

MATHEMATICS FORM II

Time allowed: 1 hour 30 minutes

Exam date: 15th May 2003

Instructions:

All questions may be attempted.

All questions are of equal value.

All necessary working must be shown.

Marks may not be awarded for careless or badly arranged work.

Calculators and templates may NOT be used.

Protractors, rulers and compasses may be used.

Collection:

Staple all your paper in one bundle.

Write your name, class and master's initials on the front.

2A: JNC

2B: MLS

2C: JMR

2D: BDD

2E: DNW

2F: REN

2G: DS

2H: REP

2I: PKH

2J: FMW

Checklist:

Writing paper required.

Candidature: 192 boys.

QUESTION ONE

(a) Calculate:

(i) $4 - (-3)$

(ii) $3 - 5 - 6$

(iii) $-63 \div 7$

(b) Calculate:

(i) 0.431×100

(ii) $(0.2)^2$

(iii) 1.5×0.03

(iv) $2.4 \div 0.4$

(c) Calculate:

(i) $\frac{3}{4} \times \frac{2}{5}$

(ii) $\frac{2}{9} \div \frac{3}{4}$

(iii) $1\frac{1}{2} + \frac{3}{5}$ _____

QUESTION TWO

- (a) Simplify $6a + 2b - 3a$.
- (b) Simplify:
- (i) $12x^3 \div 4x$
 - (ii) $(2y)^3$
 - (iii) $r^{17} \div r^{12}$
- (c) Evaluate $7 + 3x$ if $x = 2$.
- (d) Expand and simplify:
- (i) $3(2a - 1)$
 - (ii) $3x + 2(x - 4)$
- (e) Factorise fully:
- (i) $5x - 15$
 - (ii) $6xy - 9x$
- (f) Express the following as single fractions in simplest form:
- (i) $\frac{3b}{4} + \frac{b}{5}$
 - (ii) $\frac{15x}{y} \times \frac{y^2}{10x}$

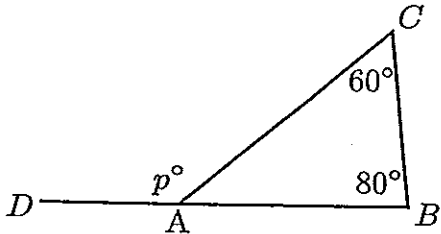
QUESTION THREE

- (a) Write 15% as a fraction in lowest terms.
- (b) Write $\frac{3}{5}$ as a percentage.
- (c) Find 12% of 40.
- (d) In a 1200-gram packet of a certain breakfast cereal there are 150 grams of fruit. What percentage of the cereal is fruit?
- (e) A certain shopkeeper buys items and then marks them up by 30% before reselling them. If he purchases an item for \$70, what price will he sell it at?
- (f) In a certain country, 80% of the yearly rainfall falls over a two-week period. If 1200 mm falls in this time, what is the yearly rainfall? (You must show a clear method in your solution.)

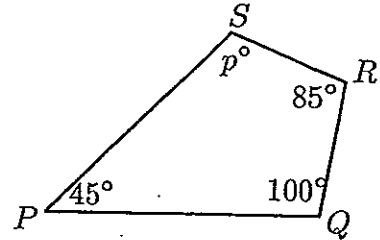
QUESTION FOUR

(a) Find the unknown angle marked p° in each of the diagrams below. You must give reasons to explain any geometry.

