SYDNEY GRAMMAR SCHOOL — YEARLY EXAMINATION 2002

MATHEMATICS FORM II

Time allowed: 2 hours

Exam date: 13th November 2002

Instructions:

All questions may be attempted.
All questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Calculators may NOT be used.

Collection:

Staple all your paper in one bundle.
Put your name, class, and master's initials on the front.

2A: GJ	2B: MLS	2C: JMR
2D: WMP	2E: JNC	2F: REN
2G: TCW	2H: BDD	2I: DS

Checklist:

A4 writing paper required.
Candidature: 174 boys.

QUESTION ONE

(a) Express $0{\cdot}27$ as a percentage.

(b) Evaluate the following:

(i) $3 + (8 - 10)$,

(ii) $\frac{3}{4} \div \frac{7}{9}$,

(iii) $24{\cdot}2 \div 11$,

(iv) $(0{\cdot}4)^2$.

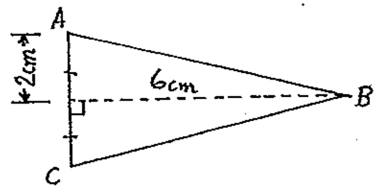
(c) Find 25% of \$16.

(d) Express $5{\cdot}3\%$ as a fraction.

(e) Convert $21{\cdot}75$ hectares to square metres.

(f) Find $\sqrt{64 + 36}$.

(g)



Find the area of $\triangle ABC$ in the diagram above.

QUESTION TWO

(a) Simplify:

(i) $3a - 7a$,

(ii) $\frac{x}{5} + \frac{x}{3}$.

(b) Simplify:

(i) $(7a)^2$,

(ii) $16x^2y \div 4x$,

(iii) $2p^2q \times 4q$.

(c) Expand $2(x - y)$.

(d) Simplify $\frac{2}{x} + \frac{1}{2x}$.

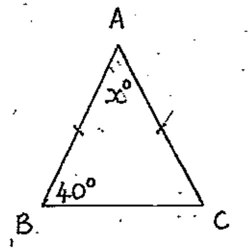
(e) Find the value of $a^2 - a$ if $a = -1$.

Exam continues next page ...

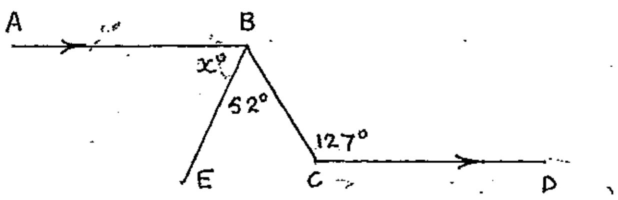
QUESTION THREE

(a) Find the value of x in each of the diagrams below. You must state all reasons.

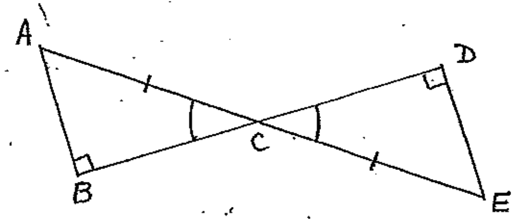
(i)



(ii)



(b)



In the diagram above, name a pair of congruent triangles, stating the congruence test used.

(c) Tear off the answer sheet to Questions 3(c) at the end of this paper. Do the constructions as set out on that page then staple it with the rest of your answers.

QUESTION FOUR

(a) Simplify the following ratios:

(i) 16 : 12,

(ii) $\frac{4}{5} : \frac{3}{8}$,

(iii) 3 metres : 250 centimetres.

(b) The ratio of John's savings to Sarah's savings is 9 : 5.

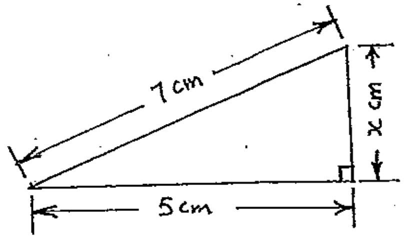
(i) If John has saved \$378, how much has Sarah saved?

(ii) If John and Sarah save \$378 between them, find how much each has saved.

(c) Concrete is made up of sand, gravel and cement. The ratio of sand to cement is 2 : 5 and the ratio of gravel to cement is 3 : 8. Find the ratio of sand to gravel.

