$$= 72 - \left[10\times2^2 - \frac{4\times2^3}{3} - 0\right]$$

$$= 42\tfrac{2}{3} \text{ units}$$

Question 8.

(a)

(i) $T_8 = a + 7d$

$= 800 + 7\times20$

$= 940$ m

(ii) 1.4 km = 1400m

$T_n = a + (n-1)d$

$800 + (n-1)20 = 14000$

$800 + 20n - 20 = 14000$

$780 + 20n = 1400$

$20n = 620$

$n = 31$

∴ the trip is number 31.

(iii) 27.3 km = 27300 m

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\frac{n}{2}\left[2\times800 + (n-1)20\right] = 2730[illegible]$$

$n(1600 + 20n - 20) = 54600$

$n(1580 + 20n) = 54600$

$20n^2 + 1580n - 54600 = 0$

$n^2 + 79n - 2730 = 0$

$$n = \frac{-79 \pm \sqrt{79^2 - 4\times1\times-2730}}{2}$$

$$= \frac{-79 \pm \sqrt{17161}}{2}$$

$$= \frac{-79 \pm 131}{2}$$

$= 26$ as $n > 0$

∴ 26 loads

(b) (i) $\alpha + \beta = -\frac{b}{a}$

$-\frac{b}{a} = 0$ For equal roots which are opp in sign

$m - 3 = 0$

$m = 3$

(ii) unequal roots $\Delta > 0$

$(m-3)^2 - 4\times1\times m > 0$

$m^2 - 6m + 9 - 4m > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$

$m < 1 \qquad m > 9$

(c)

$$\text{Area} = \int_{-4}^{-3} f(x)\,dx + \left|\int_{-3}^{0} f(x)\,dx\right|$$

Question 9

(a) (i)

$\angle ADE = 90°$ (corresp $\angle$s DE ∥ BC)

(ii) In $\triangle$AED, $\triangle$BED

DE is common

$\angle ADE = \angle BDE = 90°$ (straight $\angle$)

AD = BD given

∴ $\triangle AED \equiv \triangle BED$ SAS

(iii)

$\angle AED = \angle BED$ (corresp $\angle$s in cong Δs)

$\angle AED = \angle ECB$ (corresp $\angle$s BC ∥ DE)

$\angle BED = \angle EBC$ (alt $\angle$s BC ∥ DE)

∴ $\angle ECB = \angle EBC$

∴ $\triangle$ECB is isosceles ([illegible])

∴ BE = EC (opp sides = $\angle$ in isos $\triangle$EBC)

(b)

$$\text{Area} \doteq \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{1}{3}\left[1 + 4\times2 + 2\times5 + 4\times4 + 3\right]$$

$= 12\,^2/_3$ units.

(c) (i) $r = \frac{7.2}{12}\%$

$= 0.6\%$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A_1 = \$120\,000\left(1 + \frac{0.6}{100}\right)^1 - M$$

$= \$(120000 \times 1.006 - M)$ given

(ii) $A_2 = A_1 \times 1.006 - M$

$= (120\,000\times1.006 - M)\times1.006 - M$

$= \$120\,000\times1.006^2 - 1.006M - M$

$= \$(120\,000\times1.006^2 - M(1.006 + 1))$

(iii) $25\times12 = 300$

$$A_{300} = \$120000\times1.006^{300} - M(1.006^{299} + 1.006^{298} + \ldots + 1.006 + [illegible]$$

but $A_{300} = 0$.

$$\$120000\times1.006^{300} - M(1 + 1.006 + \ldots + 1.006^{[illegible]}) = 0$$

$$120\,000\times1.006^{300} = M(1 + 1.006 + \ldots [illegible]$$

$$\frac{M\times1(1.006^{300} - 1)}{1.006 - 1} = 120000\times1.006^{300}$$

$$M = \$120\,000\times1.006^{300}\times\frac{0.006}{1.006^{300} - 1}$$

$= \$863.51$.

