MLC SCHOOL SYDNEY
MATHEMATICS

2/3 Unit Common Paper

Year 11 Yearly

1998

Time Allowed: 2 hours

Directions to Candidates:

All questions may be attempted.

Begin each question on a new page.

Write your name clearly on each page

Marks may not be awarded for careless or badly arranged work

Do NOT write on the back of your pages

Show all necessary working for each question

Question 1

(a) Solve for x:

$$4(x - 1) - 2 = 26$$

(b) Find the value of $\sqrt{20 - tv}$ correct to 3 significant figures when

$$t = 5.3 \times 10^{-3} \quad \text{and } v = 7.8 \times 10^{-2}$$

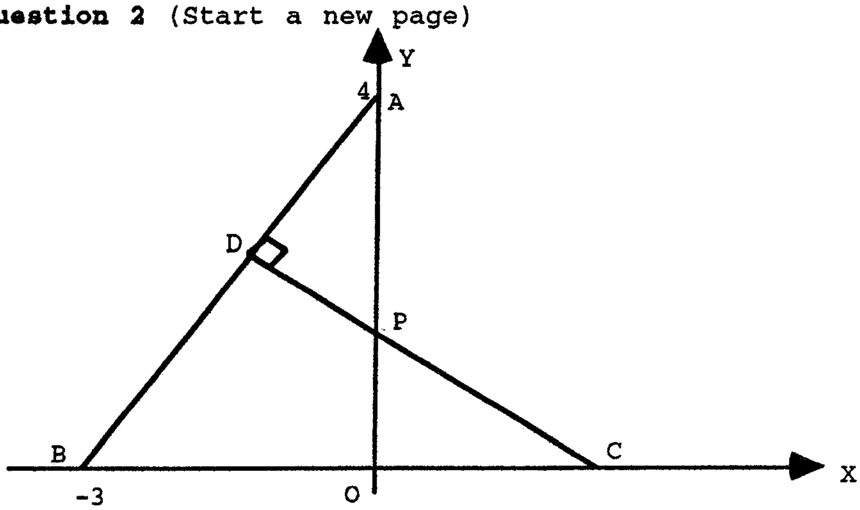
(c) Express $\dfrac{6}{7 - 2\sqrt{5}}$ with a rational denominator.

(d) Factorise $3x^2 - 2x - 5$

(e) Make a the subject of $b = \sqrt{a - 2c}$

(f) Find the value(s) of x for which $|5 - 2x| \geq 3$

Question 2 (Start a new page)



In the diagram, AB = BC and CD is perpendicular to AB. CD intersects the y axis at P.

Copy the diagram onto your answer sheet.

(a) Find the exact length of AB

(b) Explain why the co-ordinates of C are (2,0)

(c) Show the equation of CD is $3x + 4y = 6$

(d) Find the co-ordinates of P

(e) Use Pythagoras' Theorem on ΔPOC to show the length of CP is $2\frac{1}{2}$ units.

(f) If AC is $2\sqrt{5}$ and P is $(0,\frac{3}{2})$, use the Cos Rule to find $\angle APC$ to the nearest degree.

(g) On your diagram, extend the line $3x + 4y = 6$ in both directions and shade in the area defined by the inequality

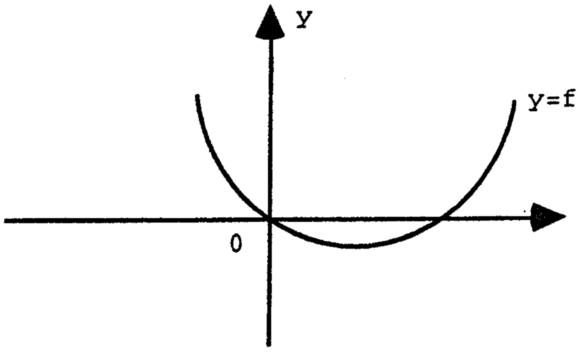
$$3x + 4y \leq 6$$

(h) Use inequalities to define the area of ΔPOC. DO NOT FIND THE AREA!

Question 3 (Start a new page)

(a) $f(x) = ax^2 + bx + c$

(i) the graph of $f(x)$ is shown below



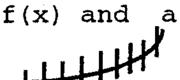
(i) Copy the diagram onto your answer sheet. Label the minimum point of $f(x)$ and draw in and label the axis of symmetry.

(ii) Is $f(x)$ positive definite, negative definite, or is it indefinite?

(iii) Is the discriminant $(b^2 - 4ac)$ greater than 0, equal to 0 or less than 0 for $f(x)$?

(iv) Find the value of c.

(v) If $(x - 2)$ is a factor of $f(x)$ and $a = 1$, find b.

