

MARCELLIN COLLEGE RANDWICK



YEAR 12  
MATHEMATICS

HSC ASSESSMENT TASK 3  
2014

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STUDENT NAME: \_\_\_\_\_ MARK \_\_\_\_\_ /30

TEACHER: \_\_\_\_\_

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TIME ALLOWED: 45 minutes  
WEIGHTING: 20 %

**Directions:**

- Answer multiple choice questions on the page provided.
- Use a new sheet for each question.
- Show all necessary working. Where more than one mark is allocated to a question, full marks may not be awarded for answers only.
- Marks may not be awarded for careless or badly arranged work.
- Calculators may be used

**Section I**  
Start each question on a new sheet of paper

**Question 1 (8 marks)**

- |   | Marks |
|---|-------|
| a) Differentiate $e^{-3x^2}$  | 1     |
| b) Find $\int \frac{4x}{x^2+1} dx$  | 2     |
| c) Find the equation of the tangent to the curve $y = e^{x^2+1}$ at the point where $x = 1$ | 2     |
| d) Evaluate $2 \log_4 6 - \log_4 9$   | 1     |
| e) Solve $e^{2x} - 5e^x + 4 = 0$ leaving your answers in exact form                         | 2     |

**Question 2 (8 marks)**

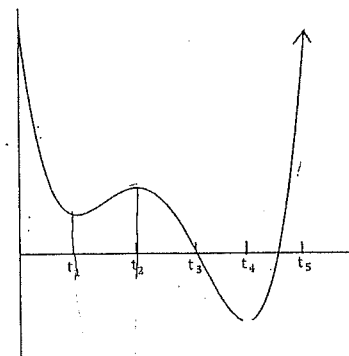
Start a new page

- |  | <b>Marks</b> |
|--|--------------|
| a) Differentiate $x^3 \cos x$  | 2            |
| b) Find $\int 3 \sin 4x \, dx$   | 2            |
| c) Solve $2 \sin \theta = \sqrt{3}$ for $0 \leq \theta \leq 2\pi$  | 2            |
| d) Calculate the exact volume of the solid of revolution formed when the region bounded by the curve $y = \sec x$ , the lines $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$ and the $x$ -axis is rotated about the $x$ -axis. | 2            |

**Question 3 (9 marks)**

Start a new page

- |   | <b>Marks</b> |
|---|--------------|
| a) A particle starts from rest 5m from a fixed point O and moves in a straight line with an acceleration of $a \, \text{ms}^{-2}$ where $a = 3t - 4$ .  |              |
| i) Find an expression for the velocity of the particle at any time  | 2            |
| ii) Find an expression for the position of the particle at any time $t$   | 2            |
| b) A town's population increases according to the equation $P = P_0 e^{kt}$ where $t$ is in years. If the population increases from 9 000 to 12 500 after five years, how many years will it take for the population to triple? (answer to 1 decimal place) | 3            |
| c) The graph below shows the displacement of a particle over time.  |              |



Sketch a graph (on the sheet provided) that shows the velocity of the particle

END OF SECTION 1

**Section II – Multiple Choice (5 marks)**

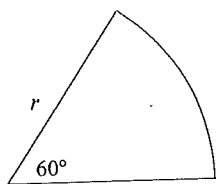
Attempt all questions

Use the Multiple Choice Answer Sheet for Questions 4 to 8

4. A particle moves along a straight line so that its distance  $x$  from a fixed point  $O$  is given by  $x = \cos t + t$  where  $t$  is the time measured in seconds. When does the particle first come to rest?

- (A)  $\frac{\pi}{4}$  seconds    (B)  $\frac{3\pi}{2}$  seconds    (C)  $\frac{3\pi}{4}$  seconds    (D)  $\frac{\pi}{2}$  seconds

5. The area of the sector below is  $10\pi$  units<sup>2</sup>. What is the value of  $r$ ?



Not to scale

- (A)  $\sqrt{60}$     (B)  $\sqrt{60}\pi$     (C)  $\sqrt{\frac{\pi}{3}}$     (D)  $\sqrt{\frac{1}{3}}$

6. What is the solution of  $4^x = 32$ ?

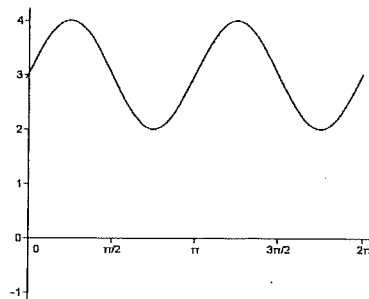
- (A)  $x = 0.4$     (B)  $x = 2.5$     (C)  $x = 3$     (D)  $x = 8$