Question 10

$$\text{(a) Area} = \int_{-2}^{1}\left[2x + 3 - (x^3 + 3x^2 + 2x - 1)\right]$$

$$= \int_{-2}^{1} 2x + 3 - x^3 - 3x^2 - 2x + 1\,dx$$

$$= \int_{-2}^{1} -x^3 - 3x^2 + 4\,dx$$

$$= \left[\frac{-x^4}{4} - x^3 + 4x \right]_{-2}^{1}$$

$$= \left[\left(-\frac{1}{4} - 1 + 4\right) - \left(\frac{-(-2)^4}{4} - (-2)^3 + 4 \times -2\right) \right]$$

$$= -\frac{1}{4} - 1 + 4 + 4 - 8 + 8$$

$$= 6\frac{3}{4} \text{ units}^2$$

(b)

(i) $(BX)^2 = 5^2 + x^2$

$(BX)^2 = 25 + x^2$

$BX = \sqrt{25 + x^2}$ given

$BH^2 = (NB)^2 + (NH)^2$

$13^2 = 5^2 + (NH)^2$

$(NH)^2 = 169 - 25$

$= 144$

$NH = 12$

$NH - x = XH$

$\therefore XH = 12 - x$ (given)

(ii) $t = \frac{(25 + x^2)^{\frac{1}{2}}}{3} + \frac{12 - x}{5}$

$$\frac{dt}{dx} = \frac{1}{2} \frac{(25 + x^2)^{-\frac{1}{2}}}{3} \times 2x - \frac{1}{5}$$

$$= \frac{x}{3}(25 + x^2)^{-\frac{1}{2}} - \frac{1}{5}$$

(iii) For minimum amount $\frac{dt}{dx} = 0$

$$\frac{x}{3}(25 + x^2)^{-\frac{1}{2}} - \frac{1}{5} = 0$$

$$\frac{x}{3\sqrt{25 + x^2}} = \frac{1}{5}$$

$$5x = 3\sqrt{25 + x^2}$$

$$\frac{5x}{3} = \sqrt{25 + x^2}$$

$$\frac{25x^2}{9} = 25 + x^2$$

$$\frac{16}{9}x^2 = 25$$

$$x^2 = \frac{225}{16}$$

$x = \frac{15}{4}$ as $x > 0$

Test

x	3	$\frac{15}{4}$	4
$\frac{dt}{dx}$	−	0	+

\ — /

$\therefore$ Minimum amount of time when $x = \frac{15}{4}$

OR

$$\frac{dt}{dx} = \frac{x}{3}(25 + x^2)^{-\frac{1}{2}} - \frac{1}{5}$$

at $x = 3\frac{3}{4}$

$$\frac{dt}{dx} = \frac{3\frac{3}{4}}{3}\left[25 + \left(3\frac{3}{4}\right)^2\right]^{-\frac{1}{2}} - \frac{1}{5}$$

$$= \frac{1}{5} - \frac{1}{5}$$

$$= 0$$

since $\frac{dt}{dx} = 0$ at $x = 3\frac{3}{4}$

$x = 3\frac{3}{4}$ is a stat pt.

Test $x = 3\frac{3}{4}$ as in method 1 $\Rightarrow$ $\frac{dt}{dx} = \frac{x}{3(25 + x^2)^{\frac{1}{2}}} - \frac{1}{5}$

OR

$$\frac{d^2t}{dx^2} = \frac{3(25 + x^2)^{\frac{1}{2}} \times 1 - x \times 3 \times \frac{1}{2}(25 + x^2)^{-\frac{1}{2}} \times 2x}{9(25 + x^2)} - 0$$

$$= \frac{3(25 + x^2)^{\frac{1}{2}} - 3x^2(25 + x^2)^{-\frac{1}{2}}}{9(25 + x^2)}$$

$$= \frac{3(25 + x^2) - 3x^2}{9(25 + x^2)^{3/2}}$$

at $x = 3\frac{3}{4}$

$$= \frac{3\left[25 + \left(3\frac{3}{4}\right)^2\right] - 3 \times \left(3\frac{3}{4}\right)^2}{9\left(25 + \left(3\frac{3}{4}\right)^2\right)^{\frac{3}{2}}}$$

$= \frac{64}{1875}$ or $0.0341\dot{3}$

> 0

$\therefore$ concave up

$\therefore$ a minimum amount of time is taken when $x = 3\frac{3}{4}$ s