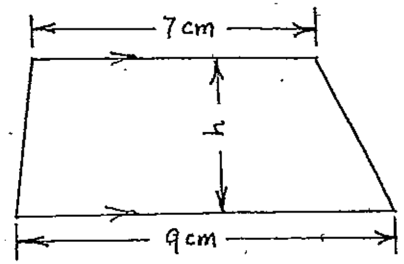
QUESTION FIVE

(a)



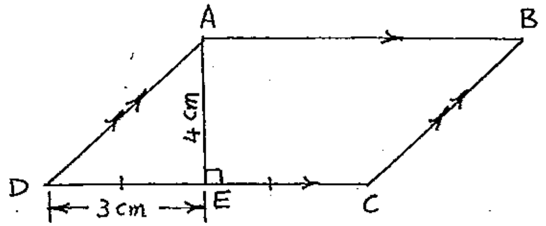
Find the value of x in the diagram above. Leave your answer in exact form.

(b)



The area of the trapezium above is $80\,\text{cm}^2$. Find its perpendicular height h.

(c)



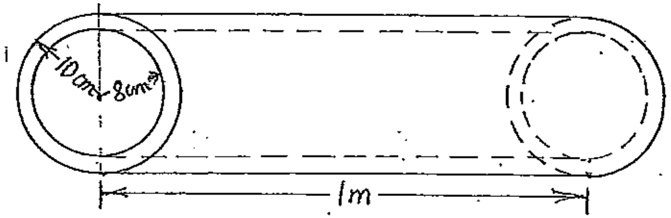
In the diagram above, $ABCD$ is a parallelogram and E is the midpoint of the side CD. AE perpendicular to DC, $AE = 4$ cm and $DE = 3$ cm.

(i) Find the area of $ABCD$.

(ii) Use Pythagoras' theorem to find the length of the side AD.

(iii) Find the perimeter of $ABCD$.

(d)



The cylindrical pipe shown above is made from metal. It has an inner radius of 8 cm and an outer radius of 10 cm and its length is 1 metre. Find the volume of concrete used to manufacture the pipe. Use the approximation $\pi \doteqdot 3{\cdot}14$.

QUESTION SIX

(a) Solve the following equations:

(i) $\frac{x}{2} = 5$,

(ii) $3t - 2 = 5t + 7$,

(iii) $3 - 7(1 - 2a) = 4(a - 3)$,

(iv) $\frac{2x - 3}{2} + \frac{x + 2}{3} = \frac{x - 1}{6}$.

(b) Solve the following inequation and graph your solution on a number line:

$7 + 2(x - 1) > 3x + 5$.

QUESTION SEVEN

(a) (i) Plot the points $A(8, 3)$, $B(8, 0)$ and $C(4, 3)$ on a number plane. Use a scale of 1 cm = 1 unit on both the x and the y axes.

(ii) Use Pythagoras' theorem to find the length of the interval BC in the $\triangle ABC$.

(iii) Indicate on your diagram the point D that will make the quadrilateral $ABCD$ a parallelogram. Write down the coordinates of this point.

(iv) Join BD and label the point of intersection of AC and BD with the letter E. Write down the coordinates of E.

(v) Given that E is the midpoint of both AC and BD, what property of a parallelogram does this confirm?

(b)

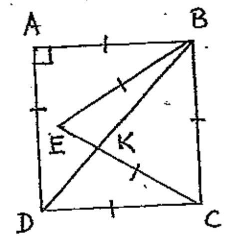
x	-1	0	2
y	-4	-1	5

The table above gives the coordinates of three points lying on a straight line. Find the equation of the line.

QUESTION EIGHT

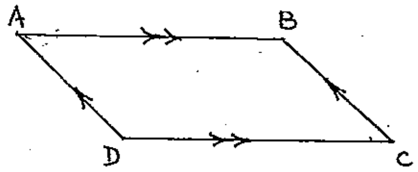
(a) Krissy swam the 25 km event at the Australian Open Swimming Championships in 6 hours and 40 minutes. What was her average speed in metres per second? Give your answer correct to the nearest whole number.

(b)



In the diagram above, $ABCD$ is a square in which the equilateral triangle BEC has been constructed. DB is a diagonal of the square and meets EC at K. Copy the diagram onto your answer sheet and find the size of $\angle EKB$, stating all reasons.

(c) Copy the diagram below and complete the proof. You need not copy the "Given", "Aim" and "Construction".



GIVEN: In the diagram above, $ABCD$ is a parallelogram with $AB \parallel DC$ and $AD \parallel BC$.

