Moriah College
בית ספר הר המוריה

YEAR 12

MATHEMATICS

PRETRIAL
TERM 1 2015

Time Allowed: 3 hours + 5mins reading time

Examiners: CO BT BR BO

Instructions:

- **USE A BLACK PEN.**
- Answer every QUESTION on a NEW PAGE.
- SHOW all working.
- Draw clear, well labelled, BIG diagrams.
- Marks may be deducted for careless or untidy work.
- Board approved calculators may be used.

Question 1 – (10 marks)

1. What is 4.0978 4 correct to three significant figures.

   A) 4.09   B) 4.10   C) 4.097   D) 4.098

2. Find $\int \pi \, dx$

   A) $\pi x + c$ B) $\dfrac{\pi x^2}{2}$ C) $\pi + c$ D) $\dfrac{\pi^2}{2} + c$

3. Solve: $(2^x)^2 = 2^8$

   A) $x = 6$ B) $x = 4$ C) $x = 2\sqrt{2}$ D) $x = 8$

4. Factorise: $3a^2 + 10a - 8$

   A) $(3a-2)(a+4)$ B) $(3a+2)(a-4)$
   C) $(3a+4)(a-2)$ D) $(3a+4)(a+2)$

5. Simplify: $\dfrac{x^2-4x}{2x-8}$

   A) $x-2$ B) $\dfrac{x-2}{4}$ C) $\dfrac{x}{2}$ D) $\dfrac{x^2-2}{-8}$

6. Solve the simultaneous equations $2x + y = 3$ and $x - 2y = 4$.

   A) $x = -1,\ y = 2$ B) $x = 1,\ y = 1$
   C) $x = 2,\ y = -1$ D) $x = -2,\ y = 1$

7. Solve for θ:
$\sqrt{3}\tan\theta+3=0$ for $0^0 \le \theta \le 360^0$

A) $\theta = 150^0, 330^0$ B) $\theta = 60^0, 240^0$
C) $\theta = 120^0, 300^0$ D) $\theta = 30^0, 210^0$

8. $\int (2x+1)^{\frac{1}{3}}\, dx$

~~A)~~ 1C $\frac{3}{4}(2x+1)^{\frac{4}{3}}+c$ B) $\cancel{-\frac{2}{3}(2x+1)^{\frac{-1}{3}}}+c$

C) $\frac{8}{3}(2x+1)^{\frac{4}{3}}+c$ D) $\frac{3}{8}(2x+1)^{\frac{4}{3}}+c$

9. Find the derivative of x^2e^{2x} with respect to x

A) $2xe^{2x}$ B) $4x^2e^{2x}$ C) $(2x+x^2)e^{2x}$ D) $(1+x)2xe^{2x}$

10. Find the values of a and b if $(\sqrt{a}+\sqrt{2})^2 = 5+2\sqrt{b}$

A) $a=25,\ b=2$ B) $a=25,\ b=6$

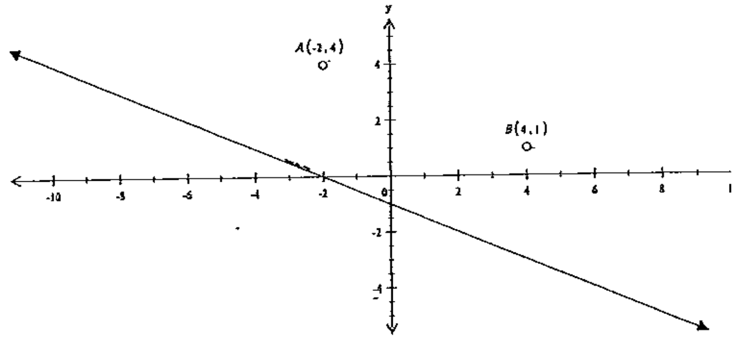
C) $a=3,\ b=6$ D) $a=3,\ b=2$

Question 11 (start each question on a new page) **(15 marks)**

a) Solve for x:
$$\frac{x-5}{3}-\frac{x+1}{4}=5$$ 3

b) Find the integers a and b such that $\frac{7}{5+3\sqrt{2}}=a-b\sqrt{2}$ 3

c) In the quadrilateral ABCD the coordinates of the points A and B are $(-2,4)$ and $(4,1)$ respectively. The equation of the line DC is $x+2y+2=0$.



