



FORM II

MATHEMATICS

Examination date

Monday 8th November 2004

Time allowed

2 hours

Instructions

- All nine questions may be attempted.
- All nine questions are of equal value.
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Calculators are not to be used.

Collection

- Write your name, class and master clearly on the front.
- Hand in all the writing paper in a single well-stapled bundle.
- Keep the printed examination paper and bring it to your next Mathematics lesson.

2A: KWM	2B: TCW	2C: GJ
2D: WMP	2E: LYL	2F: BDD
2G: PKH	2H: JNC	2I: REP

Checklist

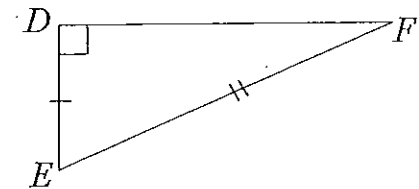
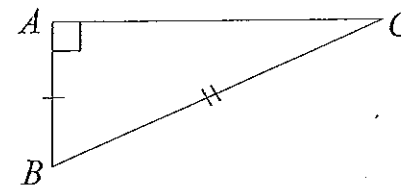
- Writing paper required.
- Candidature: 188 boys.

Examiner

JNC

QUESTION ONE Start a new page.

- (a) Simplify $16 - 8 \times 2 - 1$.
- (b) Evaluate $\sqrt{25 + 144}$.
- (c) Evaluate $4x^2$ if $x = \frac{1}{2}$.
- (d) Expand $5(3 - x)$.
- (e) Factorize $15x + 10y$.
- (f) Graph the inequality $x < -1$ on a number line.
- (g) Simplify $\frac{x}{2} + x$.
- (h) Solve $a - 4 = 11$.
- (i) Simplify $(3x^3)^2$.
- (j)

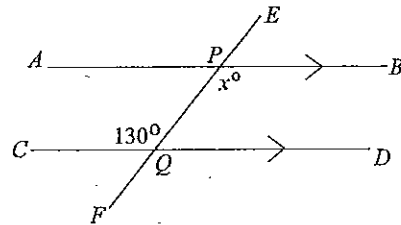


In the diagram above, which congruence test would prove $\triangle ABC \cong \triangle DEF$?

- (k) Express 37.5% as a fraction in lowest terms.
- (l) An MP3 player is sold for \$450. If it cost \$300, find the percentage profit on the cost price.
- (m) Evaluate $5.6 \div 0.8$.

QUESTION TWO Start a new page.

- (a) Expand and simplify $(x + 2)(x - 4)$.
- (b) Solve $5 - 3x = 29$.
- (c) Find the area of a circle whose diameter is 40 centimetres. Use $\pi \doteq 3.14$ and give your answer correct to the nearest cm^2 .
- (d) Divide \$600 between Bill and Ben in the ratio 7 : 5.
- (e) Expand and simplify $(x + 5)^2$.
- (f)

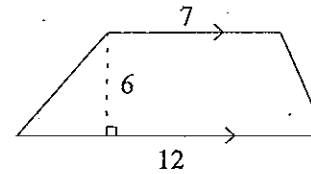


In the diagram above, find the value of x , giving reasons.

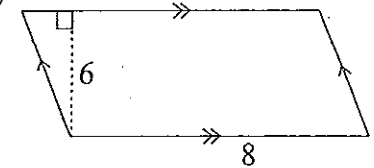
- (g) Find the median of the scores:
5, 7, 10, 9, 4.

QUESTION THREE Start a new page.

- (a) (i)

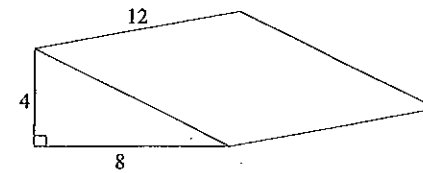


- (ii)



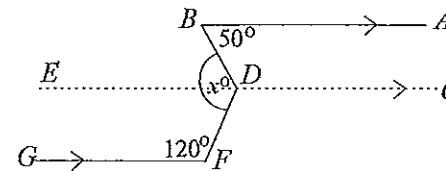
In each of the diagrams above, find the area of the figure drawn. All measurements are in centimetres.

- (b)



Find the volume of the triangular prism in the diagram above. All measurements are in centimetres.

