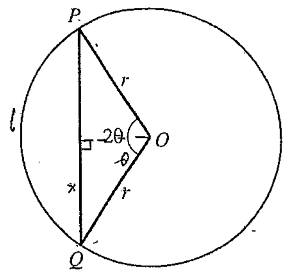
MORIAH COLLEGE – YR 12 – EXT 1 – RADIANS + GENERAL FORMULA
(2008)

Year 12 radians

Question 1



The diagram shows a circle, centre O and radius r. The chord PQ subtends an angle 2θ radians at O, where $0 < \theta < \frac{\pi}{2}$. The length of the minor arc PQ is 1.5 times the length of the chord PQ.

(a) Show that $2\theta - 3\sin\theta = 0$.

(b) Show that the area of ΔOPQ is $r^2 \sin\theta \cos\theta$.

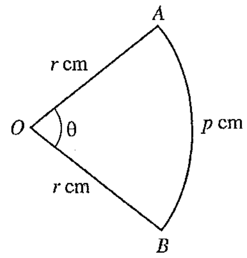
(c) Hence show that, for this value of θ,

$$\frac{\text{area of sector } OPQ}{\text{area of triangle } OPQ} = \frac{3}{2\cos\theta}.$$

(d) By considering $f(x) \equiv 2x - 3\sin x$, or otherwise, show that $1.49 < \theta < 1.50$

[14]

Question 2



The diagram shows a sector OAB of a circle, centre O and radius r cm. The length of the arc AB is p cm and $\angle AOB$ is θ radians.

(a) Find θ in terms of p and r.

(b) Deduce that the area of the sector is $\frac{1}{2}pr$ cm^2.

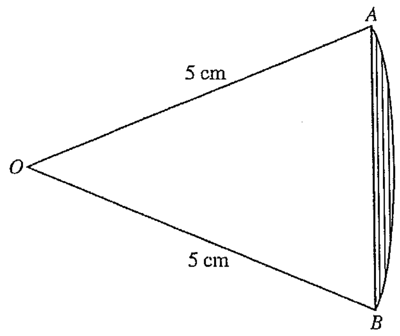
Given that $r = 4.7$ and $p = 5.3$, where each has been measured to 1 decimal place, find, giving your answer to 3 decimal places,

(c) the least possible value of the area of the sector,

(d) the range of possible values of θ.

[13]

Question 3



The diagram shows a sector OAB of a circle, centre O, of radius 5 cm and a shaded segment of the circle. Given that $\angle AOB = 0.7$ radians, calculate

(a) the area, in cm^2, of the sector OAB,

(b) the area, in cm^2 to 2 significant figures, of the shaded segment.

[5]

Question 4

$$f(x) \equiv 3 + 2\sin(2x + k)°,\ 0 \leq x < 360,$$

where k is a constant and $0 < k < 360$. The curve with equation $y = f(x)$ passes through the point with coordinates $(15, 3 + \sqrt{3})$.

(a) Show that $k = 30$ is a possible value for k and find the other possible value of k.

Given that $k = 30$,

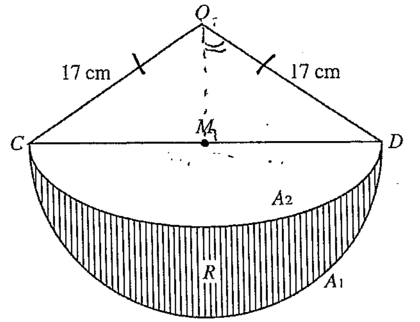
(b) solve the equation $f(x) = 1$,

(c) find the range of f,

(d) sketch the graph of $y = f(x)$, stating the coordinates of the turning points and the coordinates of the point where the curve meets the y-axis.