AIM: To prove:

(i) $\triangle ABC \equiv \triangle CDA$.

(ii) $AB = CD$ and $AD = CB$.

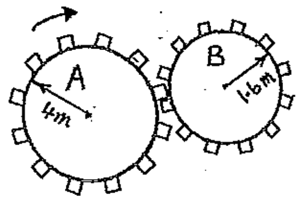
CONSTRUCTION: Construct AC.

PROOF: ....

QUESTION NINE

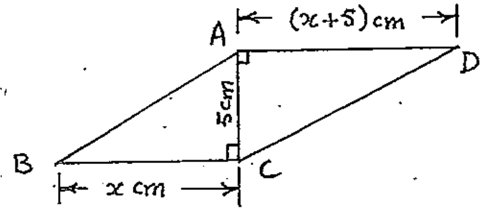
(a) Given that $R = \dfrac{P+T}{2P-T}$, find the value of T if $P = 1{\cdot}5$ and $R = 0{\cdot}6$.

(b)



In the diagram above, A and B are two cogwheels of radii 4 metres and 1·6 metres respectively. Wheel A rotates at 8 revolutions per minute in a clockwise direction and drives wheel B.
Find the rate at which B is turning, in revolutions per minute, and state the directon in which it turns. You must show all working.

(c)



In the diagram above, the ratio of the area of $\triangle ABC$ to the area of $\triangle ADC$ is $3 : 5$. Form an equation and solve it to find the value of x.

QUESTION TEN

(a) Simplify the expression $\dfrac{2x}{2x-3} - \dfrac{2x-3}{2x+3}$.

(b) Mike trains continuously for 2 hours. He spends 10 minutes out of each 15 minutes running and the remaining 5 minutes walking. If he ran for 5 minutes and walked for 10 minutes he would cover 6 km less distance. Mike runs three times as fast as he walks. Form an equation and solve it to find his walking speed.

(c) Solve the following old Hindu problem. You must show all reasoning.

"Arjuna, the son of Pritha, exasperated in combat, shot a quiver of arrows to slay Karna. With half his arrows he parried those of his antagonist; with four times the square root of the quiverful he killed his horses; with six arrows he slew Shalya the charioteer; with three he demolished the umbrella, standard and bow; and with one he cut off the head of the foe. How many were the arrows which Arjuna let fly?"

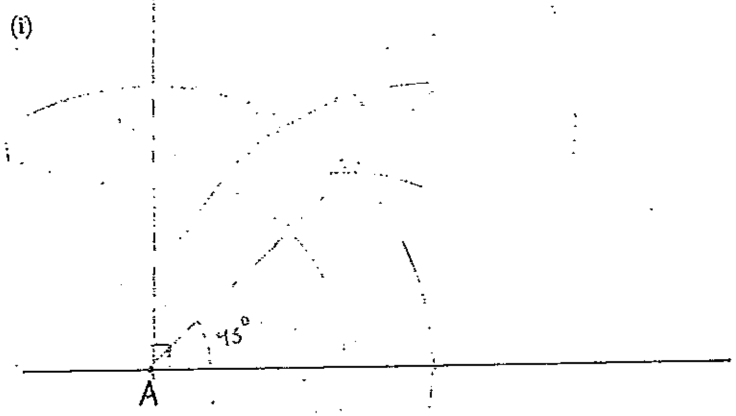
GJ

Name: .... Class: .... Master: ....

QUESTION THREE (Tear off this page and bundle it with the rest of the question)

In the following constructions use pencil, ruler and compasses only. All construction lines must be clearly drawn — do not erase them.

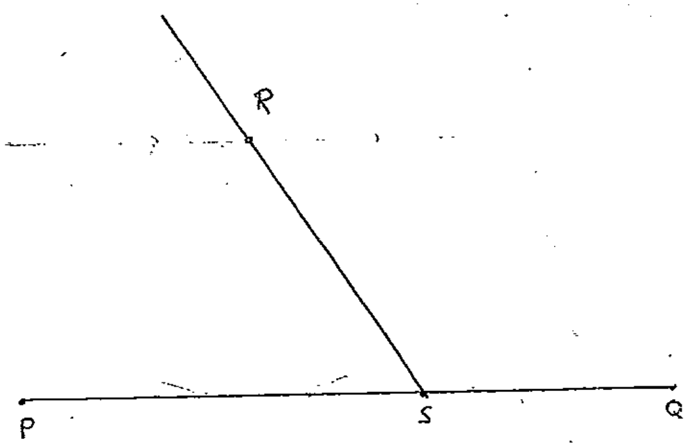
(c) (i)



(α) On the diagram above, construct an angle of 90° at the point A.