i) Find the gradients of AB and DC. Hence, explain why he quadrilateral is a trapezium. 2

ii) Find the length of AB 1

iii) The line BC is parallel to the y-axis. Find the coordinates of C. 1

iv) The line AD is parallel to the x-axis. Find the coordinates of D. 1

v) Find the perpendicular distance from B to DC 2

vi) Hence, find the area of the trapezium ABCD 2

Question 12 (start each question on a new page) (15 marks)

a) Differentiate: $\left(2e^{3x}-4\right)^7$ **2**

b) The first three terms of a sequence are $20, 15, 11\frac{1}{4}$

i) Give a reason why the sequence is geometric? **1**

ii) Find the 8^{th} term of this sequence.(give answer in index form) **1**

iii) Write an expression for the sum of n terms of this sequence. (give answer in simplified index form) **1**

iv) Find the limiting sum of this sequence. **1**

c) Find the equation of the normal to the curve $y=x+e^{2x}$ at the point where $x=0$ **4**

c) Consider the parabola $y^2=8(x+2)$

i) Find the coordinates of the vertex. **1**

ii) Find the coordinates of the focus. **1**

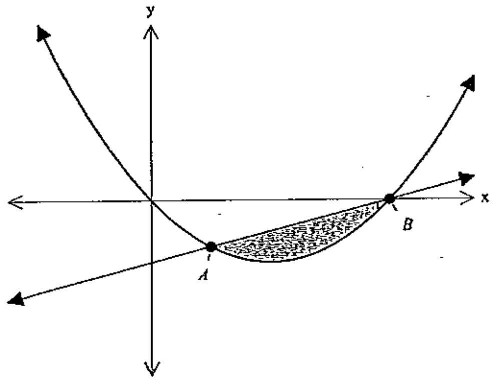
iii) Find the equation of the directrix. **1**

iv) Find the end points of the latus rectum **1**

d) Evaluate: $\int_1^2 e^{3x}\,dx$ (give answer to 2 decimal places) **2**

Question 13 (start each question on a new page) (15 marks)

a) The graphs of $y=x-4$ and $y=x^2-4x$ intersect at A and B.



i) Find the x-co-ordinates of the points of intersection of the 2 graphs. **2**

ii) Find the area of the region bounded by $y=x-4$ and $y=x^2-4x$. **3**

b) The quadratic equation $2x^2+8x+k=0$ has roots α and β. Use this information to evaluate:

i) $\alpha+\beta$ **1**

ii) Given that $\alpha^2\beta+\alpha\beta^2=6$, find the value of k. **2**

c) For what values of m does the equation $2x^2+mx+8=0$ have 2 positive, unequal real roots? **3**

d) A tourist drives $25\ km$ from town P on a bearing of 150^0T to town R. He then drives $45\ km$ on a bearing of $022^0\ T$ to town Q.

i) Draw the diagram into your examination booklets and show that $\angle PRQ=52^0$. (give reasons) **2**

ii) Find the distance from P to Q. (to 2 decimal places) **2**

Question 14 (start each question on a new page) **(15 marks)**

a) A function $f(x)$ is defined by $f(x) = x^3 - 3x^2$ for $-3 \le x \le 4$.

i) Find the x and y intercepts **2**

ii) Find the stationary points and their nature **3**

iii) Sketch the curve $y = f(x)$, clearly showing the intercepts and the point of inflexion. **2**

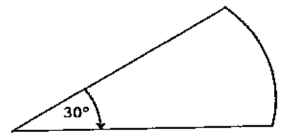
iv) Find the range of $f(x)$. **2**

b) Use Simpson's rule with 5 function values to find an approximation to $\int_0^4 xe^x \, dx$ (give answer to 2 decimal places) **3**

c) If $\sin\theta = x$, express $\dfrac{1-\cos^2\theta}{\sec^2\theta}$ in terms of x **3**

Question 15 (start each question on a new page) **(15 marks)**

a) Sami calculated that the area of the sector below is $4\pi \ cm^2$.