[17]

Question 5

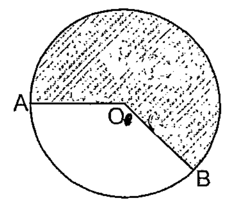


The diagram shows the triangle OCD with $OC = OD = 17$ cm and $CD = 30$ cm. The mid-point of CD is M. With centre M, a semicircular arc A_1 is drawn on CD as diameter. With centre O and radius 17 cm, a circular arc A_2 is drawn from C to D. The shaded region R is bounded by the arcs A_1 and A_2. Calculate, giving answers to 2 decimal places,

(a) the area of the triangle OCD,
(b) the angle COD in radians,
(c) the area of the shaded region R.

[12]

Question 6



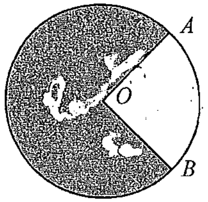
This diagram shows a circle, centre O, of radius 6 cm. The points A and B are on the circumference of the circle. The area of the shaded major sector is 80 cm^2.
Given that $\angle AOB = \theta$ radians, where $0 < \theta < \pi$, calculate

(a) the value, to 3 decimal places, of θ,

(b) the length, in cm to 2 decimal places, of the minor arc AB.

[6]

Question 7



This diagram shows a circle, centre O and radius 5 cm. The length of the minor arc AB is 6.4 cm.

(a) Calculate, in radians, the size of the acute angle AOB.

The area of the minor section AOB is R_1 cm^2 and the area of the shaded major sector AOB is R_2 cm^2.

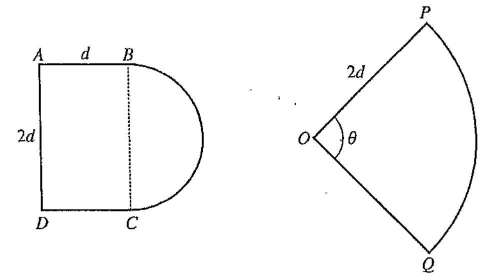
(b) Calculate the value of R_1.

(c) Calculate $R_1 : R_2$ in the form $1 : p$, giving the value of p to 3 significant figures.

[5]

Question 8

Figure 1



Shape X Shape Y

Figure 1 shows the cross-sections of two drawer handles.

Shape X is a rectangle $ABCD$ joined to a semicircle with BC as diameter. The length $AB = d$ cm and $BC = 2d$ cm.

Shape Y is a sector OPQ of a circle with centre O and radius $2d$ cm.
Angle POQ is θ radians.

Given that the areas of shapes X and Y are equal,

(a) prove that $\theta = 1 + \frac{1}{4}\pi$.

Using this value of θ, and given that $d = 3$, find in terms of π,

(b) the perimeter of shape X,

(c) the perimeter of shape Y.

(d) Hence find the difference, in mm, between the perimeters of shapes X and Y.

[12]

Year 12 Radians

Joshua Swift

Question One

(a) $d = 1.5\,\cancel{pq}$

$d = 1.5(2r\sin\theta)$

$d = 3r\sin\theta$

$r2\theta = 3r\sin\theta$

$2\theta = 3\sin\theta$

$\boxed{2\theta - 3\sin\theta = 0.}$

(b) $A = \frac{1}{2}ab\sin\theta$

$A = \frac{1}{2}r^2\sin 2\theta$

$A = \frac{1}{2}r^2\sin(\theta + \theta)$

$A = \frac{1}{2}r^2 \sin\theta\cos\theta + \sin\theta\cos\theta$

$A = \frac{1}{2}r^2(2\sin\theta\cos\theta)$

$A = \frac{\cancel{2}\sin\theta\cos\theta r^2}{\cancel{2}}$

$\boxed{A = \sin\theta\cos\theta r^2}$

(c) $\frac{\text{Area of sector OPQ}}{\text{Area of } \Delta \text{ OPQ}} = \frac{\frac{1}{2}\cancel{r^2\sin\theta}}{\cancel{r^2\sin}\theta\cos\theta} = \frac{\frac{1}{2}}{\cos\theta} = \frac{1}{2\cos\theta}.$

(d) $f(x) \equiv 2x - 3\sin x$

~~$y \equiv 2x - 3\sin x$~~

$\therefore f(\theta) = 2\theta - 3\sin\theta$

$0 = 2\theta - 3\sin\theta$

$3\sin\theta = 2\theta$

as we do not know Newtons method see graph.