- (c)

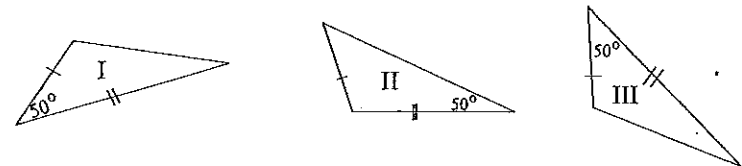


In the diagram above, find the value of x . You must give reasons.

- (d) Use ruler and compasses to complete the following construction. Do NOT erase any construction marks.

- (i) On about one third of a page draw an acute angle and label it ABC .
- (ii) Bisect $\angle ABC$ showing all constructions.

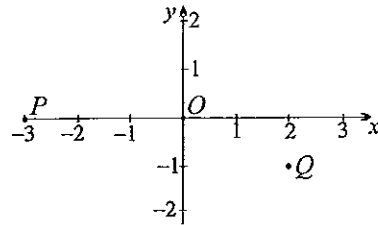
- (e)



From the three triangles drawn above, name a pair of congruent triangles and state the test used.

QUESTION FOUR Start a new page.

(a)



Write down the co-ordinates of points O , P and Q .

(b) (i) On a number plane, plot the points $A(-1, 3)$, $B(4, 3)$ and $C(-1, -1)$. Use a scale of 1 centimetre for 1 unit on both axes.

(ii) What type of angle is $\angle BAC$?

(iii) What is the distance from point A to point C ?

(iv) Find the area of $\triangle ABC$.

(c) Show by substitution that the point $(-1, 2)$ lies on the line $x - 3y = -7$.

(d) (i) For the equation $y = 2 - 3x$, copy and complete the following table.

x	-1	0	1
y			

(ii) Use a ruler and pencil to draw a number plane and accurately draw the graph of $y = 2 - 3x$. Use a scale of 1 centimetre for 1 unit on both axes.

(iii) Write down the co-ordinates of the point where the graph of $y = 2 - 3x$ crosses the y -axis.

QUESTION FIVE Start a new page.

(a) Six coins were tossed 32 times. Each time the six coins were tossed, the number of heads was recorded as follows:

5	1	0	4	3	3	3	2
4	6	3	4	2	4	4	3
2	3	4	3	1	6	4	3
3	5	1	2	3	1	3	4

(i) Copy this table onto your answer sheet and complete the Frequency and the Frequency \times Outcome columns:

Outcome	Frequency	Frequency \times Outcome
0		
1		
2		
3		
4		
5		
6		
Totals	32	

(ii) Draw a frequency histogram to represent this information. Label both axes.

(iii) Write down the range and mode.

(iv) Calculate the mean number of heads.

(v) How many times were more than 4 heads tossed?

(b) During the last season Mr Wilde's soccer team scored the following number of goals:

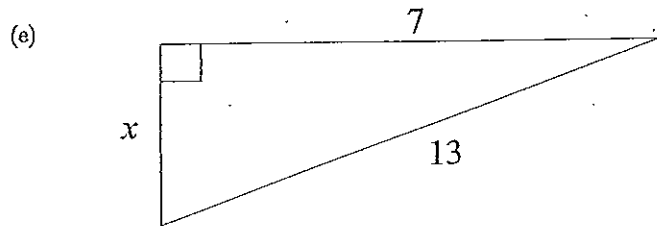
1, 1, 2, 3, 0, 1, 4, 2, 0, 1, 0, 3, 2.

(i) Construct a dot plot using this data.

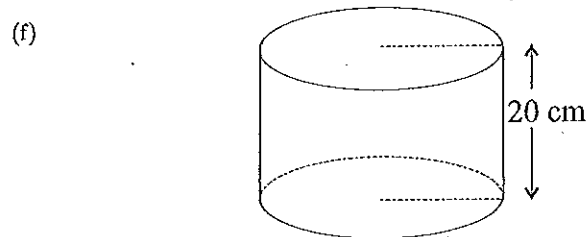
(ii) Find the median of the set of scores.

QUESTION SIX Start a new page.

- (a) Simplify the ratio 5.5 litres : 500 millilitres.
- (b) Find x if $x : 14 = 7 : 2$.
- (c) A rectangular field is 85 metres long and 48 metres wide. What would be the measurement of the width of the field on a scale drawing if the scale were 1 : 200?
- (d) In a school, the ratio of boys to girls is 2 : 3 and the ratio of girls to teachers is 8 : 1. Find the ratio of pupils to teachers.



