

MATHEMATICS FORM II

Time allowed: 1 hour 30 minutes.

Exam date: 16th May 2002

Instructions:

- All questions may be attempted.
- All questions are of equal value.
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Calculators are NOT to be used.

Collection:

- Staple all your paper in one bundle.
- Write your name, class and master's initials on the front.
- Papers will be collected in class sets:

IJA: GJ	IIB: MLS	IIC: JMR
IID: WMP	IIE: JNC	IIF: REN
IIG: TCW	IIH: BDD	III: DS

Checklist:

- Writing paper required.

QUESTION ONE

(a) Simplify the following:

(i)  $45 \div (-9)$ ,

(ii)  $5 - 6 \div 3$ ,

(iii)  $\frac{4}{5} - \frac{3}{8}$ ,

(iv)  $\frac{2}{3} \times \frac{3}{4}$ .

(b) Simplify the following:

(i)  $20.62 - 8.7$ ,

(ii)  $0.98 \times 0.2$ ,

(iii)  $2.73 \div 0.3$ .

(c) If  $a = \frac{2}{3}$ , evaluate  $2 - 3a$ .

(d) Simplify  $(1\frac{1}{2})^2 - 1\frac{1}{2}$ .

(e) Find the circumference of a circle of radius 4 cm. (Use  $\pi \doteq 3.14$ .)

QUESTION TWO

(a) Simplify the following:

(i)  $7x + 5x - 3x$ ,

(ii)  $(3ab) \times (-12a)$ ,

(iii)  $10m \div 2m$ .

(b) Simplify the following:

(i)  $6x^{12} \div x^6$ ,

(ii)  $2x^3y^3 \times 3xy^2$ ,

(iii)  $(xy^2)^3$ .

(c) Expand and simplify  $3(a - 2) + 2(a + 3)$ .

(d) Factorise the following:

(i)  $6x - 36$ ,

(ii)  $4xy - 10x$ .

(e) Simplify the following:

(i)  $\frac{4x}{3} - \frac{x}{2}$ ,

(ii)  $\frac{4a}{7b} \div \frac{a}{14b^2}$ .

QUESTION THREE

(a) Write 72% as a fraction in simplest form.

(b) Find 22% of 250 cm.

(c) Write  $6\frac{3}{4}\%$  as a decimal.

(d) Express 0.12 as a percentage.

(e) The total fat content of a 45-gram muesli bar is 5.4 grams.

What percentage of the muesli bar is fat?

(f) Bill receives 4% of \$14 000 each year for three consecutive years. Find the total amount he receives in that time.

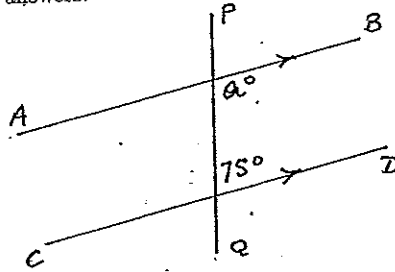
(g) In 1992 I purchased a block of land for \$120 000 and over the next ten years its value increased by 150%. What is the value of the land in 2002?

(h) The price of a shirt marked at \$64 is increased by 10%. It remains unsold, so its price is then reduced by 10%. What is its new price?

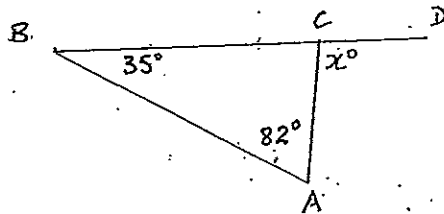
**QUESTION FOUR**

(a) Find the value of the pronumerals in each of the following diagrams. Reasons must be given for your answers.

(i)

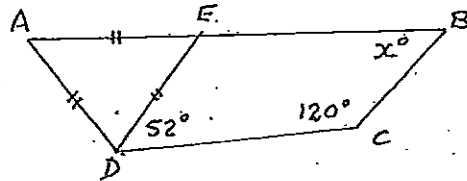


(ii)

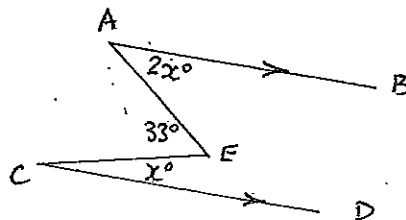


(b) Copy the following diagrams and find, stating all reasons, the value of the pronumeral.