~~θ is just short of 3.~~

Unfold to View Graph.

Question Two

(a) $p = r\theta$

$\boxed{\theta = \frac{p}{r}}$

(b) $A_{sector} = \frac{1}{2}r^2\theta$

$A_{sector} = \frac{1}{2}r^2\frac{p}{r}$

$A_{sector} = \frac{r^{\cancel{2}}p}{2\cancel{r}}$

$\boxed{A_{sector} = \frac{rp}{2}\ \text{cm}^2}$

(c) $\therefore A_{sector} = \frac{24.91}{2} = \boxed{12.455\text{cm}^2 (3dp)}$

(d) $1.128 \leq \theta \leq 2\pi$

$\boxed{\therefore 1.128 \leq \theta \leq 6.283}$

Question Three

(a) $A_{sector} = \frac{1}{2} r^2 \theta$

$A_{sector} = \frac{1}{2} \times 25 \times 0.7$

$A_{sector} = 8.75 \text{ cm}^2$

(b) $A_{segment} = \frac{1}{2} r^2 (\theta - \sin\theta)$

$A_{segment} = \frac{1}{2} \times 25 (0.7 - \sin 0.7)$

~~Asegment = ... (2.s.f)~~

$A_{segment} = 0.039 \, (2.s.f.)$

Question Four

(a) $3+\sqrt{3} = 3+2\sin(30+k)^\circ$ $\qquad (0 \leq k \leq 360)$

RHS:

$= 3 + 2\sin(30+30)^\circ$

$= 3 + 2\sin 60^\circ$

$= 3 + 2\left(\frac{\sqrt{3}}{2}\right)$

$= 3 + \sqrt{3} = \underline{LHS}$

$\therefore$, $k^\circ = 30^\circ$ is a possible solution.

Since $\sin 60 = \sin 120$; k° also can be 90°

Question Four Continued

(b) $f(x) = 1$.

$1 = 3+2\sin(2x+30)^\circ$

$2\sin(2x+30) = -2$.