(β) Hence construct an angle of 45° at A.

(ii)



On the diagram above, construct a line through the point R parallel to the interval PQ.

PENALIZE UNITS ONCE ONLY AT QUESTION 1(g).

QUESTION ONE

(a) $0{\cdot}27 = 27\%$. ☑

(b) (i) $3 + (8 - 10) = 3 - 2$
$= 1$. ☑

(ii) $\frac{3}{4} \div \frac{7}{9} = \frac{3}{4} \times \frac{9}{7}$
$= \frac{27}{28}$. ☑

(iii) $24{\cdot}2 \div 11 = 2{\cdot}2$. ☑

(iv) $(0{\cdot}4)^2 = 0{\cdot}16$. ☑

(c) 25% of \$16 = \$4. ☑

(d) $5{\cdot}3\% = \frac{53}{1000}$. ☑

(e) $21{\cdot}75$ hectares $= 217500$ square metres. ☑

(f) $\sqrt{64 + 36} = 10$. ☑

(g) Area $= 12\,\text{cm}^2$. ☑

QUESTION TWO

(a) (i) $3a - 7a = -4a$. ☑

(ii) $\frac{x}{5} + \frac{x}{3} = \frac{8x}{15}$. ☑

(b) (i) $(7a)^2 = 49a^2$. ☑

(ii) $16x^2y \div 4x = 4xy$. ☑

(iii) $2p^2q \times 4q = 8p^2q^2$. ☑

(c) $2(x - y) = 2x - 2y$. ☑

(d) $\frac{2}{x} + \frac{1}{2x} = \frac{4+1}{2x}$ ☑
$\frac{5}{2x}$. ☑

(e) $a^2 - a = (-1)^2 - (-1)$ ☑
$= 1 + 1$
$= 2$. ☑

QUESTION THREE

(a) (i) One mark for correct angle and one mark for correct reasons.
$\angle ACB = x°$ (angles opposite equal sides)
$x + 40 + 40 = 180$ (angle sum of triangle)
$x = 100$. ☑☑

(ii) One mark for correct angle and one mark for correct reason.
$x + 52 = 127$ (alternate angles, $AB \parallel CD$)
$x = 75$. ☑☑

(b) Vertices must be named in corresponding order.
$\triangle ABC \equiv \triangle EDC$. ☑
~~SAS~~ AAS test. ☑

(c) (i) One mark for constructing 90° angle. ☑
One mark for bisection to give 45°. ☑
Tolerance is ±2°.

(ii) One mark for arcs. ☑
One mark for parallel line. ☑
Tolerance is ±2°.

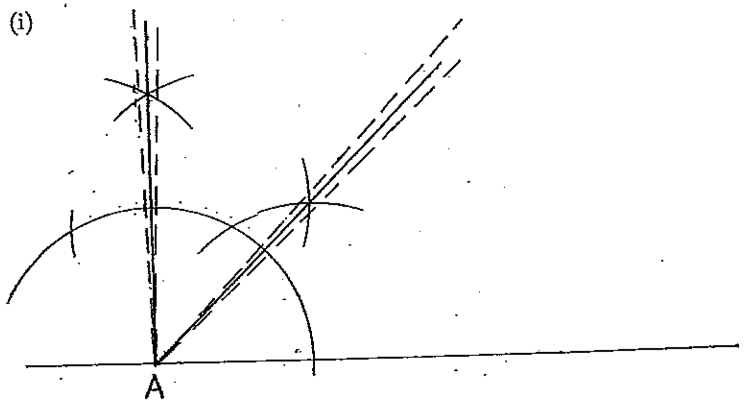
SEE NEXT PAGE FOR CONSTRUCTIONS.

Name: .... Class: .... Master: ....

QUESTION THREE (Tear off this page and bundle it with the rest of the question)

In the following constructions use pencil, ruler and compasses only. All construction lines must be clearly drawn — do not erase them.