In the diagram above, find the exact value of the pronumeral.



In the diagram above, the circumference of the base of the cylinder is 44 centimetres and its height is 20 centimetres.

- (i) Find the radius of the circular base of the cylinder. Use $\pi \doteq \frac{22}{7}$.
- (ii) Find the volume of the cylinder. Use $\pi \doteq \frac{22}{7}$.

QUESTION SEVEN Start a new page.

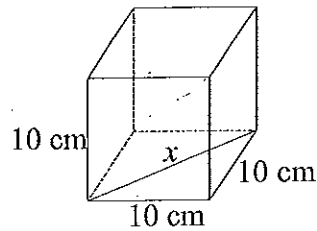
- (a) (i) For the equation $y = 1 - x^2$, copy and complete the following table of values:

x	-2	-1	0	1	2
y					

- (ii) Using a scale of 1 centimetre for 1 unit on both the x -axis and y -axis, carefully draw the graph of $y = 1 - x^2$.
- (b) Simplify the following:
 - (i) $(2x + 3)(x - 5) - (x^2 - 7x)$
 - (ii) $\frac{x + 1}{2} + \frac{2 - x}{3}$
 - (iii) $\frac{x}{3} \times \frac{a}{2} \div \frac{x}{12}$
- (c) I think of a positive number and I get the same answer whether I double it and add 225 or triple it and add 112.
 - (i) Let the unknown number be n and write an equation to represent the information in the question.
 - (ii) Solve the equation to find the number.
- (d) If a truck can be unloaded in 4 hours by 2 people, how long will it take 5 people working at the same rate? Give your answer correct to the nearest minute.

QUESTION EIGHT Start a new page.

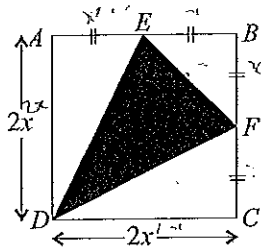
- (a) Solve the inequation $x - \frac{5x}{2} > 5$.
- (b) The average of ten numbers is 20. If one number is removed from the set, the average of the remaining numbers is 19. What number was removed?
- (c)



In the diagram above, a cube of side length 10 centimetres has been drawn.

- (i) Find the exact length of x , the diagonal of the base.
- (ii) Hence find the exact length of the longest pencil that can fit in the cube.

(d)

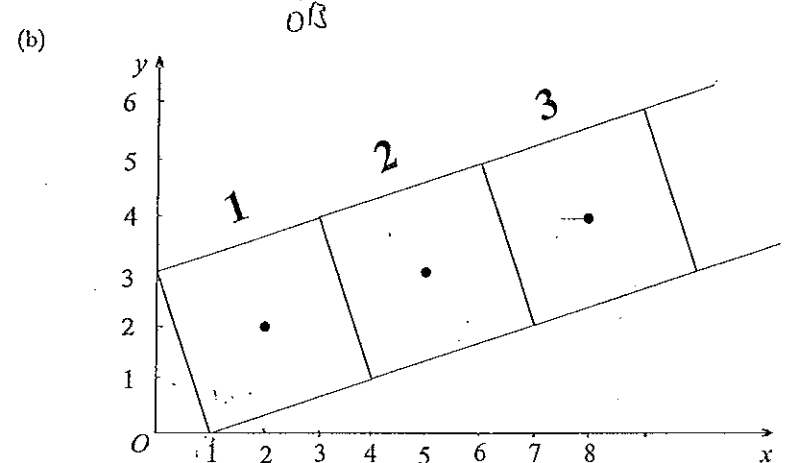


In the diagram above, $ABCD$ is a square whose sides are $2x$ units, and E and F are midpoints of the sides AB and BC respectively. What fraction of the area of the square is the area of the shaded triangle DEF ?

- (e) Sam the gardener has three hoses of different diameters. He has observed that it takes 40 minutes to fill his pond using only the red hose, one hour using only the green hose or 72 minutes using only the black hose. How long would it take to fill the pond using all three hoses together?

QUESTION NINE Start a new page.

- (a) In this question you must leave any construction marks on your diagram.
 - (i) Draw a circle with centre O and a radius of 3 centimetres.
 - (ii) Draw a chord AB .
 - (iii) Construct the perpendicular bisector of the chord AB and label the midpoint of the chord with the letter M .
 - (iv) Join OM , OA and OB and prove that $\triangle OAM \equiv \triangle OBM$.