$\sin(2x+30)^\circ = -1$

$2x + 30^\circ = 270^\circ$

$2x = 240^\circ$

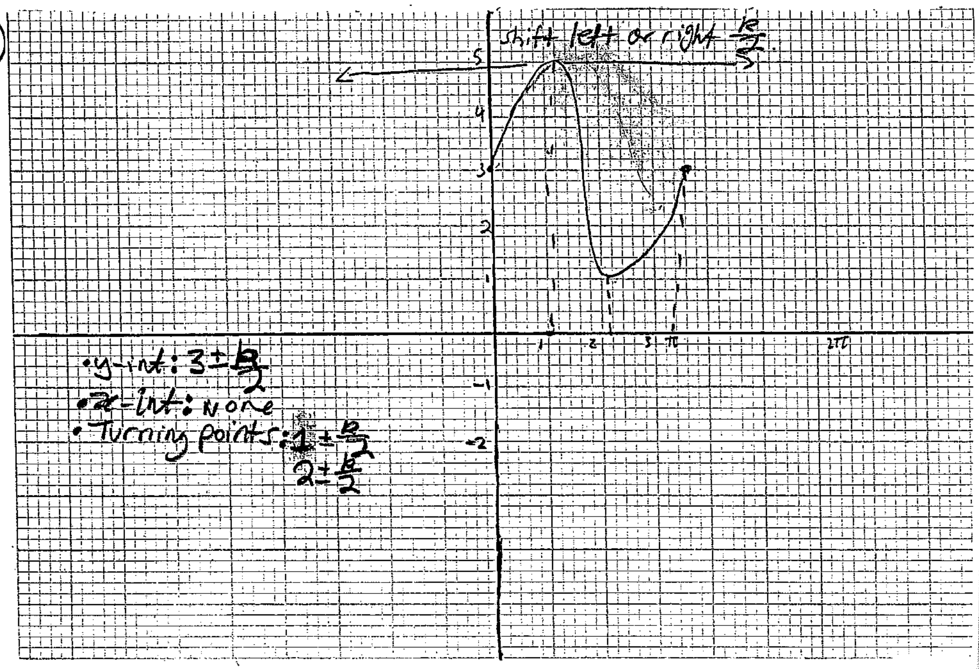
$x = 120^\circ$

(c) $f(\theta) = 3+2\sin(\theta+k)^\circ$

$f(360) = 3 + 2\sin(360+k)^\circ$

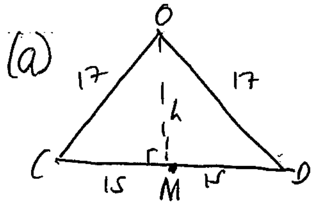
$\therefore$, $3+2\sin(\theta+k)^\circ \leq f \leq 3+\sin(360+k)^\circ$

d)



Question Five

(a)



$h^2 = 17^2 - 15^2$

$h^2 = 64$

$h = \sqrt{64}$

$h = 8$ units.

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 30 \times 8$

$A = 120 \text{ cm}^2$.

(b) $\sin(\angle COM) = \frac{15}{17}$

$\angle COM = 61.9° = 1.08^c$

$\angle COD = 2(61.9°) = 2(1.08^c)$

$\angle COD = 123.8° = 2.16^c$ (2dp).

c) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$A = 8.5(2.16 - \sin 2.16)$

$A = 11.29 \text{ cm}^2$ (2dp)

Question Six:

let $\alpha = 2\pi - \theta$

a) $\frac{1}{2}r^2\alpha = 80$

$\frac{36\alpha}{2} = 80$

$18\alpha = 80$

$\alpha = \frac{80}{18}$

$\alpha = \frac{40}{9}$

$\theta = 2\pi - \frac{40}{9}$

$\theta = 1.41^c$ (2dp)

b) $Arc_{AB} = R\theta$

$Arc_{AB} = 6 \times 1.413...$

$Arc_{AB} = 8.48$ (2dp) cm

Question Seven

(a) $6.4 = 5\theta$

$\theta = \frac{6.4}{5}$

$\theta = 1.28^c$

(b) $A_{R_1} = \frac{1}{2} r^2 \theta$

$A_{R_1} = \frac{1}{2} \times 5 \times 1.28$

$A_{R_1} = \frac{6.4}{2}$

$A_{R_1} = 3.2\,cm^2$

(c) $A_{R_2} = \pi r^2 - 3.2$

$A_{R_2} = 25\pi - 3.2$

$A_{R_2} = 75.34\,cm^2$ (2dp)

$R_1 : R_2 = \frac{R_2}{R_1} = \frac{\frac{75.34}{3.2}}{\frac{\cancel{75.34}}{3.2}} = \cancel{23.54}$ (3.s.f.)

$\therefore P = 23.5$ (3.s.f.)

Question Eight

(a) $\frac{\pi d^2}{2} + 2d^2 = 2d^2\theta$

$\frac{\pi \cancel{d^2}}{4\cancel{d^2}} + \frac{4\cancel{d^2}}{4\cancel{d^2}} = \frac{4\cancel{d^2}\theta}{\cancel{4d^2}}$

$\theta = \frac{\pi}{4} + 1.$

(b) $P_x = 6+3+6+3+3\pi$

$P_x = 18 + 3\pi$

(c) $P_y = 4d + 2d\theta$

$P_y = 12 + 6 + \frac{6\pi}{4}$

$P_y = 18 + \frac{6\pi}{4}.$

$P_y = 18 + \frac{3\pi}{2}.$

(d) $P_x - P_y = \frac{3\pi}{2}. \doteqdot 4.712\,cm$ (3dp) $\doteqdot 47.12\,mm$ (2dp)