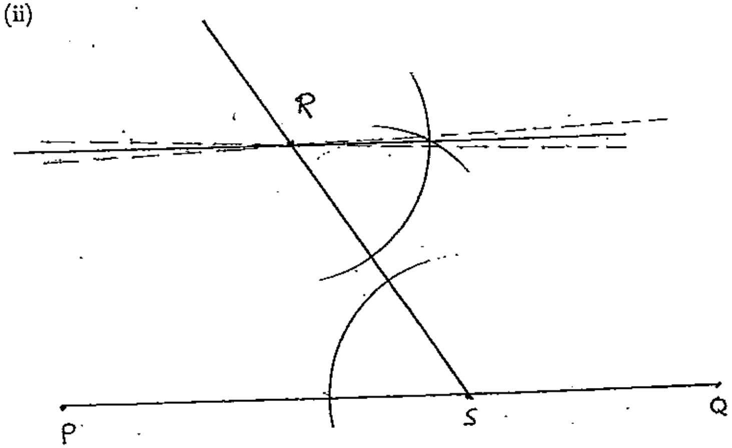
(c) (i)



(α) On the diagram above, construct an angle of 90° at the point A.

(β) Hence construct an angle of 45° at A.

(ii)



On the diagram above, construct a line through the point R parallel to the interval PQ.

QUESTION FOUR

(a) (i) $16 : 12 = 4 : 3$ ✓

(ii) $\frac{4}{5} : \frac{3}{8} = 32 : 15$ ✓

(iii) 3 metres : 250 centimetres $= 300 : 250$ ✓

$= 6 : 5.$ ✓

(b) (i) Here 9 parts is \$378.

$\boxed{\div 9}$ 1 part is \$42. ✓

$\boxed{\times 5}$ 5 parts is \$210.

Hence Sarah has saved \$210. ✓

OR

Let \$$s$ be Sarah's savings.

Then John's savings are $\dfrac{9s}{5}$.

Hence $\dfrac{9s}{5} = 378$ ✓

$\boxed{\times \frac{5}{9}}$ $s = 378 \times \dfrac{5}{9}$

$= 210.$

Thus Sarah has saved \$210. ✓

(ii) Here 14 parts is \$378.

$\boxed{\div 14}$ 1 part is \$27. ✓

$\boxed{\times 5}$ 5 parts is \$135.

$\boxed{\times 9}$ 9 parts is \$243.

Hence Sarah has saved \$135, and John has saved \$243. ✓

OR

Let \$$s$ be Sarah's savings.

Again, John's savings are $\dfrac{9s}{5}$.