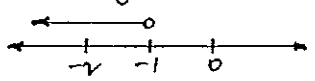
In the diagram above, the pattern of squares is continued.

- (i) What are the coordinates of the centre of the 34th square?
- (ii) What are the coordinates of the bottom left-hand vertex of the 34th square?
- (c) Fred and Peter are on a school camp. They have been given the job of peeling the potatoes for the evening meal. They are fairly slow at peeling potatoes and both peel one potato every minute. They start with the same number of potatoes to peel. After every second potato he peels, Fred throws one of his unpeeled potatoes onto Peter's pile.
 - (i) After an even number of minutes, Peter has twice as many potatoes still to be peeled as Fred. Find how many potatoes are in each boy's pile at this stage.
 - (ii) Five minutes later this ratio has increased to 7 : 3. Find the number of potatoes in each boy's pile at the beginning.

END OF EXAMINATION

FORM II 2004 YEARLY SOLUTIONS

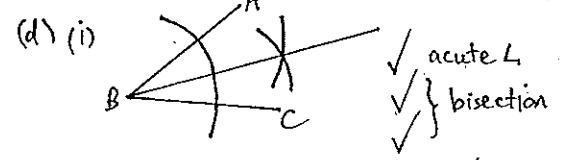
Question One

- (a) -1 ✓
- (b) 13 ✓
- (c) 1 ✓
- (d) $15 - 5x$ ✓
- (e) $5(3x + 2y)$ ✓
- (f)  ✓
- (g) $\frac{3x}{2}$ ✓
- (h) $a = 15$ ✓
- (i) $9x^6$ ✓✓
- (j) RHS ✓
- (k) $\frac{3}{8}$ ✓
- (l) 50% ✓
- (m) 7 ✓

- (e) $(x+5)^2 = x(x+5) + 5(x+5)$
 $= x^2 + 10x + 25$ ✓
- (f) $x = 130^\circ$ (Alternate \angle ; $AB \parallel CD$) ✓✓
- (g) Median is 7 ✓

Question 3

- (a) (i) $A = \frac{6}{2}(12 + 17)$
 $= 57 \text{ cm}^2$ ✓✓
- (ii) $A = 8 \times 6$
 $= 48 \text{ cm}^2$ ✓
- (b) $V = Ah$
 $= (\frac{1}{2} \times 8 \times 4) \times 12$
 $= 192 \text{ cm}^3$ ✓
- (c) $\angle EDB = 50^\circ$ (Alternate \angle ; $AB \parallel CE$) ✓✓
 $\angle FDE = 60^\circ$ (co-interior \angle ; $FG \parallel CE$) ✓✓
 $\therefore x = 50 + 60$
 $= 110^\circ$ ✓

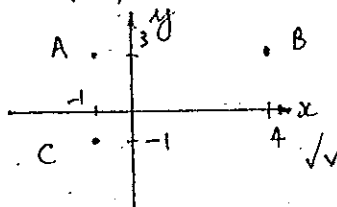


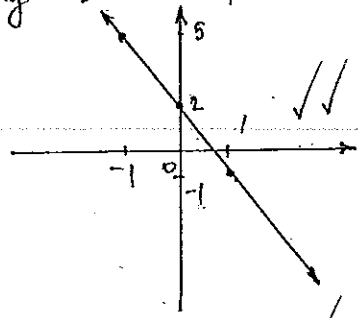
(e) $\triangle I \cong \triangle III$ (SAS) ✓✓

Question Two

- (a) $x(x-4) + 2(x-4)$
 $= x^2 - 4x + 2x - 8$
 $= x^2 - 2x - 8$ ✓
- (b) $-3x = 24$ ✓
 $\therefore x = -8$ ✓
- (c) $A = 3.14 \times 20^2$
 $= 3.14 \times 400$
 $= 1256 \text{ cm}^2$ ✓
- (d) Bill receives $\frac{7}{12} \times 600 = \350 ✓✓
 and Ben receives \$250 ✓