(vi) On your diagram, hatch in  or go over in red pen, those parts of the graph of $f(x)$ for which $f(x) > 0$

b) Find A, B, C if $x^2 \equiv A(x-2)^2 - B(x-1) + 5C$.

c) Solve the inequality $x^2 - 1 < 0$ and plot your solutions on the number plane.

Question 4 (Start a new page)

(a) If $\cos\theta = \frac{1}{\sqrt{2}}$ in the domain $0 \leq \theta \leq 360$, give the possible values of θ.

(b) If θ is an acute angle and $\tan\theta = a$, express $\cos\theta$ and $\sin\theta$ in terms of a.

(c) Given that $\sin\theta = -\frac{1}{2}$ and $\cos\theta = -\frac{\sqrt{3}}{2}$. What is $\tan\theta$ expressed exactly as a ratio?

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(d)

(i) Solve the equation $x^2 - 2x + 1 = 0$

(ii) We know the identity $\sin^2\theta + \cos^2\theta = 1$. Use this identity to show that $\tan^2\theta + 1 = \sec^2\theta$.

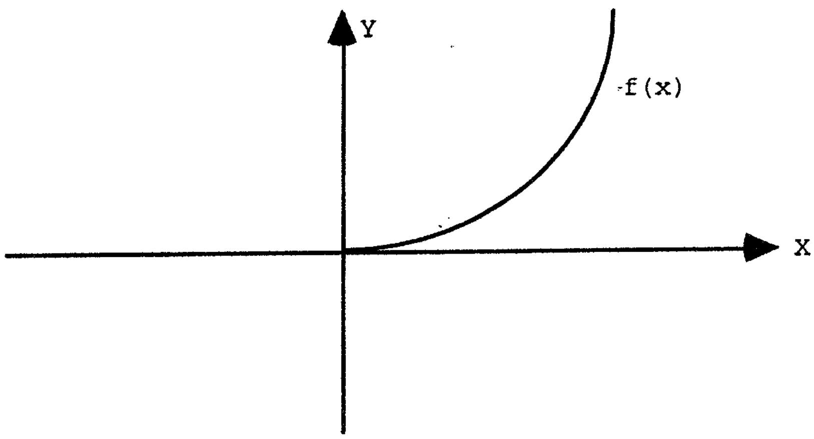
(iii) Given that $\tan^2\theta + 1 = \sec^2\theta$, by substituting for $\sec^2\theta$ in the expression $\sec^2\theta - 2\tan\theta = 0$, rewrite $\sec^2\theta - 2\tan\theta = 0$ so that you have an equation in terms of $\tan\theta$

(iv) By using your results from part (i) or otherwise, find what values of θ solve this equation for the domain $0 \leq \theta \leq 360$.

Question 5 (Start a new page)

(a)

(i) Copy the graph below, making sure the diagram is at least $\frac{1}{3}$ of a page, is neat and labelled properly.



(ii) In the graph you have just copied, f(x) is an ODD function. Using this information, extend your graph on the left hand side of the Y axis.

(b) $h(x) = x^2 - 4x + 4$

(i) What is the domain of h(x)?

(ii) By factorising or otherwise, find the values of x which solve the equation $x^2 - 4x + 4 = 0$.

(iii) Can h(x) ever be negative? Why?

(iv) What is the range of h(x)?

(c) The function g(x) is defined as :

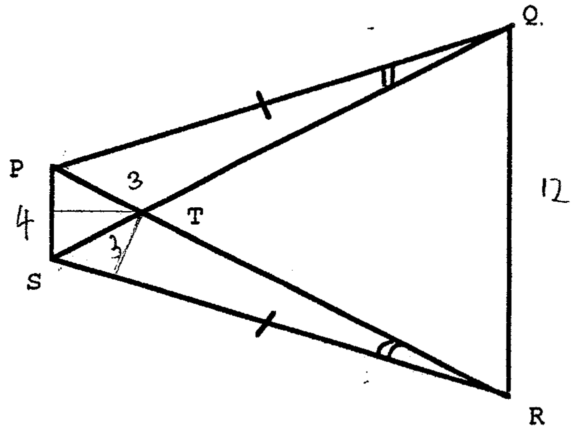
$$g(x) = \begin{cases} 2x & \text{for } -4 \leq x < 0 \\ 9 - x^2 & \text{for } 0 \leq x \leq 3 \end{cases}$$

(i) Find the value of g(0)

(ii) Sketch y = g(x)

Question 6 (Start a new page)

In the diagram below, PQRS is a quadrilateral with PQ=SR and $\angle PQS = \angle PRS$



(a) Prove that $\Delta PQT \equiv \Delta SRT$

(b) PS = 4, QR = 12, and PT = 3, find TR

(c) Why is PS || QR ?

