Name. ____________ Teacher. ____________

MORIAH COLLEGE

Year 11

MATHEMATICS PRELIM

5th September 2014

Time Allowed: 2 hours

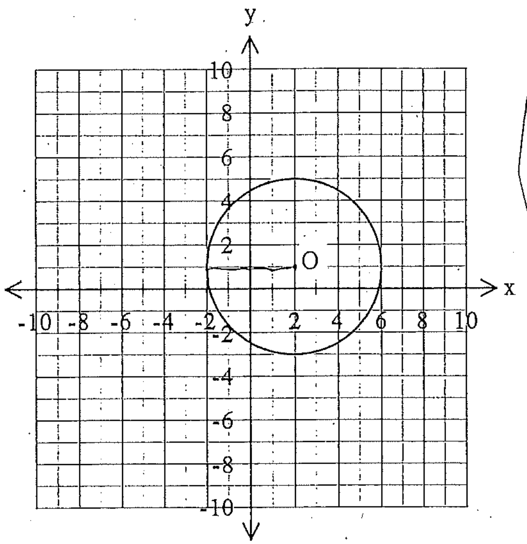
Examiners: P. Brown, L Bornstein

Instructions:

- Answer every question.
- Show all necessary working. Draw clear, well labelled diagram.
- There are 10 multiple choice question each worth 1 mark.
- There are 4 long questions each worth 15 marks.
- Marks may be deducted for careless or untidy work
- Board approved calculators may be used

1.	Simplify the expression $\frac{x^2+4x+4}{x+2}$. (A) $x+2$ (B) $\frac{x^2+2x+2}{x}$ (C) $3x+4$ (D) 1
2.	What is the angle of inclination that the line $3x-5y-7=0$ makes with the positive direction of the x-axis? (A) $149^0 2'$ (B) $59^0 2'$ (C) $120^0 28'$ (D) $30^0 58'$
3.	Make r the subject of the formula: $a=rs+1$ (A) $r=a-s$ (B) $r=\frac{a-1}{s}$ (C) $r=\frac{a-s}{s}$ (D) $r=\frac{a+s}{s}$
4.	Find the exact value of $\cos 210^0$ (A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{\sqrt{3}}{2}$
5.	What is the value of $f(-1)$ if $f(x)=x^3-4x$ (A) -3 (B) -5 (C) 3 (D) 5
6.	For the arithmetic series 3, 6, 9, ... which expression could be used to evaluate S_{15} ? (A) $3+14(3)$ (B) $15[3+14(3)]$ (C) $\frac{15}{2}[3+14(3)]$ (D) $\frac{15}{2}[6+14(3)]$
7.	The gradient of the normal to the curve $f(x)=3x^3-4x+2$ at the point (-1, 3) is: (A) 5 (B) -5 (C) $-\frac{1}{5}$ (D) $-\frac{1}{3}$

8. Which of the equations given below describes the circle centre O, shown?



(A) $x^2 - 4x + y^2 - 2y = 21$ (B) $(x-2)^2 + (y-1)^2 = 16$

(C) $x^2 + y^2 = 16$ (D) $(x+2)^2 + (y+1)^2 = 16$

9. If $f(x) = \frac{2}{x}$ and $g(x) = 3 - x$; find the values of x if $f(x) = g(x)$.

(A) $x = 1$ *and* $x = 2$ (B) $x = -1$ *and* $x = 2$
(C) $x = -1$ *and* $x = -2$ (D) $x = 1$ *and* $x = -2$

10. What is the solution to the equation $\sin(\frac{\vartheta}{2} + 30^0) = \cos\vartheta$

(A) $\vartheta = 40^0$ (B) $\vartheta = 60^0$ (C) $\vartheta = 80^0$ (D) $\vartheta = 100^0$

Question 11 (start in a new booklet) (15 marks)

a. Evaluate $\frac{3.7^2}{\cos^2 54^0 18'}$ correct to three significant figures. **2**

b. Fully simplify $\frac{1}{2\sqrt{3}-4}$ by rationalising the denominator **2**

c. Express the following in simplest terms **3**

$$\frac{x^2-3x-4}{x^2-1} \times \frac{x^2-x}{x^2-2x-8}.$$

d. Solve $2\cos x + 1 = 0$ for $0^0 \le x \le 360^0$ **3**

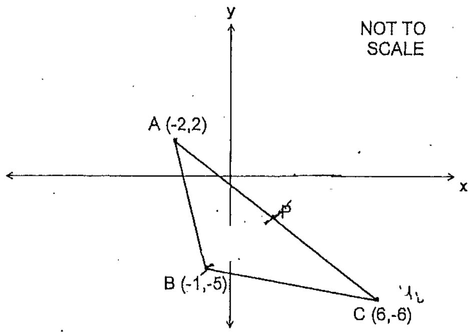
e. Solve the following inequality and sketch the solution on the number line. **3**

$|3x - 2| \ge 4$

f. Prove that: $\sin^2\theta.\cos^2\theta + \sin^4\theta = \sin^2\theta$ **2**

Question 12 (start in a new booklet) (15 marks)

a.