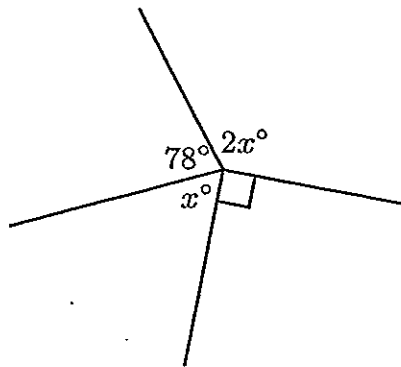
(i)



(ii)



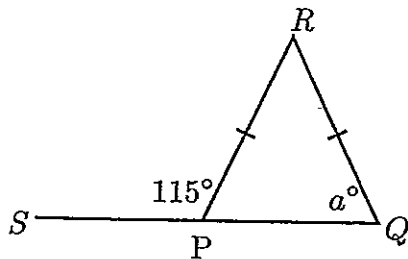
(b)



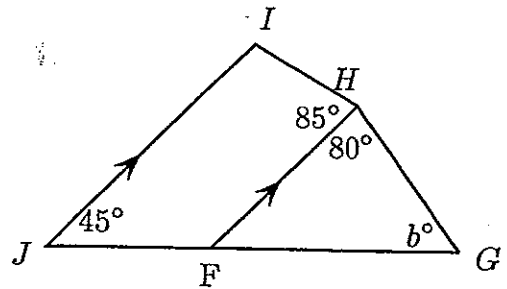
Form an equation and solve it to find the unknown angle marked x° in the diagram above, giving reasons to explain any geometry.

(c) Find the unknown angles a° and b° marked in the diagrams below. You must give reasons to explain any geometry.

(i)



(ii)



QUESTION FIVE

(a) Solve:

(i) $2x - 7 = 4$

(ii) $a + 7 = 6a - 1$

(b) Suppose that $F = 32 + \frac{9}{5}C$.

(i) Find F if $C = 20$.

(ii) Find C if $F = 95$.

(c) The length of a rectangle is 3 cm more than its width. Let x cm be the width.

(i) Write down an expression for the length in terms of x .

(ii) The perimeter of the rectangle is 32 cm. Construct and then solve an equation in order to find the dimensions of the rectangle.

QUESTION SIX

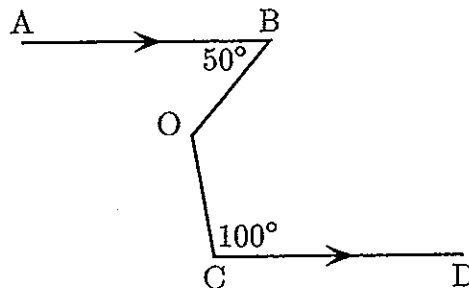
(a) Simplify $\frac{(3x^2)^3}{9x}$.

(b) Expand and simplify $3x(2x - 4) + 2x$.

(c) A certain school is noted for its success at soccer. The school has five hundred pupils, of which 60% are male. If 25% of the girls and 45% of the boys play soccer, how many pupils play the game?

(d) Simplify $9\frac{3}{8} \div \left((1\frac{1}{2})^2 + \frac{1}{4} \right)$.

(e)

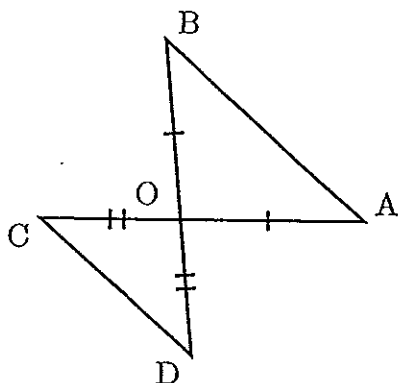


Copy the diagram above and find the reflex $\angle BOC$. Be careful to give all reasons.

QUESTION SEVEN

- (a) Solve the equation $3(4x - 3) - 2(3x - 1) = 47$.
- (b) I have two glasses of equal size. Glass A is empty and glass B contains some water. Half of the water in B is poured into A. This is repeated twice more— each time half of the remaining water in B is poured into A.
After these three pourings, A is half full. How much water is left in B (as a fraction of a full glass)? You must show a clear method in your answer.

(c)



In the diagram above, $OA = OB$ and $OC = OD$.

Prove that $AB \parallel CD$, being careful to give reasons. (HINT: Let $\angle ODC = x$.)