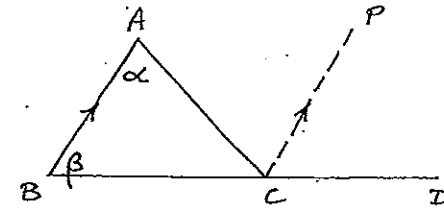
(i)



(ii)



(c)



GIVEN: In the diagram above,  $ABC$  is a triangle with the side  $BC$  produced to  $D$ .  
Let  $\angle BAC = \alpha$  and  $\angle ABC = \beta$ .

AIM: To prove that  $\angle ACD = \alpha + \beta$ .

CONSTRUCTION: Construct the ray  $CP$  from  $C$  parallel to  $AB$ .

PROOF: .....

Complete this proof. (Do not copy the question.)

**QUESTION FIVE**

(a) Solve the following equations:

(i)  $3x + 2 = 20$ ,

(ii)  $7x - 13 = x + 5$ ,

(iii)  $\frac{x}{3} - \frac{3x}{2} = -1$ ,

(iv)  $3(a - 2) + 2(a - 3) = a - 7$ .

(b) (i) Given that  $A = \frac{h(a+b)}{2}$ , find the value of  $A$  when  $h = 4$ ,  $a = 2$ , and  $b = 3$ .

(ii) Given that  $s = ut + \frac{1}{2}at^2$ , find the value of  $a$  when  $s = 90$ ,  $u = \frac{2}{3}$  and  $t = 9$ .

(c) (i) Write down an expression for the average of the two numbers  $x$  and  $y$ .

(ii) The average of two numbers is 55, and one is 24 more than the other. Form an equation and solve it to find the numbers.

QUESTION SIX

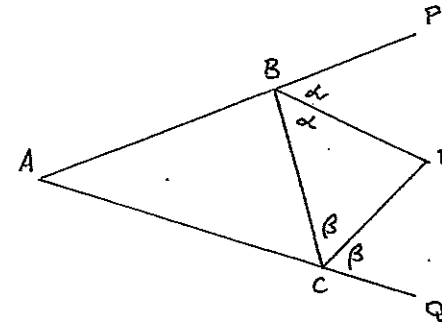
- (a) Evaluate  $\frac{1.08 \times 2.5 \times 2.8}{1.2 \times 45 \times 0.5}$ .
- (b) Simplify  $(2x^2y)^3 \times 8xy^3 \div 16xy^5$ .
- (c) Evaluate  $\left( (1\frac{2}{3})^2 + \frac{1}{2} \right) \div \left( 1\frac{1}{4} + \frac{1}{2} \right)$ .
- (d) Given that  $m(x - y) = 2x$ , find the value of  $x$  when  $m = 1\frac{3}{5}$ , and  $y = \frac{3}{4}$ .
- (e) A two-dollar coin has a diameter of 2 cm. Twenty-five two-dollar coins are laid on the bottom of a flat rectangular tray of length 16 cm and breadth 10 cm. Find what percentage of the area of the tray is left uncovered.

Leave your answers in terms of  $\pi$ .

Question Seven follows on the next page.

QUESTION SEVEN

(a)



In the diagram above,  $ABC$  is a triangle with the side  $AB$  produced to the point  $P$  and the side  $AC$  produced to the point  $Q$ .

The bisectors of  $\angle CBP$  and  $\angle BCQ$  intersect at the point  $D$ .

Copy the diagram, let  $\angle CBD = \alpha$  and  $\angle BCD = \beta$ , and answer the following questions, stating all reasons.