$A(-2,2)$, $B(-1,-5)$ and $C(6,-6)$ are vertices of a triangle and P lies on the line AC.

i) Show that the midpoint P of AC has coordinates $(2,-2)$. **1**

ii) Find the gradient of BP. **1**

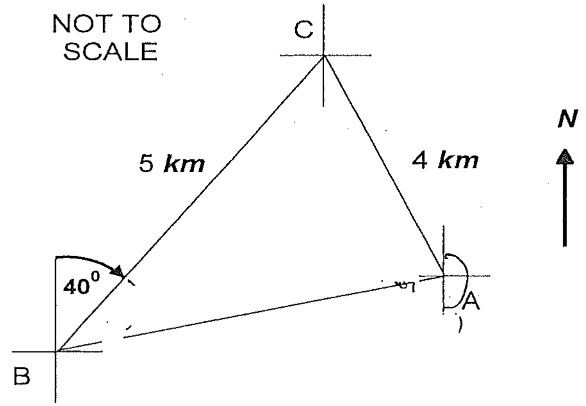
iii) Show that BP is perpendicular to AC. **1**

iv) Show that the line BP has equation $x-y-4=0$. **2**

v) Find the coordinates of D if P is the midpoint of BD. **2**

vi) Find the perpendicular distance from A to BP **2**

b.



$A, B,$ and C are markers in an orienteering course. $AC = 4\,km$ and $BC = 5\,km$. The bearing of C from B is 40^0T. The bearing of B from A is $260^0 T$

i) Copy the diagram into your books and, clearly mark all information onto your diagram. Hence show that $\angle CBA = 40^0$ **2**

ii) Use the Sine Rule to show that $\angle CAB = 53^0$ to the nearest degree. **2**

iii) Hence find the bearing of C from A. **2**

Question 13 (start in a new booklet) (15 marks)

a Find: $\lim_{x \to 1} \dfrac{x^2 + 3x - 4}{x - 1}$ 1

b Differentiate each of the following expressions with respect to x.

(fully simplify all answers)

i) $3x^3 + 3x^2 - 2x + 1$ 1

ii) $(2x-1)^6$ 1

iii) $\dfrac{x+1}{3x-2}$ 2

iv) $\dfrac{2}{\sqrt{x}} + 2\sqrt{x}$ (give solution without negative fractional indices) 2

c Consider the curve given by $y = x^3 - 3x^2 - 9x + 20$.

i) Find the coordinates of the two stationary points and determine their nature. 4

ii) Find the point of inflexion. 2

iii) Sketch the curve, showing any other relevant features if $-2 \le x \le 5$ 2

Question 14 (start in a new booklet) (15 marks)

a. Solve for θ if $2\cos^2\theta - 3\sin\theta = 0$ in the domain $0^0 \le x \le 360^0$ 3

b. Given $y = \sqrt{1+x^2}$, show that $\dfrac{dy}{dx} = \dfrac{x}{y}$ 2

c. The tangent to $y = ax^3 + bx - 2$ at $(-1,2)$ is horizontal. Find a and b. 3

d. A function f(x) is defined as

$$f(x) = \begin{cases} 5x-4 & for \;\; x \ge 0 \\ px+q & for \;\; x < 0 \end{cases}$$

Find p and q, so that $f(x)$ is an even function. 2

e. Draw a sketch of the function $y = f(x)$ given that $f(x)$ has the following properties:

$f(x)$ is defined over the domain $-1 \le x \le 1$
$f(x)$ is negative for $x < 0$ and is positive for $x > 0$.
$f(x)$ is increasing everywhere. 2
$f(x)$ is an odd function

f. Write 3 terms of an arithmetic sequence whose product is 315 and whose sum is 21. 3

END OF TEST

STUDENT NUMBER/NAME:............................

Section I

Yr 11 2U 2014 prelim.

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

	A	B	C	D
1	X			
2				X
3		X		
4				X
5			X	
6				X
7			X	
8		X		
9	X			
10	X			

#1a. 40.20317 ✓

$\therefore 40.2$ (to 3 s.f) ✓ 2.

b. $\dfrac{1}{(2\sqrt{3}-4)} \times \dfrac{2\sqrt{3}+4}{2\sqrt{3}+4} = \dfrac{2\sqrt{3}+4}{12-16} = \dfrac{2\sqrt{3}+4}{-4}$ ✓

$= \dfrac{\sqrt{3}+2}{-2}$ ✓ 2.

c. $\dfrac{(x-4)(x+1)}{(x-1)(x+1)} \times \dfrac{x(x-1)}{(x-4)(x+2)} = \dfrac{x}{x+2}$ ✓ 3.

d. $\cos x = -\frac{1}{2}$

rel $\angle = 60°$ ✓

2nd	3rd
$x = 120°$ ✓	$x = 240°$ ✓

3.

e. Consider $|3x-2| = 4$.

either $3x-2 = 4$ or $-(3x-2) = 4$

$3x = 6$ $\quad 3x-2 = -4$

$x = 2$ ✓ $\quad 3x = -2$

$x = -\frac{2}{3}$. 3.