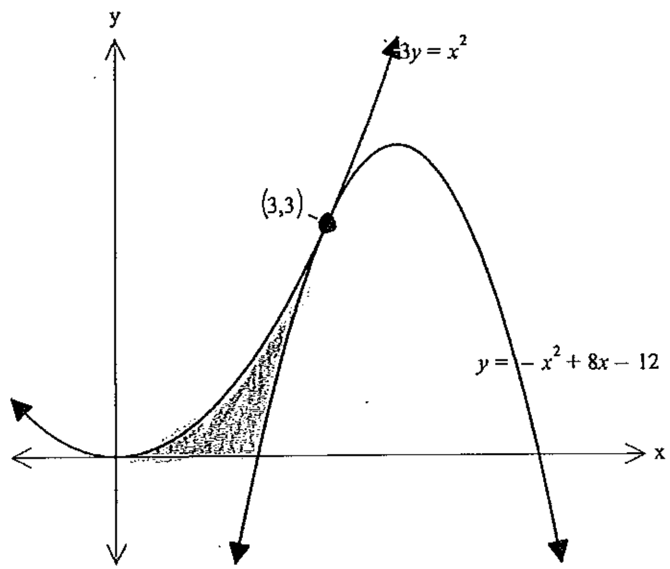


i) Find the radius of the sector. **1**

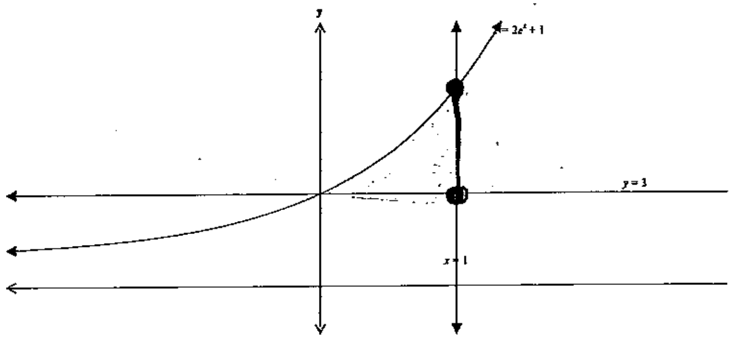
ii) Find the perimeter of the sector. **2**

b) The graphs of $3y = x^2$ and $y = -x^2 + 8x - 12$ are shown on the same system of axes. These curves meet at $(3,3)$ as shown.

Calculate the area enclosed by the curves $3y = x^2$, $y = -x^2 + 8x - 12$ and the x-axis. **4**



d) The area enclosed by the curve $y = 2e^x + 1$ and the lines $x = 1$ and $y = 3$ is shaded as shown in the diagram.

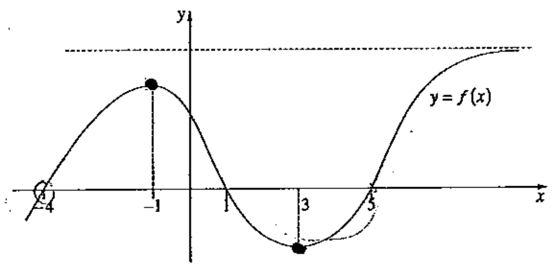


i) Show that the volume of the solid formed when this shaded region is rotated about the x-axis can be expressed as

$$V = 4\pi\int_0^1 \left(e^{2x} + e^x - 2\right)\, dx.$$ 2

ii) Calculate the exact volume of the solid formed. 2

e) The diagram shows the graph of $y = f(x)$



(i) For which values of x is the derivative $y = f'(x)$, negative? 1

(ii) What happens to $f'(x)$ for large values of x? 1

(iii) Sketch the graph of $y = f'(x)$ on the attached tear-off sheet 2

Question 16 (start each question on a new page) (15 marks)

a) Differentiate $(x^4+8)^5$ hence find $\int_0^1 x^3\left(x^4+8\right)^4\, dx$. 3

b) If $\tan^2\theta + 2\sec^2\theta = 5$, find the value of $\sin^2\theta$. 2

c) $K(k, k - e^{-k})$, $L(-4,-3)$ and $M(5,9)$

Show that the area of ΔKLM is $A = \frac{3}{2}\left(3e^{-k} + k + 7\right)$ 3

d) A farmer is fencing a paddock using P metres of fencing. The paddock is to be in the shape of a sector of a circle with radius r and sector angle θ.

i) Show that the length of the fencing required to fence the perimeter of the paddock is $P = r(\theta + 2)$. 1

ii) Show that the area of the sector is $A = \frac{1}{2}\text{Pr} - r^2$. 1

iii) Find the radius of the sector, in terms of P, that will maximize the area of the paddock. 2

iv) Find the angle θ, that gives the maximum area of the paddock. 1

v) Explain why it is only possible to construct a paddock in the shape of a sector if $\frac{P}{2(\pi+1)} < r < \frac{P}{2}$. 2