7. What is the value of  $\int_0^1 (e^{2x} + 1) dx$

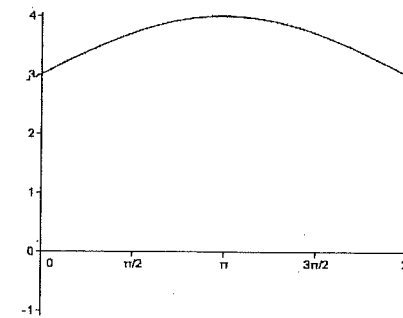
- (A)  $e^2$     (B)  $\frac{1}{2}(e^2 + 1)$     (C)  $\frac{1}{2}e^2$     (D)  $e^2 + 1$

8. Which graph represents the function  $y = 3 + \sin 2x$  for  $0 \leq x \leq 2\pi$ .

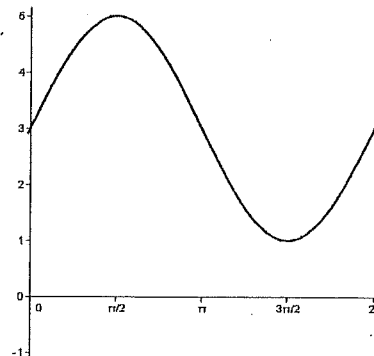
(A)



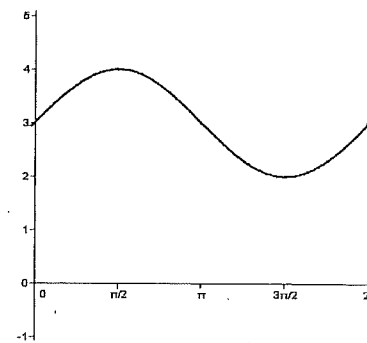
(B)



(C)



(D)



a)  $\frac{d}{dx} e^{-3x^2} = -6xe^{-3x^2}$  ✓

b)  $\int \frac{4x}{x^2+1} dx$   
 $= 2 \int \frac{2x}{x^2+1} dx$   
 $= 2 \ln(x^2+1) + C$  ✓✓

c)  $y = e^{x^2+1}$   
 $\frac{dy}{dx} = 2x e^{x^2+1}$

when  $x = 1$

$\frac{dy}{dx} = 2(1)e^{(1)^2+1}$   
 $= 2e^2$

$y = e^{(1)^2+1}$   
 $= e^2$

$y - y_1 = m(x - x_1)$

$y - e^2 = 2e^2(x - 1)$   
 $= 2e^2x - 2e^2$   
 $= 2e^2x - e^2$

$0 = 2e^2x - y - e^2$  ✓✓

d)  $2 \log_4 6 - \log_4 9$

$= \log_4 6^2 - \log_4 9$

$= \log_4 \left(\frac{36}{9}\right)$

$= \log_4 4 = 1$  ✓

e) let  $e^x = u$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4$

$u = 1$

$e^x = 4$

$x = \ln 4$

$e^x = 1$

$x = \ln 1$

$= 0$

$\therefore x = \ln 4, 0$  ✓✓

a)  $\frac{d}{dx} x^3 \cos x$

$v = x^3$

$v' = 3x^2$   
 $u = \cos x$

$u' = -\sin x$

$y = vu' + uv'$

$= x^3(-\sin x) + 3x^2 \cos x$

$= x^2(-x \sin x + 3 \cos x)$

$= x^2(3 \cos x - x \sin x)$  ✓✓

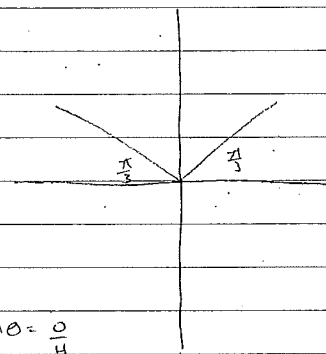
b)  $\int 3 \sin 4x dx$

$= 3 \left( \frac{-\cos 4x}{4} \right) + C$

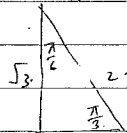
$= -\frac{3}{4} \cos 4x + C$  ✓✓

c)  $2 \sin \theta = \sqrt{3}$

$\sin \theta = \frac{\sqrt{3}}{2}$



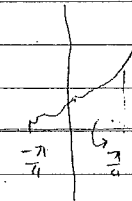
$\sin \theta = \frac{0}{1}$



$\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3}$

$= \frac{\pi}{3}, \frac{2\pi}{3}$  ✓✓

d)