- (d) Archie earns four-fifths as much as Bert before Bert gets a \$5000 increase in his yearly wage. Afterwards, two-thirds of Bert's wage differs from Archie's wage by only \$2000. How much do the two men earn after Bert's wage increase?

BDD

QUESTION ONE

- (a) (i) $4 - (-3) = 7$, ✓
(ii) $3 - 5 - 6 = -8$, ✓
(iii) $-63 \div 7 = -9$. ✓
- (b) (i) $0.431 \times 100 = 43.1$, ✓
(ii) $(0.2)^2 = 0.04$, ✓
(iii) $1.5 \times 0.03 = 0.045$, ✓✓
(iv) $2.4 \div 0.4 = 6$. ✓✓
- (c) (i) $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ ✓
 $= \frac{3}{10}$ ✓
- (ii) $\frac{2}{9} \div \frac{3}{4} = \frac{2}{9} \times \frac{4}{3}$ ✓
 $= \frac{8}{27}$, ✓
- (iii) $1\frac{1}{2} + \frac{3}{5} = \frac{3}{2} + \frac{3}{5}$
 $= \frac{15+6}{10}$ ✓
 $= \frac{21}{10}$, ✓
 $= 2\frac{1}{10}$.

QUESTION TWO

- (a) $6a + 2b - 3a = 3a + 2b$. ✓
- (b) (i) $12x^3 \div 4x = 3x^2$, ✓
(ii) $(2y)^3 = 8y^3$, ✓
(iii) $r^{17} \div r^{12} = r^5$. ✓
- (c) If $x = 2$, $7 + 3x = 7 + 3 \times 2$ ✓
 $= 13$. ✓
- (d) (i) $3(2a - 1) = 6a - 3$, ✓
(ii) $3x + 2(x - 4) = 3x + 2x - 8$ ✓
 $= 5x - 8$. ✓

(e) (i) $5x - 15 = 5(x - 3)$, ✓
 (ii) $6xy - 9x = 3x(2y - 3)$. ✓

(f) (i) $\frac{3b}{4} + \frac{b}{5} = \frac{15b + 4b}{20}$ ✓
 $= \frac{19b}{20}$, ✓

(ii) $\frac{15x}{y} \times \frac{y^2}{10x} = \frac{3}{1} \times \frac{y}{2}$ ✓
 $= \frac{3y}{2}$. ✓

QUESTION THREE

(a) $15\% = \frac{15}{100}$ ✓
 $= \frac{3}{20}$. ✓

(b) $\frac{3}{5} \doteq \frac{3}{5} \times 100\%$ ✓
 $= 60\%$. ✓

(c) $12\% \text{ of } 40 = 40 \times \frac{12}{100}$ ✓
 $= 4 \times \frac{12}{10}$
 $= 4.8$. ✓

(d) $150\text{g as a percentage of } 1200\text{g} = \frac{150}{1200} \times 100\%$ ✓
 $= \frac{150}{12}\%$ ✓
 $= \frac{50}{4}\%$
 $= 12.5\%$. ✓

Thus 12.5% is the cereal is fruit.

(e) $130\% \text{ of } \$70 = \$70 \times \frac{130}{100}$ ✓
 $= \$7 \times 13$
 $= \$91$ ✓

Hence the shopkeeper sells it for \$91.

- (f) 80% of the rainfall is 1200mm ✓
 so 10% of the rainfall is 150mm ✓
 thus 100% of the rainfall is 1500mm ✓
 Hence the yearly rainfall is 1500mm.