END OF TEST

Mathematics

Pre-trial 2015 Solutions

Question 11

Q11 a) $\frac{x-5}{3} - \frac{x+1}{4} = 5$

$4(x-5) - 3(x+1) = 60$

$4x - 20 - 3x - 3 = 60$

$x - 23 = 60$

$x = 83$ ✓

Q11 b) $\frac{7}{5+3\sqrt{2}} = a - b\sqrt{2}$

$\frac{7}{5+3\sqrt{2}} \times \frac{5-3\sqrt{2}}{5-3\sqrt{2}}$

$= \frac{7(5-3\sqrt{2})}{25-18}$

$= \frac{\cancel{7}(5-3\sqrt{2})}{\cancel{7}}$ $\therefore a = 5$ ✓ $b = 3$ ✓

Q11 c) i) AB $(-2,4)$ $(4,1)$

$m = \frac{4-1}{-2-4}$

$m = \frac{3}{-6} = -\frac{1}{2}$ ✓

DC $x + 2y + 2 = 0$

$2y = -x - 2$

$y = -\frac{x}{2} - 1$

$m = -\frac{1}{2}$ ✓

$m_1 = m_2$ $\therefore$ lines are parallel

One set of parallel lines makes a trapezium.

11c) ii) length AB

$(-2,4)$ $(4,1)$

$d = \sqrt{(-2-4)^2 + (4-1)^2}$

$d = \sqrt{36+9}$

$d = \sqrt{45}$

$d = 3\sqrt{5}$ ✓

iii) BC parallel to y-axis $C(4,y)$

$x + 2y + 2 = 0$

$4 + 2y + 2 = 0$

$2y = -6$

$y = -3$ ✓

co-ordinates $C = (4,-3)$ ✓

iv) AD parallel to x-axis $D(x,4)$

$x + 2y + 2 = 0$

$x + 8 + 2 = 0$

$x = -10$

co-ordinates $D = (-10,4)$ ✓

v) perpendicular distance

DC $= x + 2y + 2 = 0$

B $= (4,1)$

$\perp d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$\perp d = \frac{|1(4)+2(1)+2|}{\sqrt{1^2+2^2}}$

$\perp d = \frac{8}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$ u ✓

11 c vi) Area trapezium ABCD

$A = \frac{(a+b)}{2} \times h$

$= \left(\frac{3\sqrt{5}+7\sqrt{5}}{2}\right) \times \frac{8\sqrt{5}}{5}$

$= \frac{120+280}{10}$

$= 40$ units^2 ✓

$(4,-3)$ $(-10,4)$

$d = \sqrt{(4--10)^2 + (-3-4)^2}$

$d = \sqrt{196+49}$

$d = \sqrt{245}$ (49×5)

$d = 7\sqrt{5}$

Question 12

12 a) $y = (2e^{3x} - 4)^7$

$y' = 7(2e^{3x}-4)^6 \times 6e^{3x}$

$y' = 42e^{3x}(2e^{3x}-4)^6$ ✓

12 b) i) $20, 15, 11\frac{1}{4}$

$\frac{15}{20} = \frac{11.25}{15}$

$\frac{3}{4} = \frac{3}{4}$ ✓

ii) $T_8 = ar^{n-1}$

$= 20 \cdot (0.75)^7$ ✓

iii) $S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{20(1-(0.75)^n)}{\frac{1}{4}}$