$V = \pi \int_{-\pi/4}^{\pi/4} y^2 dx$

$y = \sec^2 x$

$V = \pi \int_{-\pi/4}^{\pi/4} \sec^2 x dx$

$= \pi \left[ \tan x \right]_{-\pi/4}^{\pi/4}$

$= \pi \left[ \tan \frac{\pi}{4} - \left[ \tan -\frac{\pi}{4} \right] \right]$

$= \pi [1] - [-1]$

$= \pi [1+1]$

$= 2\pi$  units<sup>2</sup> ✓✓

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Question 3

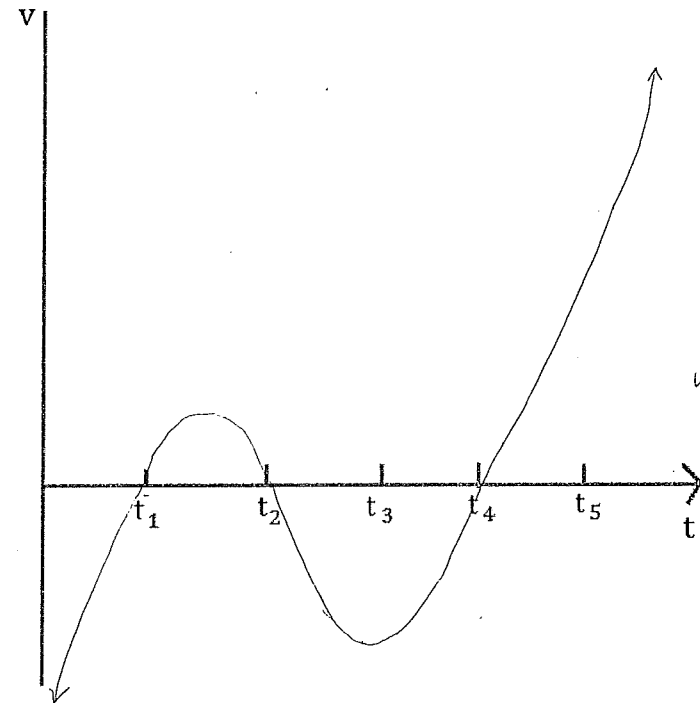
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Name \_\_\_\_\_

YEAR 12  
MATHEMATICS  
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QUESTION 3 c) ANSWER SHEET

(2 marks)



✓ (2)

a). i)  $a = 3t - 4$   
 $v = \frac{3t^2}{2} - 4t + c$

when  $t = 0, v = 0$

$$0 = \frac{3(0)^2}{2} - 4(0) + c$$

$$= 0 - 0 + c$$

$$0 = c$$

$$\therefore v = \frac{3t^2}{2} - 4t$$

ii)  $x = \frac{3t^3}{6} + \frac{4t^2}{2} + c$

when  $t = 0, x = 5$

$$5 = \frac{3(0)^3}{6} + \frac{4(0)^2}{2} + c$$

$$5 = c$$

$$\therefore x = \frac{1}{2}t^3 + 2t^2 + 5$$

b).  $A = 9000$

$$P = 9000 e^{kt}$$

when  $t = 5, P = 12500$

$$12500 = 9000 e^{k(5)}$$

$$\therefore \frac{25}{18} = e^{5k}$$

$$\ln\left(\frac{25}{18}\right) = 5k$$

$$k = 0.0657\dots$$

when  $P = 3(9000)$   
 $= 27000$

$$27000 = 9000 e^{0.0657\dots(t)}$$

$$3 = e^{0.0657\dots(t)}$$

$$\ln 3 = 0.0657\dots(t)$$

$$16.72\dots = t$$

16.7 years. ///

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Name \_\_\_\_\_

YEAR 12  
MATHEMATICS  
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MULTIPLE CHOICE ANSWER SHEET

Section II (5 marks)

Allow about 5 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

- 4.             A         B         C         D ✓
  
- 5.             A         B         C         D ✓
  
- 6.             A         B         C         D ✓
  
- 7.             A         B         C         D ✓
  
- 8.             A         B         C         D ✓