(d) What is the mathematical name of the quadrilateral PQRS?

(e) What would we need to know in order to calculate the area of quadrilateral PQRS? DO NOT FIND THE AREA!

2 UNIT YR 11 1998 - SOLUTIONS.

Answers:

1 (a)

$$4(x-1) - 2 = 26$$
$$4(x-1) = 28$$
$$(x-1) = 7$$
$$x = 8$$

(b) $\sqrt{20 - tv} = 4.47$

(c) $\dfrac{6}{7 - 2\sqrt{5}} = \dfrac{6(7 + 2\sqrt{2})}{29}$

(d) $3x^2 - 2x - 5 = (3x - 5)(x + 1)$

(e) $b = \sqrt{a - 2c}$

$b^2 = a - 2c$

$a = b^2 + 2c$

(f) (i) $|5 - 2x| \geq 3$

$5 - 2x \geq 3$ $\qquad$ $-(5 - 2x) \geq 3$

$2 \geq 2x$ $\qquad$ $-5 + 2x \geq 3$

$1 \geq x$ $\qquad$ $2x \geq 8$

$\qquad$ $x \geq 4$

OR

(ii) $|5 - 2x| \geq 3$ square both sides

$25 - 20x + 4x^2 \geq 9$

$16 - 20x + 4x^2 \geq 0$

$4 - 5x + x^2 \geq 0$

$(4 - x)(1 - x) \geq 0$

$1 \geq x$ or $x \geq 4$

Answers:

2 (a) By Pythagoras, in ΔABO, $AB^2 = BO^2 + AO^2$

$AB = \sqrt{9 + 16}$

$AB = 5$

Some students familiar with the 3,4,5 triangle may be able to write this straight down.

(b) Since AB = BC = 5, and since BO is 3 units, OC must be 2 units. C is on the x axis so the co-ordinates of C must be (2, 0)

(c) CD ⊥ AB. The gradient of AB $= \dfrac{4 - 0}{0 - (-3)}$

$= \dfrac{4}{3}$

∴ gradient of CD $= -\dfrac{3}{4}$

In intercept gradient form the straight line CD is:

$y = mx + b$

substituting $-\dfrac{3}{4}$ for m

$y = -\dfrac{3}{4}x + b$

CD passes through the point (2, 0) so substituting

$0 = -\dfrac{3}{2} + b$

$b = \dfrac{3}{2}$

so $y = -\dfrac{3}{4}x + \dfrac{3}{2}$

and $3x + 4y = 6$

Alternatively, they could show that the gradient of $3x + 4y = 6$ is $-\dfrac{3}{4}$ which is perpendicular to AB and also that the line passes through C.

(d) $P = \dfrac{3}{2}$

(e) $CP = \sqrt{OP^2 + OC^2}$

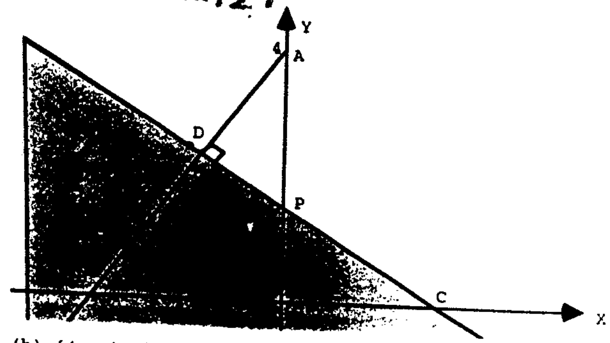
$= \sqrt{(\frac{3}{2})^2 + 2^2}$

$= \sqrt{\frac{25}{4}}$

2 (f) $\cos APC = \dfrac{AP^2 + PC^2 - AC^2}{2AP.PC}$

$= -0.6$

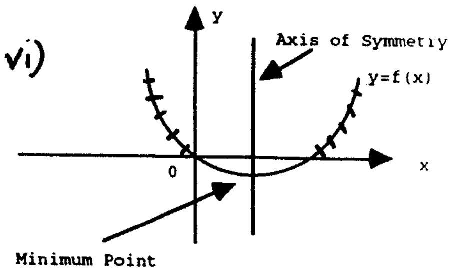
$\angle APC = 127°$



(h) $\{(x,y): 3x + 4y \leq 6 \cup 0 \leq y \cup 0 \leq x\}$

Answers:

3a) (i)



(ii) f(x) is indefinite

(iii) the discriminant > 0

(iv) c = 0

(v) $f(2) = 0$ if $(x - 2)$ is a factor

$4 + 2b = 0$

$b = -2$

b)