$= 80(1-(0.75)^n)$ ✓

iv) limiting sum

$S = \frac{a}{1-r}$ $|r| < 1$

$= \frac{20}{1-3/4}$

$= 80$ ✓

12 b) $y = x + e^{2x}$

$y' = 1 + 2e^{2x}$

$x = 0$

$= 1 + 2e^0$

$= 3$

$m = 3$ at $x = 0$

$m(\text{normal}) = -\frac{1}{3}$ ✓ When $x = 0$

$y = 0 + e^{2(0)}$

$y = 1$ $(0, 1)$

$-\frac{1}{3}(x - 0) = y - 1$

$-1(x - 0) = 3y - 3$

$-x = 3y - 3$

$0 = 3y + x - 3$ ✓

12 c) $y^2 = 8(x + 2)$

i) vertex $(-2, 0)$

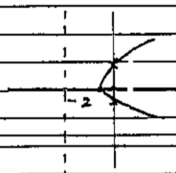
ii) focus $(0, 0)$ focal length $= 2$

iii) directrix $y = -4$

iv) latus rectum/end points

$4a = 8$

$(0, 4)$ $(0, -4)$ ✓



12 d) $\int_1^2 e^{3x}\, dx$

$= \left[\frac{e^{3x}}{3}\right]_1^2$

$= \frac{e^6}{3} - \frac{e^3}{3}$ ✓ $= \frac{e^6 - e^3}{3}$

127.78

Question 13

13 a) $y = x - 4$ $y = x^2 - 4x$

i) $x - 4 = x^2 - 4x$

$0 = x^2 - 5x + 4$

$0 = (x - 4)(x - 1)$

$x = 4$ or $x = 1$

When $x = 4$ When $x = 1$

$y = 0$ $y = -3$

$A = (1, -3)$ $B(4, 0)$ ✓

ii) $y = x - 4$ $y = x^2 - 4x$

$\int_1^4 x - 4 - (x^2 - 4x)\, dx$

$\int_1^4 x - 4 - x^2 + 4x\, dx$

$\int_1^4 -x^2 + 5x - 4\, dx$

$= \left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^4$

$= \left(-\frac{4^3}{3} + \frac{5(16)}{2} - 4(4)\right) - \left(-\frac{1}{3} + \frac{5}{2} - 4\right)$

$= \left(\frac{8}{3} + \frac{11}{6}\right)$

$= \frac{9}{2} = 4\frac{1}{2}\ u^2$ ✓

13 b) $2x^2 + 8x + k = 0$

i) $\alpha + \beta = -\frac{b}{a}$ $\alpha\beta = \frac{c}{a}$

$= -4$ $= \frac{k}{2}$

ii) $\alpha^2\beta + \alpha\beta^2 = 6$

$\alpha\beta(\alpha + \beta) = 6$

$\frac{k}{2}(-4) = 6 = -2k = 6$ ✓

$= \underline{k = -3}$

13 c) $2x^2 + mx + 8 = 0$

$b^2 - 4ac > 0$

$m^2 - 4(2)(8) > 0$

$m^2 > 64$

$m > 8$, $m < -8$

$m < -8$

if $m > 8$

$2x^2 + 10x + 8$

$x^2 + 5x + 4$

$(x + 4)(x + 1)$

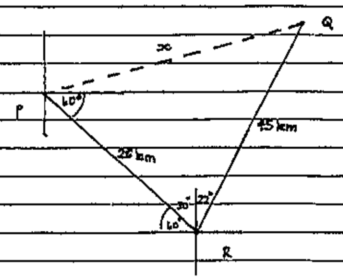
$x = -4$ $x = -1$

13d) opposite page

13 d) i) $\angle PRQ = 52°$

Alternate angles

Right angle



ii) $x^2 = 25^2 + 45^2 - 2(25)(45)\cos 52°$

$x = \sqrt{1264 \ldots}$

$= 35.56$ km ✓

Question 14

14 a) $f(x) = x^3 - 3x^2$

let $x = 0$ — let $y = 0$

$y = 0$ — $0 = x^3 - 3x^2$

y intercept $(0,0)$ — $0 = x^2(x-3)$

$x = 0$ or $x = 3$

x-intercept = $(3, 0)$

ii) S.P $m = 0$

$y = x^3 - 3x^2$

$y' = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

S.P $x = 0$ or $x = 2$

When $x = 2$

$y = 2^3 - 3(2^2)$

$y = 8 - 12$

$y = -4$

$(2, -4)$

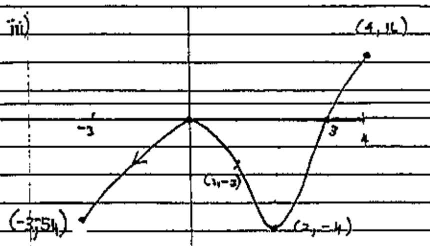
x	-1	0	1
y'	/	—	\
	9		-3

max

x	1	2	3
y	\	—	/
	-3		9

min

iii)



Point of Inflexion

$y'' = 6x - 6$

$0 = 6x - 6$

$1 = x$

$y = 1 - 3$

$y = -2$

$(1, -2)$

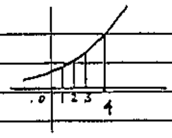
iv) range — when $x = -3$ — when $x = 4$

$y = (-3)^3 - 3(-3)^2$ — $y = (4)^3 - 3(4)^2$

$y = -27 - 27$ — $y = 16$

Range $-54 \le y \le 16$

14 b) $\int_0^4 x e^x \, dx$



x	y		
0	0	1	0
1	e	4	$4e$
2	$2e^2$	2	$4e^2$
3	$3e^3$	4	$12e^3$
4	$4e^4$	1	$4e^4$
			499...