- (i) Show that  $\angle BAC = 2\alpha + 2\beta - 180^\circ$ .
- (ii) Find  $\angle BAC$ , given that  $\angle BDC = 80^\circ$ .
- (b) A town's population decreased by 400 people during one year. In the next year, the population increased by 6%, but the number of residents was still 40 fewer than before the decrease. Form an equation and solve it to find the population at the beginning of the first year.
- (c) Four singers take part in a musical round of four equal lines, each finishing after singing the round through three times. The second singer begins when the first singer begins the second line, the third singer begins when the first singer begins the third line, the fourth singer begins when the first singer begins the fourth line.

Find the fraction of the total singing time that all four singers are singing at the same time. You must show all reasoning.

GJ

Units: Penalize once only at Q1(e) or Q3(b).

QUESTION ONE

- (a) (i)  $45 \div (-9) = -5$ .
- (ii)  $5 - 6 \div 3 = 5 - 2$   
 $= 3$ .
- (iii)  $\frac{4}{5} - \frac{3}{8} = \frac{32 - 15}{40}$ ,  
 $= \frac{17}{40}$ .
- (iv)  $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ .
- (b) (i)  $20.62 - 8.7 = 11.92$ .
- (ii)  $0.98 \times 0.2 = 0.196$ .
- (iii)  $2.73 \div 0.3 = 27.3 \div 3$    
 $= 9.1$ .
- (c)  $2 - 3a = 2 - 3 \times \frac{2}{3}$    
 $= 0$ .
- (d)  $(1\frac{1}{2})^2 - 1\frac{1}{2} = 2\frac{1}{4} - 1\frac{1}{2}$    
 $= \frac{3}{4}$ .
- (e)  $C = 2\pi r$   
 $\doteq 2 \times 3.14 \times 4$    
 $\doteq 25.12 \text{ cm}$ .

QUESTION TWO

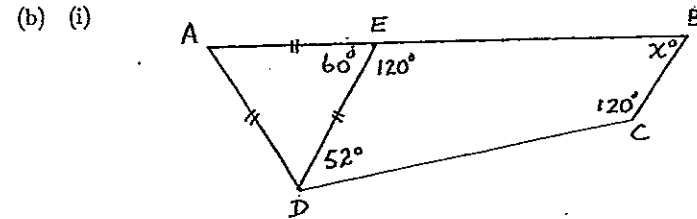
- (a) (i)  $7x + 5x - 3x = 9x$ .
- (ii)  $(3ab) \times (-12a) = -36a^2b$ .
- (iii)  $10m \div 2m = 5$ .
- (b) (i)  $6x^{12} \div x^6 = 6x^6$ .
- (ii)  $2x^3y^3 \times 3xy^2 = 6x^4y^5$ .
- (iii)  $(xy^2)^3 = x^3y^6$ .
- (c)  $3(a - 2) + 2(a + 3) = 3a - 6 + 2a + 6$    
 $= 5a$ .
- (d) (i)  $6x - 36 = 6(x - 6)$ .
- (ii)  $4xy - 10x = 2x(2y - 5)$ .
- (e) (i)  $\frac{4x}{3} - \frac{x}{2} = \frac{8x - 3x}{6}$    
 $= \frac{5x}{6}$ .
- (ii)  $\frac{4a}{7b} \div \frac{a}{14b^2} = \frac{4a}{7b} \times \frac{14b^2}{a}$    
 $= 8b$ .

**QUESTION THREE**

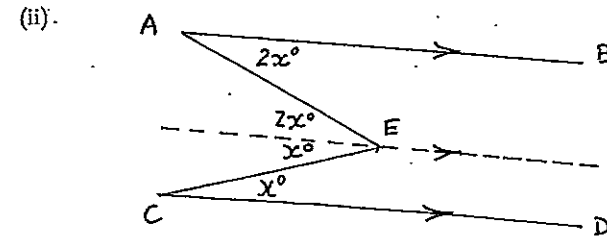
- (a)  $72\% = \frac{72}{100}$   
 $= \frac{18}{25}$  ✓
- (b)  $22\% \times 250 = \frac{22}{100} \times 250$  ✓  
 $= 55 \text{ cm.}$  ✓
- (c)  $6\frac{3}{4}\% = 6.75 \div 100$  ✓  
 $= 0.0675.$  ✓
- (d)  $0.12 = 12\%.$  ✓
- (e) Percentage fat  $= \frac{5.4}{45} \times 100\%$  ✓  
 $= \frac{0.6}{5} \times 100\%$   
 $= 12\%.$  ✓
- (f) Total amount  $= 14\,000 \times \frac{4}{100} \times 3$  ✓  
 $= 560 \times 3$   
 $= \$1680.$  ✓
- (g) New value  $= 2.5 \times 120\,000$  ✓  
 $= \$300\,000.$  ✓
- (h) New price  $= 90\% \times 110\% \times 64$   
 $= \frac{9}{10} \times \frac{11}{10} \times 64$  ✓✓  
 $= \$63.36.$  ✓