QUESTION FOUR

- (a) (i) $p = 60 + 80$
 $= 140^\circ$ ✓ (exterior opposite angle of $\triangle ABC$) ✓
- (ii) $p + 85 + 100 + 45 = 360$ ✓ (angle sum of quadrilateral) ✓
 so $p + 230 = 360$
 $p = 130^\circ$. ✓
- (b) $2x + 78 + x + 90 = 360^\circ$ ✓ (revolution) ✓
 $3x + 168 = 360$
 $3x = 192$
 $x = 64^\circ$ ✓
- (c) (i) $115 + \angle RPQ = 180$ ✓ (straight angle) ✓
 so $\angle RPQ = 65^\circ$. ✓
 Thus $\angle RQP = \angle RPQ$ (equal angles opposite equal sides) ✓
 $= 65^\circ$
- (ii) $\angle HFG = \angle IJF$ (corresponding angles, $IJ \parallel HF$) ✓
 $= 45^\circ$ ✓
 But now
 $45 + 80 + b = 180$ ✓ (angle sum of $\triangle HFG$) ✓
 so $b = 55^\circ$.

QUESTION FIVE

- (a) (i) $2x - 7 = 4$
 $2x = 11$. ✓
 $x = 5.5$. ✓
- (ii) $\boxed{-7} \quad a + 7 = 6a - 1$
 $\boxed{-6a} \quad a = 6a - 8$ ✓
 $\boxed{\div -5} \quad -5a = -8$ ✓
 $a = \frac{8}{5}$ ✓
 $a = 1\frac{3}{5}$.

(b) (i) $F = 32 + \frac{9}{5}C$ ✓
 so $F = 32 + \frac{9}{5} \times 20$ (if $C = 20$)
 $F = 32 + 36$ ✓
 $F = 68$ ✓

(ii) $F = 32 + \frac{9}{5}C$
 so $95 = 32 + \frac{9}{5} \times C$ ✓ (if $F = 95$)
 $63 = \frac{9}{5}C$ ✓
 $\frac{9}{5}C = 63$
 $C = \frac{5}{9} \times 63$ ✓
 $C = 35$ ✓

(c) (i) The length is $x + 3$ cm. ✓

(ii) $x + x + 3 + x + x + 3 = 32$ ✓
 $4x + 6 = 32$
 $4x = 26$ ✓
 $x = 26 \div 4$
 $x = 6.5$ ✓

The dimensions of the rectangle are 6.5cm × 9.5cm. ✓

QUESTION SIX

(a) $\frac{(3x^2)^3}{9x} = \frac{27x^6}{9x}$ ✓
 $= 3x^5$ ✓

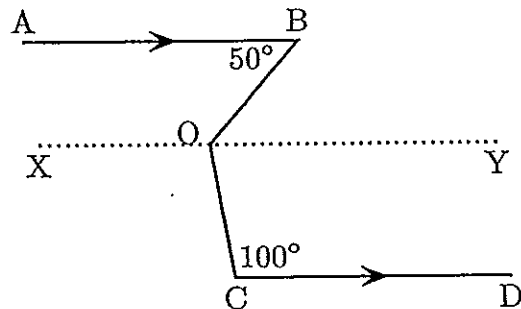
(b) $3x(2x - 4) + 2x = 6x^2 - 12x + 2x$ ✓
 $= 6x^2 - 10x$ ✓

(c) Of the 500 pupils, 60% of 500=300 are male. ✓
 Thus 200 are female.
 Now 25% of girls + 45% of boys = 25% of 200 + 45% of 300 ✓
 $= 50 + 300 \times \frac{45}{100}$ ✓
 $= 50 + 135$
 $= 185.$ ✓

Thus 185 pupils play soccer.

$$\begin{aligned}
 \text{(d)} \quad 9\frac{3}{8} \div \left((1\frac{1}{2})^2 + \frac{1}{4} \right) &= \frac{75}{8} \div \left((\frac{3}{2})^2 + \frac{1}{4} \right) \\
 &= \frac{75}{8} \div \left(\frac{9}{4} + \frac{1}{4} \right) \checkmark \\
 &= \frac{75}{8} \times \frac{4}{10} \checkmark \\
 &= \frac{15}{2} \times \frac{1}{2} \\
 &= \frac{15}{4} \\
 &= 3\frac{3}{4} \checkmark
 \end{aligned}$$

(e)



✓ Construction

Construct $XY \parallel AB$ and through O , as in the diagram above.