$x^2 = A(x-2)^2 - B(x-1) + 5C.$

if x = 0 $\qquad 0 = 4A + B + 5C$ ----------[1]

if x = 2 $\qquad 4 = - B + 5C$ ----------[2]

if x = 1 $\qquad 1 = A + 5C$ ----------[3]

[1] + [2] $\qquad 4 = 4A + 10C$

$\qquad 2 = 2A + 5C$ ----------[4]

[4]-[3] $\qquad 1 = A$ subst in [4]

$\qquad C = 0$

$\qquad B = -4$

OR

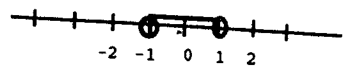
$x^2 = A(x-2)^2 - B(x-1) + 5C.$

$x^2 = Ax^2 - (4A + B)x + 4A + B + 5C$

so A = 1, B = -4, C = 0

c) $|x| = 1$

$-1 < x < 1$



Answers:

4(a) $\theta = 45,\ 315$

(b) $\cos\theta = \dfrac{1}{\sqrt{1 + a^2}}$ $\sin\theta = \dfrac{a}{\sqrt{1 + a^2}}$

(c) $\tan = \dfrac{1}{\sqrt{3}}$

(d)

(i) $x = 1$

(ii) $\sin^2\theta + \cos^2\theta = 1$ divide both sides by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

(iii) $\sec^2\theta - 2\tan\theta = 0$

$$\tan^2\theta + 1 - 2\tan\theta = 0$$

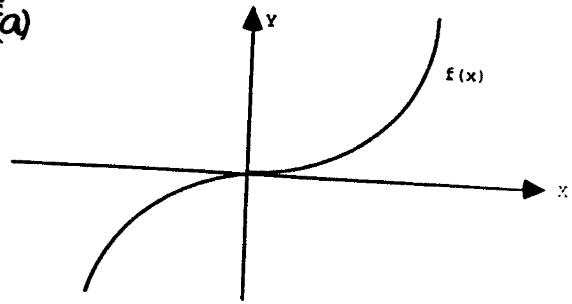
$$\tan^2\theta - 2\tan\theta + 1 = 0$$

(iv) from (i) $\tan\theta = 1$

$$\theta = 45,\ 225$$

(a)

5(a)



(b) (i) $\infty \le x \le \infty$

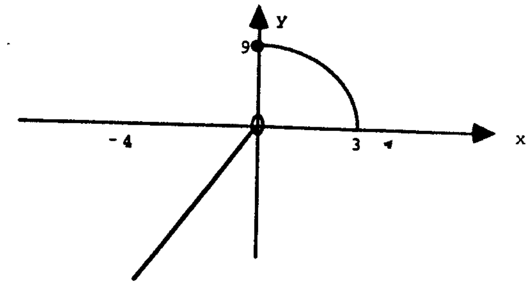
(ii) $x = 2$

(iii) $h(x)$ is a perfect square, $\therefore h(x) \ge 0$

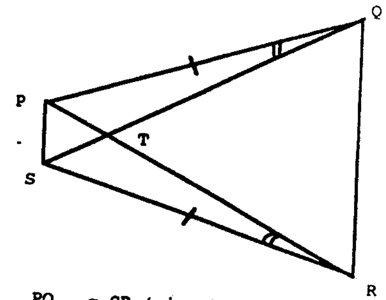
(iv) $0 \le y$

(c) (i) $g(0) = 9$

5cont'd.



6.



(a)

$PQ = SR$ (given)

$\angle PQS = \angle PRS$ (given)

$\angle PTQ = \angle STR$ (vertically opposite)

$\therefore \Delta PQT \equiv \Delta SRT$ (AAS)

(b) In ΔQRT, $TQ = TR$ (corresponding sides of congruent triangles, $\Delta PQT \equiv \Delta SRT$ shown above)

$\therefore \Delta QRT$ is an isosceles Δ

$\therefore \angle TQR = \angle QRT$ (base angles of isosceles Δ)

Similarly in ΔPTS, $\angle SPT = \angle PST$

But $\angle PTS = \angle QTR$ (vertically opposite)

so $\angle SPT = \angle PST = \frac{1}{2}(180 - \angle PTS)$ (angle sum of triangle)

and similarly $\angle TQR = \angle QRT = \frac{1}{2}(180 - \angle PTS)$

Hence $\Delta PQT \,|||\, \Delta SRT$

$$\therefore \frac{PS}{QR} = \frac{PT}{TR} = \frac{4}{12}$$

$$\text{so } \frac{TR}{3} = \frac{12}{4}$$

$$\therefore TR = 9$$

(c) Since $\Delta PQT \,|||\, \Delta SRT$

$\angle PSQ = \angle SQR$, alternate angles

$\therefore PS \parallel QR$

(d) trapezium

(e) The perpendicular distance between PS and QR