**QUESTION FOUR**

- (a) (i)  $a = 105$  (co-interior angles,  $AB \parallel CD$ ). ✓  
 (ii)  $x = 35 + 82$  (exterior angle of  $\triangle ABC$ )  
 $= 117.$  ✓



- $\angle AED = 60^\circ$  (angle of an equilateral triangle) ✓  
 $\angle BED = 120^\circ$  (straight angle) ✓  
 $x = 360 - (120 + 52 + 120)$  (angle sum of quadrilateral  $DCBE$ )  
 $= 68.$  ✓



- Construct  $XE \parallel AB \parallel CD.$  ✓  
 $\angle AEX = 2x^\circ$  (alternate angles,  $AB \parallel XE$ ) ✓  
 $\angle CEX = x^\circ$  (alternate angles,  $CD \parallel XE$ ) ✓  
 $2x + x = 33$   
 $x = 11.$  ✓

- (c)  $\angle ACP = \alpha$  (alternate angles,  $AB \parallel PC$ ) ✓  
 $\angle PCD = \beta$  (corresponding angles,  $AB \parallel PC$ ) ✓  
 $\angle ACD = \angle ACP + \angle PCD$   
 $= \alpha + \beta.$  ✓

**QUESTION FIVE**

**Question Five ctd.**

(a) (i)  $3x + 2 = 20$

$3x = 18$  ✓

$x = 6$ . ✓

(ii)  $7x - 13 = x + 5$

$6x = 18$  ✓

$x = 3$ . ✓

(iii)  $\frac{x}{3} - \frac{3x}{2} = -1$

$2x - 9x = -6$  ✓

$-7x = -6$

$x = \frac{6}{7}$ . ✓

(iv)  $3(a - 2) + 2(a - 3) = a - 7$

$3a - 6 + 2a - 6 = a - 7$  ✓

$5a - 12 = a - 7$

$4a = 5$

$a = 1\frac{1}{4}$ . ✓/accept  $\frac{5}{4}$

(b) (i)  $A = \frac{h(a+b)}{2}$

$= \frac{4(2+3)}{2}$  ✓

$= 10$ . ✓

(ii)  $s = ut + \frac{1}{2}at^2$

$90 = \frac{2}{3} \times 9 + \frac{1}{2} \times a \times 81$  ✓

$90 = 6 + \frac{1}{2} \times a \times 81$

$180 = 12 + 81a$

$81a = 168$

$a = \frac{168}{81}$

$= 2\frac{2}{27}$ . ✓/accept  $\frac{56}{27}$

(c) (i)  $\frac{x+y}{2}$ . ✓

(ii) Let the numbers be  $x$  and  $x + 24$ .

$\frac{x + x + 24}{2} = 55$  ✓

$2x + 24 = 110$

$2x = 86$

$x = 43$ .