$\frac{1}{3} \times 499\ldots \quad 166.62$ ✓

c) $\sin\theta = x$

$$\frac{1 - \cos^2\theta}{\sec^2\theta} = \frac{\sin^2\theta}{\sec^2\theta}$$

$$= \frac{\sin^2\theta \times \cos^2\theta}{\frac{1}{\cos^2\theta} \times \cos^2\theta}$$

$= \sin^2\theta \cos^2\theta$

$= \sin^2\theta (1 - \sin^2\theta)$

$= x^2 (1 - x^2)$ ✓

$= x^2 - x^4$

Question 15

15 a) i) $30° = \frac{\pi}{6}$

Area of sector $= \frac{1}{2}\theta r^2$

$4\pi = \frac{1}{2}\left(\frac{\pi}{6}\right) r^2$

$4\pi = \frac{\pi}{12} r^2$

$48 = r^2$

$\sqrt{48} = r$ ✓

Perimeter $L = \theta \times r$

$L = \frac{\pi}{6}\sqrt{48} + 2(\sqrt{48})$

$L = \frac{\pi}{6} \times 4\sqrt{3} + 2(4\sqrt{3})$

$= \frac{2\pi\sqrt{3}}{3} + 8\sqrt{3}$ u

15 b) opposite page

15 b) $\int_0^3 \text{blue} - \int_2^3 \text{red}$

$= \int_0^3 \frac{x^2}{3} \, dx$

$= \left[\frac{x^3}{9}\right]_0^3$

$= 3$

$= \int_2^3 -x^2 + 8x - 12 \, dx$

$= \left[-\frac{x^3}{3} + \frac{8x^2}{2} - 12x\right]_2^3$

$= \left(-\frac{27}{3} + 36 - 36\right) - \left(-\frac{8}{3} + 16 - 24\right)$

$= -9 - \left(-\frac{32}{3}\right)$

$= \frac{5}{3}$

$3 - \frac{5}{3} = \frac{4}{3}$ u² ✓

15 d) i) $y = 2e^{x} + 1 \qquad x = 1 \qquad y = 3$

$V = \pi \int_0^1 (2e^{x}+1)^2 - 9 \quad dx$

$= \pi \int_0^1 4e^{2x} + 2e^{x} + 2e^{x} + 1 - 9 \quad dx$

$= 4\pi \int_0^1 (e^{2x} + e^{x} - 2) \quad dx$ ✓

ii) $4\pi \left[\frac{e^{2x}}{2} + e^{x} - 2x \right]_0^1$

$V = 4\pi\left(\left(\frac{e^2}{2} + e - 2\right) - \left(\frac{1}{2} + 1\right)\right)$

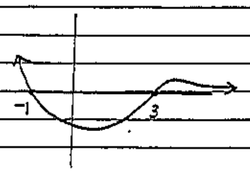
$V = 4\pi \left(\frac{e^2}{2} + e - 2 - \frac{1}{2} - 1\right)$

$V = 4\pi \left(\frac{e^2}{2} + e - 3\frac{1}{2}\right) u^3$ ✓

e) i) $-1 < x < 3$ ✓

ii) $f'(x)$ tends to 0 ✓ as $x \to \infty$, $f(x) \to 0^+$

iii)