Question 4

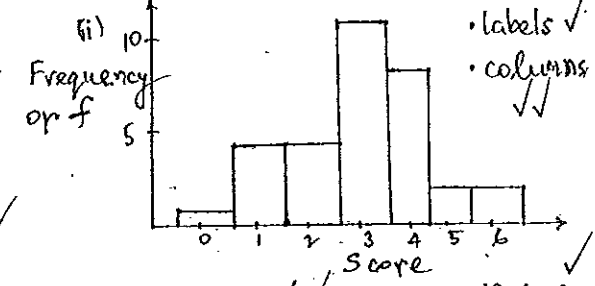
- (a) $O = (0,0)$ $R = (2,-1)$ ✓✓
 $P = (-3,0)$
- (b)  ✓✓
- (i) ✓
- (ii) Right angle ✓
- (iii) 4 (cm) ✓
- (iv) Area = $\frac{1}{2} \times 4 \times 5$
 $= 10 \text{ (cm}^2)$ ✓
- (e) $-1 - 3x(2) = -1 - 6$
 $= -7$
 $= \text{RHS}$ ✓✓

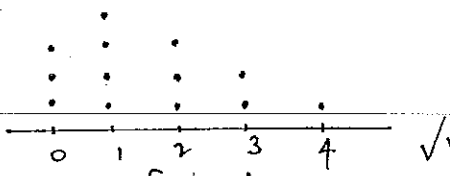
- (or similar)
- (d) x -1 0 1 ✓✓
 (i) y 5 2 -1 ✓✓
- (ii)  ✓✓
- (iii) (0, 2) ✓

Question Five

x	f	fx
0	1	0
1	4	4
2	4	8
3	11	33
4	8	32
5	2	10
6	2	12
Totals	32	99

✓✓✓✓
 -1/error



- (ii) Range = 6 ✓
- (iv) $\bar{x} = \frac{99}{32} (= 3\frac{3}{32})$ ✓
- (v) 4 times ✓
- (b) (i)  ✓✓
 No. of Goals
- (ii) Median = 1 ✓

Question Six

(a) $5500 : 500 = 11 : 1$ ✓

(b) $\frac{x}{14} = \frac{7}{2}$
 $x = 49$ ✓ ✓

(c) $x : 4800 = 1 : 200$ ✓

$x = 24$ cm (width) ✓

- Penalise lack of units
 - Penalise incorrect units
- { Answer of 24 receives 1/2 }

(d) $B:G = 2:3$ and $G:T = 8:1$

LCM is 24

so $B:G = 16:24$ and $G:T = 24:3$ ✓

∴ $B:G:T = 16:24:3$

Then Students: Teachers = 40:3 ✓

(e) $x^2 + 7^2 = 13^2$ ✓

$x^2 = 169 - 49$

$x = \sqrt{120}$ ✓

(f) (i) $(C =) 44 = 2\pi r$ ✓

∴ $r = \frac{44}{2 \times 22}$

$r = 7$ cm ✓

(ii) $V = \pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 20$ ✓

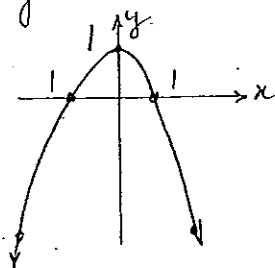
$= 3080$ cm³ ✓

Question Seven

(a) (i) $x \quad -2 \quad -1 \quad 0 \quad 1 \quad 2$

$y \quad -3 \quad 0 \quad 1 \quad 0 \quad -3$ ✓ ✓

(ii)



(b) (i) $2x^2 - 7x - 15 = -x^2 + 7x$ ✓
 $= x^2 - 15$ ✓

(ii) $\frac{3(x+1) + 2(2-x)}{6}$ ✓

$= \frac{x+7}{6}$ ✓

(iii) $\frac{ax}{6} \times \frac{12}{x} = 2a$ ✓

(c) (i) $2n + 225 = 3n + 112$ ✓

(ii) $n = 113$ ✓

(d) Trucks unloaded by 1 person in 8 hrs. ✓

∴ 5 people take $\frac{8}{5}$ h ✓
 $= 1$ h 36 min ✓

(Full marks for $\frac{8}{5}$ h or 1.6 h)

Question 8

(b) $\bar{x} = 20$ $n = 10$ ∴ $\sum x = 200$ ✓

one removed:

$\bar{x} = 19$ $n = 9$ ∴ $\sum x = 171$ ✓

∴ no number removed = $200 - 171 = 29$ ✓

(a) $x - \frac{5x}{2} > 5$

$-\frac{3x}{2} > 5$ ✓

∴ $x < -\frac{10}{3}$ ✓

(c) (i) $x^2 = 10^2 + 10^2$

$x = \sqrt{200}$ ✓

(ii) $y^2 = 10^2 + x^2$

$y^2 = 10 + 200$ ✓

$y = \sqrt{300}$ ✓

(d) Area of Square = $(2x)^2 = 4x^2$ ✓

Area of $\triangle DEF = 4x^2 - (2x \times \frac{1}{2} \times 2x^2 + \frac{1}{2} \times x^2)$ ✓
 $= \frac{3x^2}{2}$