The numbers are 43 and 67. ✓

**QUESTION SIX**

(a)  $\frac{1.08 \times 2.5 \times 2.8}{1.2 \times 45 \times 0.5} = \frac{108 \times 25 \times 28}{12 \times 45 \times 5 \times 100}$

$= \frac{9 \times 28}{9 \times 100}$

$= 0.28$ . ✓/✓/✓ -1 each error

(b)  $(2x^2y)^3 \times 8xy^3 \div 16xy^5 = 8x^6y^3 \times 8xy^3 \div 16xy^5$  ✓

$= 64x^7y^6 \div 16xy^5$  ✓

$= 4x^6y$ . ✓

(c)  $\left(\left(\frac{12}{3}\right)^2 + \frac{1}{2}\right) \div \left(1\frac{1}{4} + \frac{1}{2}\right) = \left(\left(\frac{6}{3}\right)^2 + \frac{1}{2}\right) \div \left(\frac{5}{4} + \frac{1}{2}\right)$

$= \left(\frac{25}{9} + \frac{1}{2}\right) \div \left(\frac{5}{4} + \frac{2}{4}\right)$

$= \frac{50+9}{18} \div \frac{7}{4}$

$= \frac{59}{18} \times \frac{4}{7}$

$= \frac{118}{63}$

$= 1\frac{55}{63}$ . ✓/✓/✓ -1 each error, accept  $\frac{118}{63}$

(d)  $m(x - y) = 2x$

$1\frac{3}{5}(x - \frac{3}{4}) = 2x$  ✓

$\frac{8}{5}x - \frac{6}{5} = 2x$

$8x - 6 = 10x$

$2x = -6$

$x = -3$ . ✓/✓ -1 each error after substitution.

(e) Area of coins =  $25 \times \pi \times 1^2$

$= 25\pi \text{ cm}^2$ . ✓

Area of tray =  $160 \text{ cm}^2$ . ✓

Percentage covered =  $\frac{25\pi}{160} \times 100\%$

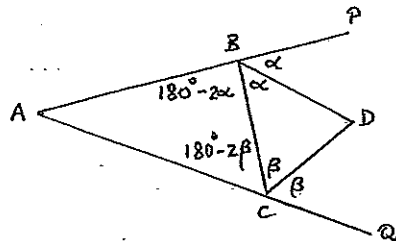
$= \frac{125\pi}{8}\%$

Percentage uncovered =  $\left(100 - \frac{125\pi}{8}\right)\%$

$= \left(\frac{800 - 125\pi}{8}\right)\%$ . ✓

QUESTION SEVEN

(a)



(i)  $\angle CBA = 180^\circ - 2\alpha$  (straight angle)  $\checkmark$   
 $\angle BCA = 180^\circ - 2\beta$  (straight angle)  $\checkmark$   
 $\angle BAC = 180^\circ - \angle CBA - \angle BCA$  (angle sum of  $\triangle ABC$ )  $\checkmark$  (reason;  $\checkmark$  algebra to result)  
 $= 180^\circ - (180^\circ - 2\alpha) - (180^\circ - 2\beta)$   
 $= 180^\circ - 180^\circ + 2\alpha - 180^\circ + 2\beta$   
 $= 2\alpha + 2\beta - 180^\circ$ .

(ii)  $\angle BDC = 80^\circ$   
 $\alpha + \beta = 100^\circ$  (angle sum of  $\triangle BDC$ )  $\checkmark$   
 $\angle BAC = 2\alpha + 2\beta - 180^\circ$   
 $= 2(\alpha + \beta) - 180^\circ$   
 $= 2 \times 100^\circ - 180^\circ$   
 $= 20^\circ$ .  $\checkmark$

(b) Let the population of the town at the beginning of the first year be  $x$ .  
 Population after decrease =  $x - 400$ .

Population after increase =  $(x - 400) \times 1.06$ .

New population =  $x - 40$ .

$(x - 400) \times 1.06 = x - 40$   $\checkmark$  (setup;  $\checkmark$  using 6%;  $\checkmark$  equation)

$1.06x - 424 = x - 40$

$0.06x = 384$

$x = \frac{384}{0.06}$

$= 6400$ .  $\checkmark$

The population at the beginning of the first year is 6400.

(c) The fourth singer sings three lines after the first singer, F, has finished his twelve lines.  
 This means that a total of fifteen lines will be sung.  $\checkmark$

All singers are singing in line four of F's first round and in all of F's second and third rounds; a total of nine lines.  $\checkmark\checkmark$

Required fraction =  $\frac{9}{15}$

$= \frac{3}{5}$ .  $\checkmark$