Hence $s + \dfrac{9s}{5} = 378$ ✓

$\boxed{\times 5}$ $5s + 9s = 378 \times 5$

$\boxed{\div 14}$ $s = 135.$

Thus Sarah has saved \$135, and John has saved \$243. ✓

(iii) $x : (x + y) = 9 : 14$

$x : 378 = 9 : 14$

$x = \frac{9}{14} \times 378$ ✓

$x = 243.$

John saves \$243 and Sarah saves \$135. ✓

(c) sand : cement $= 2 : 5$

$= 16 : 40$

gravel : cement $= 3 : 8$

$= 15 : 40$

sand : gravel $= 16 : 15.$ ✓✓

QUESTION FIVE

(a) $x^2 + y^2 = 7^2$

$x^2 + 25 = 49$ $\boxed{\surd}$

$x^2 = 24$

$x = \sqrt{24}.$ $\boxed{\surd}$

(b) $80 = \frac{1}{2}(7+9)h$ $\boxed{\surd}$

$80 = 8h$

$h = 10\,\text{cm}$ $\boxed{\surd}$

(c) (i) Area $= 6 \times 4$

$= 24\,\text{cm}^2.$ $\boxed{\surd}$

(ii) $AD^2 = 3^2 + 4^2$

$= 25$

$AD = 5\,\text{cm}.$ $\boxed{\surd}$

(iii) Perimeter $= 2 \times 5 + 2 \times 6$

$= 22\,\text{cm}.$ $\boxed{\surd}$

(d) Volume $= \pi \times 10^2 \times 100 - \pi \times 8^2 \times 100$

$= \pi \times 100 \times 100 - \pi \times 64 \times 100$ $\boxed{\surd}$

$= 10000\pi - 6400\pi$

$= 3600\pi$

$\doteqdot 3600 \times 3{\cdot}14$ $\boxed{\surd}$

$= 11304\,\text{cm}^3.$ $\boxed{\surd}$

QUESTION SIX

(a) (i) $\frac{x}{2} = 5$

$x = 10.$ $\boxed{\surd}$

(ii) $3t - 2 = 5t + 7$

$2t = -9$

$t = -4\frac{1}{2}.$ $\boxed{\surd\surd}$

(iii) $3 - 7(1 - 2a) = 4(a - 3)$

$3 - 7 + 14a = 4a - 12$

$10a = -8$

$a = -\frac{4}{5}.$ $\boxed{\surd\surd}$

(iv) $\frac{2x-3}{2} + \frac{x+2}{3} = \frac{x-1}{6}$

$3(2x - 3) + 2(x + 2) = x - 1$

$6x - 9 + 2x + 4 = x - 1$

$8x - 5 = x - 1$

$7x = 4$

$x = \frac{4}{7}.$ $\boxed{\surd\surd}$

(b) $7 + 2(x - 1) > 3x + 5$

$7 + 2x - 2 > 3x + 5$

$2x + 5 > 3x + 5$

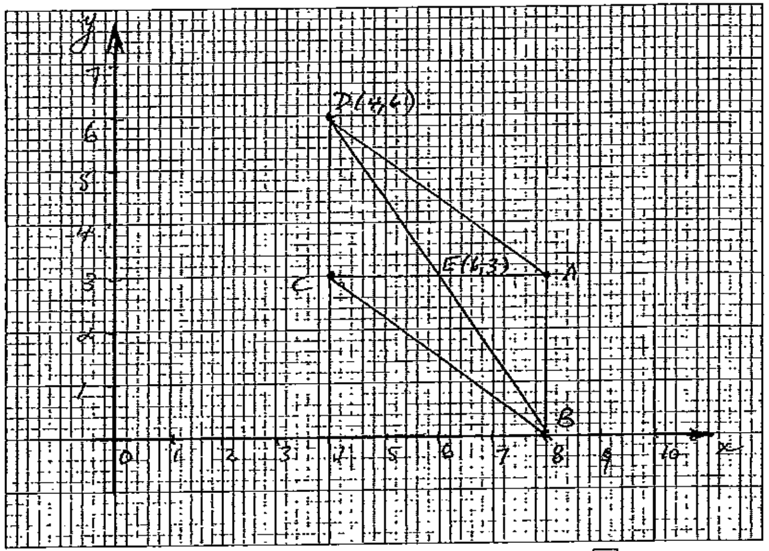
$0 > x$

$x < 0.$ $\boxed{\surd\surd}$

$\boxed{\surd}$

QUESTION SEVEN

(a) (i)



Points A, B and C. [√√] Axes and labels [√]

(ii) $AC = 4$

$AB = 3$

$BC^2 = 4^2 + 3^2$

$= 25$

$BC = 5.$ [√√]

(iii) $D(4,6)$. [√point on graph and coordinates]

Allow $(4,0)$ and $(12,0)$.

(iv) $E(6,3)$. [√allow ±0·5 in x-value]

(v) The diagonals of a parallelogram bisect one another. [√]

(b) $y = 3x - 1$. [√for 3 √for -1]

QUESTION EIGHT

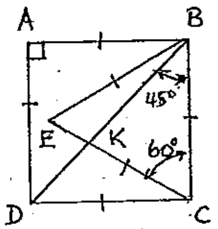
(a) Average speed $= \dfrac{25 \times 1000}{400 \times 60}$ [√]

$= \dfrac{25}{24}$

$\doteqdot 1.$

Krissy's average speed is 1 metre per second. [√]

(b)



Deduct one mark for each error or for each omission.

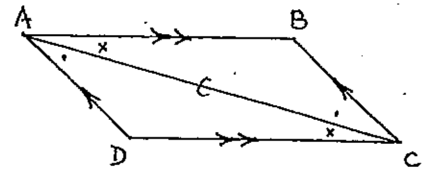
$\angle DBC = 45°$ (diagonal of a square bisects angle) [√]

$\angle ECB = 60°$ (angle of an equilateral triangle) [√]

$\angle EKB = 60° + 45°$ (exterior angle of $\triangle EKB$)

$= 105°.$ [√]

(c)



Deduct one mark for each error or for each omission.