Then $\angle BOX = 130^\circ$ (cointerior angles, $AB \parallel XY$) ✓

$\angle XOC = 100^\circ$ (alternate angles, $XY \parallel CD$) ✓

Hence reflex $\angle BOC = 130^\circ + 100^\circ$ (adjacent angles)
 $= 230^\circ$ ✓

QUESTION SEVEN

$$\begin{aligned}
 \text{(a)} \quad 3(4x - 3) - 2(3x - 1) &= 47 \\
 12x - 9 - 6x + 2 &= 47 \checkmark
 \end{aligned}$$

$$\boxed{+7} \quad 6x - 7 = 47$$

$$\boxed{\div 6} \quad 6x = 54$$

$$x = 9 \checkmark$$

Hence $x = 9$.

(b) Let x be the fraction of a full glass in B. As the pouring is carried out, the fractions in each glass vary as ...

in A	in B
0	x
$\frac{1}{2}x$	$\frac{1}{2}x$
$(\frac{1}{2} + \frac{1}{4})x$	$\frac{1}{4}x$
$(\frac{1}{2} + \frac{1}{4} + \frac{1}{8})x$	$\frac{1}{8}x$

✓
✓

Thus $(\frac{1}{2} + \frac{1}{4} + \frac{1}{8})x = \frac{1}{2}$

i.e. $\frac{7}{8}x = \frac{1}{2}$

$x = \frac{1}{2} \times \frac{8}{7}$

$x = \frac{4}{7}$

At the end of the pourings, the amount left in B is $\frac{1}{8} \times \frac{4}{7} = \frac{1}{14}$ of a full glass.

- (c) We are given $\angle ODC = x$. Thus we need to show that $\angle OBA = x$ also, so that the alternate angles are equal.

First, $\angle OCD = x$ (equal angles opposite equal sides)

so $\angle COD = 180 - 2x$ (angle sum of $\triangle COD$)

but then $\angle BOA = \angle COD$ (vertically opposite angles)

$= 180^\circ - 2x$.

Now we also know that $\angle OBA = \angle OAB$ (equal angles opposite equal sides).

Thus $\angle OAB + \angle OBA + 180^\circ - 2x = 180^\circ$ (angle sum of $\triangle AOB$)

-180

$2\angle OBA - 2x = 0$

Hence

$\angle OBA = x$.

But this means that $\angle ODC = \angle OBA$ and hence that $DC \parallel BA$, since the alternate angles are equal.

- (d) Let x be Bert's wage after his increase, so that $x - 5000$ is his original wage. Archie's wage is thus $\frac{4}{5}(x - 5000)$.

After the increase, $\frac{2}{3}$ of Bert's new wage is EITHER 2000 more OR 2000 less than Archie's — the question doesn't say which. Thus EITHER

$\frac{4}{5}(x - 5000) - \frac{2}{3}x = 2000$

OR

$\frac{2}{3}x - \frac{4}{5}(x - 5000) = 2000$.

Solving the first equation;

$\frac{4}{5}(x - 5000) - \frac{2}{3}x = 2000$

$\frac{4}{5}x - 4000 - \frac{2}{3}x = 2000$

$\frac{2}{15}x = 6000$

$2x = 90000$

$x = 45000$

Solving the second equation;

$\frac{2}{3}x - \frac{4}{5}(x - 5000) = 2000$

$\frac{2}{3}x - \frac{4}{5}x + 4000 = 2000$

$-\frac{2}{15}x = -2000$

$2x = 30000$

$x = 15000$

Hence the two possibilities are that following the wage increase Archie earns \$32 000 and Bert \$45 000 OR that Archie earns \$8 000 and Bert \$15 000.