Question 16

16 a) $(x^4+8)^5$

$\frac{dy}{dx} = 5(x^4+8)^4 \times 4x^3$

$= 20x^3(x^4+8)^4$

$\int_0^1 x^3 (x^4+8)^4 \quad dx$

$\frac{1}{20} \int_0^1 20x^3 (x^4+8)^4 \quad dx$

$= \frac{1}{20} \left[(x^4+8)^5 \right]_0^1$

$= \frac{1}{20} (9^5 - 8^5)$ ✓

$\frac{26281}{20} \qquad 1314\frac{1}{20}$

16 b) $\tan^2\theta + 2\sec^2\theta = 5 \qquad$ Find $\sin^2\theta$

$(\sec^2\theta - 1) + 2\sec^2\theta = 5$

$\sec^2\theta + 2\sec^2\theta = 6$

$3\sec^2\theta = 6$

$\sec^2\theta = 2$

$\frac{1}{\cos^2\theta} = 2$

$\frac{1}{1-\sin^2\theta} = 2$

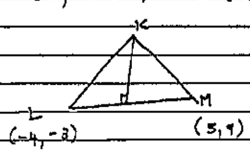
$1 = 2(1-\sin^2\theta)$

$1 = 2 - 2\sin^2\theta$

$-1 = -2\sin^2\theta$

$\frac{1}{2} = \sin^2\theta$

16 c) $K\ (k, k - e^{-k}) \qquad L\ (-4, -3) \qquad M\ (5, 9)$



Area $= \frac{1}{2} b \times h$

Distance $LM = \sqrt{(-4-5)^2 + (-3-9)^2}$

$= \sqrt{81 + 144}$

$= \sqrt{225}$

$= 15$ units ✓

Equation of LM. $m = \frac{-3-9}{-4-5}$

$= \frac{-12}{-9}$

$= +\frac{4}{3}$

$+\frac{4}{3}(x-5) = y - 9$

$+4(x-5) = 3y - 27$

$+4x - 20 = 3y - 27$

$4x - 3y + 7 = 0$

$\perp d = \left| \frac{ax + by + c}{\sqrt{a^2+b^2}} \right| \qquad 4x - 3y + 7 = 0 \qquad (k, k - e^{-k})$

$= \frac{4(k) - 3(k - e^{-k}) + 7}{\sqrt{16+9}}$

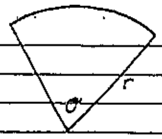
$= \frac{4k - 3k + 3e^{-k} + 7}{5} \qquad = \frac{k + 3e^{-k} + 7}{5}$ ✓

16 c) Area $\frac{1}{2} b \times h$

$= \frac{1}{2} \times \cancel{15}^{3} \times \left(\frac{k + 3e^{-k} + 7}{\cancel{5}_1}\right)$

$\frac{3}{2}(3e^{-k} + k + 7)$ ✓

16 d)



i) Perimeter = Arc length + 2r

$P = \theta r + 2r$

$P = r(\theta + 2)$

ii) Area of sector $= \frac{1}{2}\theta r^2$

$P = r(\theta + 2)$

$\frac{P}{r} = \theta + 2$

$\frac{P}{r} - 2 = \theta$

$A = \frac{1}{2}\left(\frac{P}{r} - 2\right) r^2$

$A = \frac{1}{2}(Pr - 2r^2)$

$A = \frac{1}{2}Pr - r^2$ ✓

iii) radius in terms of P

$A = \frac{1}{2}Pr - r^2$

$A' = \frac{1}{2}P - 2r$

$0 = \frac{1}{2}P - 2r$ for S.P

$\frac{1}{2}P = 2r$

$\frac{1}{4}P = r$ ✓

check max

r	$\frac{1}{8}P$	$\frac{1}{4}P$	P
	/	—	\
A'	$\frac{1}{4}P$		$-1\frac{1}{2}P$

$\therefore$ max

16 d iv) $P = r(\theta + 2)$ $r = \frac{1}{4}P$

$\frac{P}{r} = \theta + 2$

$\frac{P}{r} - 2 = \theta$

$\frac{4}{P} \times \frac{P}{1} - 2 = \theta$

$\frac{4}{P} \times \frac{P}{4}$

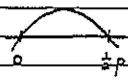
$4 - 2 = \theta$

$2^c = \theta$ ✓

d v) $A > 0$

$\frac{1}{2}Pr - r^2 > 0$

$r(\frac{1}{2}P - r) > 0$



$0 < r < \frac{P}{2}$ (right hand side)

$r = \frac{P}{2}$

$2r = P$

If the perimeter was 2x radius there is no arc.

Sector of Area < Area of circle

$\frac{1}{2}Pr - r^2 < \pi r^2$ $(\div r)$

$\frac{1}{2}P - r < \pi r$

$\frac{1}{2}P < \pi r + r$

$P < 2\pi r + 2r$

$P < 2r(\pi + 1)$

$\frac{P}{2(\pi+1)} < r$ left hand side