In $\triangle ADC$ and $\triangle ABC$,

$\angle DAC = \angle ACB$ (alternate angles, $AD \parallel BC$) [√]

$\angle BAC = \angle ACD$ (alternate angles, $AB \parallel CD$) [√]

$AC = AC$ (common side) [√]

so $\triangle ACD \equiv \triangle CAB$ (AAS) [√]

so $AB = CD$ (matching sides of congruent triangles)

and $AD = CB$ (matching sides of congruent triangles) [√]

QUESTION NINE

(a)
$$R = \frac{P+T}{2P-T}$$
$$0{\cdot}6 = \frac{1{\cdot}5+T}{3-T} \quad \boxed{\checkmark}$$
$$1{\cdot}8 - 0{\cdot}6T = 1{\cdot}5 + T \quad \boxed{\checkmark}$$
$$0{\cdot}3 = 1{\cdot}6T$$
$$16T = 3$$
$$T = \frac{3}{16} \quad \boxed{\checkmark}$$

(b) Let x be the number of revolutions per minute for cogwheel B.

Metres travelled per minute by a tooth on A $= 2 \times \pi \times 4 \times 8$
$= 64\pi$. $\boxed{\checkmark}$

Metres travelled per minute by a tooth on B $= 2 \times \pi \times 1{\cdot}6 \times x$
$= 3{\cdot}2\pi x$ $\boxed{\checkmark}$

so $64\pi = 3{\cdot}2\pi x$
$x = 20$. $\boxed{\checkmark}$

Wheel B rotates at 20 r.p.m. in an anticlockwise direction. $\boxed{\checkmark}$

(c) Area $\triangle ABC = \frac{1}{2} \times x \times 5$
$= \frac{5x}{2}$

Area $\triangle ADC = \frac{1}{2} \times (x+5) \times 5$
$= \frac{5(x+5)}{2}$ $\boxed{\checkmark \text{for either } \triangle ABC \text{ or } \triangle ADC}$

$$\frac{5x}{2} : \frac{5(x+5)}{2} = 3 : 5 \quad \boxed{\checkmark}$$
$$\frac{x}{x+5} = \frac{3}{5}$$
$$5x = 3x + 15$$
$$2x = 15$$
$$x = 7\tfrac{1}{2}. \quad \boxed{\checkmark}$$

QUESTION TEN

(a)
$$\frac{2x}{2x-3} - \frac{2x-3}{2x+3} = \frac{2x(2x+3)-(2x-3)^2}{(2x-3)(2x+3)} \quad \boxed{\checkmark}$$
$$= \frac{4x^2+6x-4x^2+12x-9}{(2x-3)(2x+3)} \quad \boxed{\checkmark}$$
$$= \frac{18x-9}{4x^2-9} \quad \boxed{\checkmark \text{accept denominator in factored form}}$$

(b) Let Mike's walking speed be x km/h so his running speed is $3x$ km/h.
There are 8 lots of 15 minutes in 2 hours.

Situation 1.

Running time $= 8 \times \frac{1}{6}$
$= \frac{4}{3}$ hours.

Running distance $= \frac{4}{3} \times 3x$ km.
$= 4x$ km.

Walking time $= 8 \times \frac{1}{12}$
$= \frac{2}{3}$ hour.

Walking distance $= \frac{2x}{3}$ km.

Total distance $= 4x + \frac{2x}{3}$
$= \frac{14x}{3}$ km. $\boxed{\checkmark}$

Situation 2.

Running time $= 8 \times \frac{1}{12}$
$= \frac{2}{3}$ hour.

Running distance $= \frac{2}{3} \times 3x$
$= 2x$ km.

Walking time $= 8 \times \frac{1}{6}$
$= \frac{4}{3}$ hours.

Walking distance $= \frac{4}{3} \times x$
$= \frac{4x}{3}$ km.

Total distance $= \frac{4x}{3} + 2x$
$= \frac{10x}{3}$ km. $\boxed{\checkmark}$

Now $\frac{14x}{3} - \frac{10x}{3} = 6$ $\boxed{\checkmark}$
$4x = 18$
$x = 4\frac{1}{2}$

So Mike's walking speed is $4\frac{1}{2}$ km/h. $\boxed{\checkmark}$

(c) Let the quiver hold n arrows.
$$\tfrac{1}{2}n + 4\sqrt{n} + 6 + 3 + 1 = n \quad \boxed{\checkmark}$$
$$n + 8\sqrt{n} + 20 = 2n$$
$$n - 20 = 8\sqrt{n}. \quad \boxed{\checkmark}$$

Guess and check square numbers to give $n = 100$.
There were 100 arrows in the quiver. $\boxed{\checkmark}$

GJ