$x=-2 \quad -\frac{2}{3} \quad x=0 \quad 2 \quad x=3$

$x \leq -\frac{2}{3}$ ✓ $\quad x \geq 2$ ✓

f. LHS:

$\sin^2\theta\cos^2\theta + \sin^4\theta$

$= \sin^2\theta(\cos^2\theta + \sin^2\theta)$ ✓

$= \sin^2\theta(1)$

$= \sin^2\theta$ ✓

RHS:

$\sin^2\theta$

2.

$\therefore$ LHS = RHS.

#13a $\lim_{x\to 1} \frac{(x+4)\cancel{(x-1)}}{\cancel{(x-1)}} = 5$ ✓ 1.

b i) $y = 3x^3 + 3x^2 - 2x + 1$

$y' = 9x^2 + 6x - 2.$ ✓ 1.

ii) $y = (2x-1)^6$

$y' = 6(2x-1)^5 \cdot 2$ ✓ 1

$y' = 12(2x-1)^5.$

iii) $y = \frac{x+1}{3x-2}$

$u = x+1 \mid v = 3x-2$

$u' = 1 \mid v' = 3.$

$y' = \frac{(3x-2)(1) - 3(x+1)}{(3x-2)^2}$ ✓

$y' = \frac{3x-2 - 3x - 3}{(3x-2)^2}$ ✓ 2.

$y' = \frac{-5}{(3x-2)^2}$

iv) $y = 2x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}$

$y' = -x^{-\frac{3}{2}} + 1x^{-\frac{1}{2}}$ ✓ 2.

$y' = -\frac{1}{\sqrt{x^3}} + \frac{1}{\sqrt{x}}.$ ✓

c. $y = x^3 - 3x^2 - 9x + 20.$

S.P: $y' = 0$

$y' = 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3 \mid x = -1.$

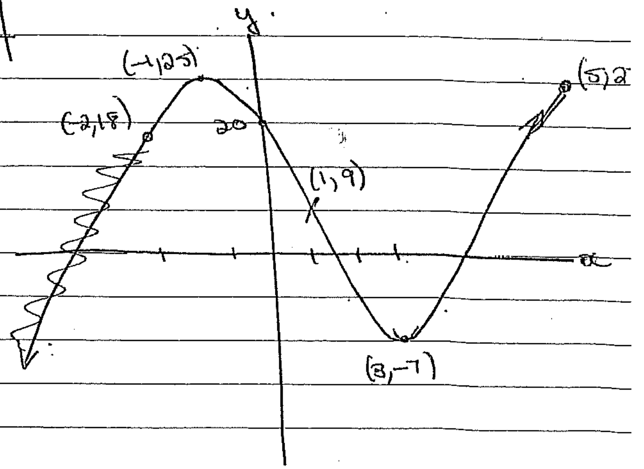
$y = -7 \mid y = 25.$

$(3,-7)$ min. $(-1,25)$. max

Nature:

x	-2	-1	0	3	4
y'	15	0	-9	0	15

max min.



P.O.I: $y'' = 0$

$y'' = 6x - 6 = 0$

$x = 1$

$y = 9$

possible POI $(1, 9)$.

x	0	1	2
y''	-6	0	6

∴ concavity changes.

$x = -2 \mid x = 5$

$y = 18 \mid y = 25.$

#14a. $2\cos^2\theta - 3\sin\theta = 0.$

$2(1-\sin^2\theta) - 3\sin\theta = 0.$

$2 - 2\sin^2\theta - 3\sin\theta = 0.$

$0 = 2\sin^2\theta + 3\sin\theta - 2$ ✓

$0 = (2\sin\theta - 1)(\sin\theta + 2)$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\cancel{\sin\theta = -2}$ $\qquad$ 3.

rel ∠ = 30° $\qquad$ no soln. ✓

1st	2nd ✓
θ = 30°	θ = 150°

b. $y = ax^3 + bx - 2$

$(-1,2)$: $2 = a(-1)^3 + b(-1) - 2$

$2 = -a - b - 2$

$a + b = -4$ —— (1). ✓

$y' = 3ax^2 + b = 0$

at $x = -1$.

$3a(-1)^2 + b = 0.$

$3a + b = 0$

$b = -3a$ —— (2). ✓

subst (2) in (1).