Fraction of areas = $\frac{\frac{3x^2}{2}}{4x^2}$ ✓

$= \frac{3}{8}$ ✓

(e) Red = 40 = $2^3 \times 5$ ✓

Green = 60 = $2^2 \times 3 \times 5$ ✓

Black = 72 = $2^3 \times 3^2$ ✓

LCM = $2^3 \times 3^2 \times 5 = 360$

So red hose fills pool 9 times in 360 min.

green " " 6 " " "

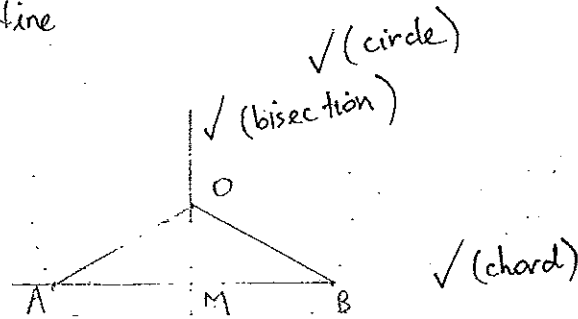
black " " 5 " " "

With all hoses used together the pool is filled 20 times and thus $\frac{360}{20} = 18$ minutes to fill pool once ✓

• Answer only earns 1/3. ✓

Question Nine

(a) (i)



(iv) $OA = OB$ (radii)
 $AM = BM$ (construction)
 OM is common
 $\therefore \triangle OAM \cong \triangle OBM$ (S.S.S)

(b) (i) The pattern of coordinates of centres is:

$$\left. \begin{array}{l} x \quad 2 \quad 5 \quad 8 \\ y \quad 2 \quad 3 \quad 4 \end{array} \right\} \Rightarrow \begin{array}{l} x = 3n - 1 \\ y = n + 1 \end{array}$$
 for some working

So square 34 has $x = 3 \times 34 - 1 = 101$ and $y = 34 + 1 = 35$

\therefore centre is $(101, 35)$ ✓ *Answers only*

(ii) Similarly the coordinates of LH vertex are acceptable here in (b).

$$\left. \begin{array}{l} x \quad 1 \quad 4 \quad 7 \\ y \quad 0 \quad 1 \quad 2 \\ n \quad 1 \quad 2 \quad 3 \end{array} \right\} \begin{array}{l} x = 3n - 2 \\ y = n - 1 \end{array}$$

So square 34 has $x = 3 \times 34 - 2 = 100$ and $y = 34 - 1 = 33$

\therefore LH vertex is $(100, 33)$ ✓

(ii) OR (Nicer) LH vertex of each square is 1 less horizontally and 2 less vertically so coordinates are $(100, 33)$.

(c) (i) Number of potatoes still to be peeled after n minutes:

Minutes	Fred	Peter
	x	$2x$
1	$x - 1$	$2x - 1$
2	$x - 3$	$2x - 1$
3	$x - 4$	$2x - 2$
4	$x - 6$	$2x - 2$
5	$x - 7$	$2x - 3$

• 3rd mark requires both Fred's Peter's piles i.e. 40 and 80.
 • Answers for both piles with no working earns 1/3.

$$\frac{2x-3}{x-7} = \frac{7}{3}$$

$$6x - 9 = 7x - 49$$

$$x = 40$$

At this stage Fred had 40 and Peter had 80 potatoes. ✓

(ii) Let P be the number of potatoes each had at the beginning. After n minutes Peter had twice as many as Fred so

$$2(p - n - \frac{n}{2}) = p - n + \frac{n}{2}$$

$$2(p - \frac{3n}{2}) = p - \frac{n}{2}$$

$$\therefore p = \frac{5n}{2}$$

$$\text{and } (p - \frac{3n}{2}) = 40$$

$$\text{So } \frac{5n}{2} - \frac{3n}{2} = 40$$

$$\therefore n = 40 \text{ and } p = 100$$

Each boy had 100 potatoes at the beginning. ✓

• Answer only earns 1/3.