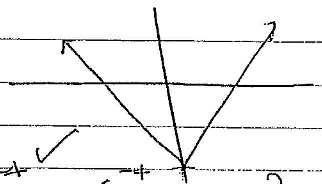
$a - 3a = -4$

$-2a = -4$ $\qquad$ 3.

$a = 2.$ ✓

$b = -6.$

c. $F(x) = \begin{cases} 5x - 4 & x \geq 0 \\ px + q & x < 0. \end{cases}$



$q = -4$ ✓

$p = -5$ ✓ $\qquad$ 2.

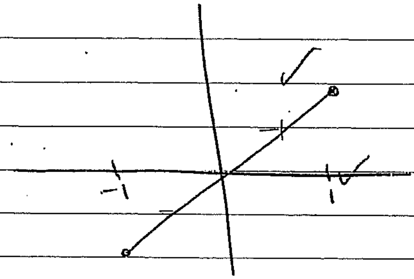
d. $y = (1+x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x$ ✓

$= \frac{x}{\sqrt{1+x^2}}$ ✓ $\qquad$ 2.

$= \frac{x}{y}$

e.



2.

f) terms: $a-d,\ a,\ a+d.$

$a(a-d)(a+d) = 315$ $\qquad$ $a - d + a + a + d = 21$

$7(7-d)(7+d) = 315$ $\qquad$ $3a = 21$

$49 - d^2 = 45$ $\qquad$ $a = 7$ ✓

$49 - 45 = d^2$

$4 = d^2$

$d = \pm 2.$ ✓

$7-2,\ 7,\ 7+2$ $\qquad$ 3.

$5,\ 7,\ 9.$ ✓

45mins

#12 i) $P = \text{midpt}(AC) = \left(\frac{-2+6}{2}, \frac{2-6}{2}\right) = (2,-2)$ ✓ 1

ii) $m(BP) = \frac{-2--5}{2--1} = \frac{3}{3} = 1$ ✓ 1

iii) $m(AC) = \frac{-6-2}{6--2} = \frac{-8}{8} = -1.$

since $1 \times -1 = -1.$ ✓ 1

$BP \perp AC.$ ✓

iv) eq. of BP:

$y - -5 = 1(x - -1)$ ✓

$y + 5 = x + 1$ 2.

$0 = x - y - 4$ ✓

v) $(2,-2) = \text{midpt } BD.$

$(2,-2) = \left(\frac{-1+x}{2}, \frac{-5+y}{2}\right)$

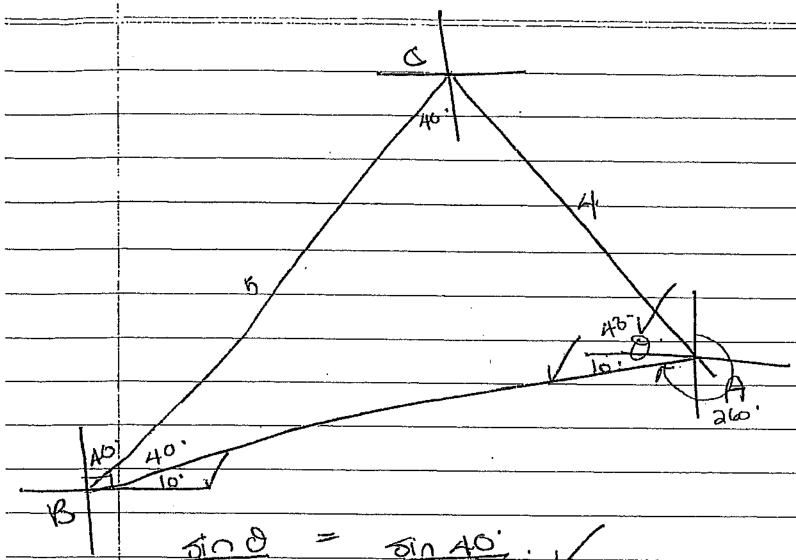
$\frac{-1+x}{2} = 2$	$\frac{-5+y}{2} = -2$
$-1 + x = 4$	$-5 + y = -4$
$x = 5.$ ✓	$y = 1.$ ✓

$D(5,1)$ 2.

vi) $A(-2,2)$ BP: $x - y - 4 = 0.$

$\perp d = \left|\frac{-2-2-4}{\sqrt{1^2+(-1)^2}}\right|$ ✓ 2.

$= \frac{|-8|}{\sqrt{2}} = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 2\sqrt{2}$ ✓



$\frac{\sin\theta}{5} = \frac{\sin 40°}{4}.$ ✓

$\sin\theta = \frac{5 . \sin 40°}{4.}$

$\sin\theta = 0.803$

$\theta = \sin^{-1}(0.803)$ ✓

$\theta = 53°.$ 6.

∴ bearing is 313°T ✓ or N 47° W.